

- 1    **a**  $\log_{10} 1000 = 3$       **b**  $\log_3 81 = 4$       **c**  $\log_2 256 = 8$       **d**  $\log_7 1 = 0$   
      **e**  $\log_3 \frac{1}{27} = -3$       **f**  $\log_{32} \frac{1}{2} = -\frac{1}{5}$       **g**  $\log_{19} 19 = 1$       **h**  $\log_{36} 216 = \frac{3}{2}$
- 2    **a**  $5^3 = 125$       **b**  $2^4 = 16$       **c**  $10^5 = 100\,000$       **d**  $23^0 = 1$   
      **e**  $9^{\frac{1}{2}} = 3$       **f**  $10^{-2} = 0.01$       **g**  $2^{-3} = \frac{1}{8}$       **h**  $6^1 = 6$
- 3    **a**  $= \log_7 7^2$   
       $= 2$       **b**  $= \log_4 4^3$   
       $= 3$       **c**  $= \log_2 2^7$   
       $= 7$       **d**  $= \log_3 3^3$   
       $= 3$   
      **e**  $= \log_5 5^4$   
       $= 4$       **f**  $= \log_8 8^1$   
       $= 1$       **g**  $= \log_7 7^0$   
       $= 0$       **h**  $= \log_{15} 15^{-1}$   
       $= -1$   
      **i**  $= \log_3 3^{-2}$   
       $= -2$       **j**  $= \lg 10^{-3}$   
       $= -3$       **k**  $= \log_{16} 16^{\frac{1}{4}}$   
       $= \frac{1}{4}$       **l**  $= \log_4 4^{\frac{3}{2}}$   
       $= \frac{3}{2}$   
      **m**  $= \log_9 9^{\frac{5}{2}}$   
       $= \frac{5}{2}$       **n**  $= \log_{100} 100^{-\frac{3}{2}}$   
       $= -\frac{3}{2}$       **o**  $= \log_{25} 25^{\frac{3}{2}}$   
       $= \frac{3}{2}$       **p**  $= \log_{27} 27^{-\frac{2}{3}}$   
       $= -\frac{2}{3}$
- 4    **a**  $5^x = 25$   
       $x = 2$       **b**  $2^6 = x$   
       $x = 64$       **c**  $x^3 = 64$   
       $x = 4$       **d**  $10^{-3} = x$   
       $x = \frac{1}{1000}$   
      **e**  $x^{\frac{2}{3}} = 16$   
       $x = 64$       **f**  $5^x = 1$   
       $x = 0$       **g**  $x^1 = 9$   
       $x = 9$       **h**  $10^x = 10^{12}$   
       $x = 12$   
      **i**  $\log_x 7 = \frac{1}{2}$   
       $x^{\frac{1}{2}} = 7$   
       $x = 49$       **j**  $4^{1.5} = x$   
       $x = 8$       **k**  $x^{-\frac{1}{3}} = 0.1$   
       $x = 1000$       **l**  $\log_8 x = -\frac{1}{3}$   
       $8^{-\frac{1}{3}} = x$   
       $x = \frac{1}{2}$
- 5    **a**  $= \log_a (4 \times 7)$   
       $= \log_a 28$       **b**  $= \log_a (10 \div 5)$   
       $= \log_a 2$       **c**  $= \log_a 6^2$   
       $= \log_a 36$   
      **d**  $= \log_a (9 \div \frac{1}{3})$   
       $= \log_a 27$       **e**  $= \log_a 25^{\frac{1}{2}} + \log_a 3^2$   
       $= \log_a 5 + \log_a 9$   
       $= \log_a (5 \times 9)$   
       $= \log_a 45$       **f**  $= \log_a 48 - \log_a 2^3 - \log_a 9^{\frac{1}{2}}$   
       $= \log_a 48 - \log_a 8 - \log_a 3$   
       $= \log_a [48 \div (8 \times 3)]$   
       $= \log_a 2$
- 6    **a**  $= 5 \log_q x$       **b**  $= \frac{15}{2} \log_q x$       **c**  $= \log_q x^{-1}$   
       $= -\log_q x$       **d**  $= \log_q x^{\frac{1}{3}}$   
       $= \frac{1}{3} \log_q x$   
      **e**  $= 4 \log_q x^{-\frac{1}{2}}$   
       $= -2 \log_q x$       **f**  $= 2 \log_q x + 5 \log_q x$   
       $= 7 \log_q x$       **g**  $= \log_q x^{-2} + \log_q x^{-3}$   
       $= -2 \log_q x - 3 \log_q x$   
       $= -5 \log_q x$       **h**  $= 6 \log_q x - 2 \log_q x$   
       $= 4 \log_q x$

7    **a** =  $\lg(5 \times 4)$   
       =  $\lg 20$

**b** =  $\lg(12 \div 6)$   
       =  $\lg 2$

**c** =  $\lg 2^3$   
       =  $\lg 8$

**d** =  $\lg 3^4 - \lg 9$   
       =  $\lg 81 - \lg 9$   
       =  $\lg(81 \div 9)$   
       =  $\lg 9$

**e** =  $\lg 16^{\frac{1}{2}} - \lg 32^{\frac{1}{5}}$   
       =  $\lg 4 - \lg 2$   
       =  $\lg(4 \div 2)$   
       =  $\lg 2$

**f** =  $\lg 10 + \lg 11$   
       =  $\lg(10 \times 11)$   
       =  $\lg 110$

**g** =  $\lg \frac{1}{50} + \lg 10^2$   
       =  $\lg \frac{1}{50} + \lg 100$   
       =  $\lg(\frac{1}{50} \times 100)$   
       =  $\lg 2$

**h** =  $\lg 10^3 - \lg 40$   
       =  $\lg 1000 - \lg 40$   
       =  $\lg(1000 \div 40)$   
       =  $\lg 25$

8    **a** =  $\log_3(54 \div 2)$   
       =  $\log_3 27$   
       =  $\log_3 3^3$   
       = 3

**b** =  $\log_5(20 \times 1.25)$   
       =  $\log_5 25$   
       =  $\log_5 5^2$   
       = 2

**c** =  $\log_2 2^4 + \log_3 3^3$   
       =  $4 + 3$   
       = 7

**d** =  $\log_6(24 \times 9)$   
       =  $\log_6 216$   
       =  $\log_6 6^3$   
       = 3

**e** =  $\log_3(12 \div 4)$   
       =  $\log_3 3$   
       = 1

**f** =  $\log_4(18 \div 9)$   
       =  $\log_4 2$   
       =  $\log_4 4^{\frac{1}{2}}$   
       =  $\frac{1}{2}$

**g** =  $\log_9(4 \times 0.25)$   
       =  $\log_9 1$   
       = 0

**h** =  $\lg 2^2 + \lg 25$   
       =  $\lg 4 + \lg 25$   
       =  $\lg(4 \times 25)$   
       =  $\lg 100$   
       =  $\lg 10^2$   
       = 2

**i** =  $\log_3 8^{\frac{1}{3}} - \log_3 18$   
       =  $\log_3 2 - \log_3 18$   
       =  $\log_3(2 \div 18)$   
       =  $\log_3 \frac{1}{9}$   
       =  $\log_3 3^{-2}$   
       = -2

**j** =  $\log_4 64^{\frac{1}{3}} + (2 \times \log_5 5^2)$   
       =  $\log_4 4 + (2 \times 2)$   
       =  $1 + 4$   
       = 5

**k** =  $\frac{1}{2} \log_5 \frac{25}{16} + \log_5 10^2$   
       =  $\log_5 (\frac{25}{16})^{\frac{1}{2}} + \log_5 100$   
       =  $\log_5 \frac{5}{4} + \log_5 100$   
       =  $\log_5 (\frac{5}{4} \times 100)$   
       =  $\log_5 125$   
       =  $\log_5 5^3$   
       = 3

**l** =  $\log_3 5 - \log_3 6^2 - \log_3 \frac{15}{4}$   
       =  $\log_3 [5 \div (36 \times \frac{15}{4})]$   
       =  $\log_3 \frac{1}{27}$   
       =  $\log_3 3^{-3}$   
       = -3

$$1 \quad \begin{array}{lll} \mathbf{a} = \log_{10} a + \log_{10} b & \mathbf{b} = \log_{10} a + \log_{10} b^7 & \mathbf{c} = \log_{10} a^3 - \log_{10} b \\ & = \log_{10} a + 7 \log_{10} b & = 3 \log_{10} a - \log_{10} b \\ \mathbf{d} = \log_{10} a + \log_{10} b^{\frac{1}{2}} & & = \log_{10} a + \frac{1}{2} \log_{10} b \\ \mathbf{e} = 2 \log_{10} ab & \mathbf{f} = -\log_{10} ab & \mathbf{g} = \log_{10} a^{\frac{3}{2}} + \log_{10} b^{\frac{5}{2}} \\ = 2 \log_{10} a + 2 \log_{10} b & = -\log_{10} a - \log_{10} b & = \frac{3}{2} \log_{10} a + \frac{5}{2} \log_{10} b \\ \mathbf{h} = 3(\log_{10} a^2 - \log_{10} b^{\frac{1}{3}}) & & = 6 \log_{10} a - \log_{10} b \end{array}$$

$$2 \quad \begin{array}{lll} \mathbf{a} = \log_q 8^2 & \mathbf{b} = \log_q 8^{\frac{1}{3}} & \mathbf{c} = \log_q 16 - \log_q q \\ = 2y & = \frac{1}{3}y & = \log_q 16 - \log_q q \\ & & = \log_q 8^{\frac{4}{3}} - 1 \\ & & = \frac{4}{3}y - 1 \\ \mathbf{d} = \log_q 4 + \log_q q^3 & & = \log_q 4 + \log_q q^3 \\ = \log_q 8^{\frac{2}{3}} + 3 & & = \log_q 8^{\frac{2}{3}} + 3 \\ = \frac{2}{3}y + 3 & & = \frac{2}{3}y + 3 \end{array}$$

$$3 \quad \begin{array}{lll} \mathbf{a} = \lg(2 \times 3^2) & \mathbf{b} = \lg(2^5 \times 3) & \mathbf{c} = \lg 9 - \lg 16 \\ = \lg 2 + 2 \lg 3 & = 5 \lg 2 + \lg 3 & = \lg 3^2 - \lg 2^4 \\ = a + 2b & = 5a + b & = 2 \lg 3 - 4 \lg 2 \\ & & = 2b - 4a \\ \mathbf{d} = \lg(2 \times 3) - \lg 2^3 & & = \lg(2 \times 3) - \lg 2^3 \\ = \lg 2 + \lg 3 - 3 \lg 2 & & = \lg 2 + \lg 3 - 3 \lg 2 \\ = \lg 3 - 2 \lg 2 & & = \lg 3 - 2 \lg 2 \\ = b - 2a & & = b - 2a \\ \mathbf{e} = \frac{1}{2} \lg 6 & \mathbf{f} = \frac{3}{2} \lg 2^4 + \frac{1}{2} \lg 3^4 & \mathbf{g} = 4 \lg 3 - 3(\lg 2 + \lg 3) \\ = \frac{1}{2}(\lg 2 + \lg 3) & = 6 \lg 2 + 2 \lg 3 & = 4 \lg 3 - 3 \lg 2 - 3 \lg 3 \\ = \frac{1}{2}(a + b) & = 6a + 2b & = \lg 3 - 3 \lg 2 \\ & & = b - 3a \\ \mathbf{h} = \lg(6 \times 10) + \lg(2 \times 10) - 2 & & = \lg(6 \times 10) + \lg(2 \times 10) - 2 \\ = \lg 6 + 1 + \lg 2 + 1 - 2 & & = \lg 6 + 1 + \lg 2 + 1 - 2 \\ = \lg 2 + \lg 3 + \lg 2 & & = \lg 2 + \lg 3 + \lg 2 \\ = 2a + b & & = 2a + b \end{array}$$

$$4 \quad \begin{array}{lll} \mathbf{a} = \log_5 10 - \log_5 2 & \mathbf{b} = \log_{12} 16 + \log_{12} 9 & \mathbf{c} = \log_4 8 \\ = \log_5 5 & = \log_{12} 144 & = \log_4 4^{\frac{3}{2}} \\ = 1 & = 2 & = \frac{3}{2} \\ \mathbf{d} = \frac{\log_7 3^4}{\log_7 3} & \mathbf{e} = \log_{27} \frac{12^3}{72^2} & \mathbf{f} = \frac{\log_{11} 5^2}{-\log_{11} 5} \\ = \frac{4 \log_7 3}{\log_7 3} & = \log_{27} \frac{12 \times 12 \times 12}{6 \times 12 \times 6 \times 12} & = \frac{2 \log_{11} 5}{-\log_{11} 5} \\ = 4 & = \log_{27} \frac{1}{3} = -\frac{1}{3} & = -2 \end{array}$$

$$5 \quad \begin{array}{lll} \mathbf{a} \quad x = 3^{1.8} & \mathbf{b} \quad x = 5^{-0.3} & \mathbf{c} \quad x - 3 = 8^{2.1} \\ x = 7.22 & x = 0.617 & x = 3 + 8^{2.1} \\ & & x = 81.8 \\ \mathbf{d} \quad \frac{1}{2}x + 1 = 4^{3.2} & \mathbf{e} \quad \log_2 3y = 5.3 & \mathbf{f} \quad \log_6(1 - 5t) = -0.6 \\ x = 2(4^{3.2} - 1) & 3y = 2^{5.3} & 1 - 5t = 6^{-0.6} \\ x = 167 & y = \frac{1}{3} \times 2^{5.3} & t = \frac{1}{5}(1 - 6^{-0.6}) \\ & y = 13.1 & t = 0.132 \end{array}$$

$$6 \quad \begin{array}{lll} \mathbf{a} = \log_2 x^5 & \mathbf{b} = \log_2(x^2 + 4x) & \mathbf{c} = \log_2 x^2 + \log_2 x \\ & & = \log_2 x^3 \\ \mathbf{d} = \log_2(x - 2)^3 - \log_2 x^4 & \mathbf{e} = \log_2 \frac{x^2 - 1}{x + 1} & \mathbf{f} = \log_2 x - 2 \log_2 x + \frac{2}{3} \log_2 x \\ = \log_2 \frac{(x - 2)^3}{x^4} & = \log_2 \frac{(x + 1)(x - 1)}{x + 1} & = -\frac{1}{3} \log_2 x \\ & = \log_2(x - 1) & = \log_2 x^{-\frac{1}{3}} \end{array}$$

- 7 a  $\log_3 5x = \log_3 (2x + 3)$   
 $5x = 2x + 3$   
 $x = 1$
- c  $\log_4 \frac{x}{x-1} = \log_4 3 + \log_4 2 = \log_4 6$   
 $\frac{x}{x-1} = 6$   
 $x = 6x - 6$   
 $x = \frac{6}{5}$
- e  $\log_6 x^2 = \log_6 5(2x - 5)$   
 $x^2 = 5(2x - 5)$   
 $x^2 - 10x + 25 = 0$   
 $(x - 5)^2 = 0$   
 $x = 5$
- 8 a  $\log_x y = 2 \Rightarrow y = x^2$   
sub.  $x^3 = 27$   
 $x = 3$   
 $\therefore x = 3, y = 9$
- c  $\log_y 32 = -\frac{5}{2} \Rightarrow y^{-\frac{5}{2}} = 32$   
 $\Rightarrow y = 32^{-\frac{2}{5}} = \frac{1}{4}$   
sub.  $\log_2 x = 3 - 2 \log_2 \frac{1}{4}$   
 $\log_2 x = 3 - (-4) = 7$   
 $x = 2^7 = 128$   
 $\therefore x = 128, y = \frac{1}{4}$
- e  $\log_a x + \log_a 3 = \frac{1}{2} \log_a y \Rightarrow 3x = y^{\frac{1}{2}}$   
 $\Rightarrow y = 9x^2$   
sub.  $3x + 9x^2 = 20$   
 $9x^2 + 3x - 20 = 0$   
 $(3x + 5)(3x - 4) = 0$   
for real  $\log_a x, x > 0 \therefore x = \frac{4}{3}$   
 $\therefore x = \frac{4}{3}, y = 16$
- b  $\log_9 10x = \frac{3}{2}$   
 $10x = 9^{\frac{3}{2}} = 27$   
 $x = 2.7$
- d  $\log_5 \frac{5x}{x+2} = \log_5 \frac{x+6}{x}$   
 $\frac{5x}{x+2} = \frac{x+6}{x}$   
 $5x^2 = (x+2)(x+6) = x^2 + 8x + 12$   
 $x^2 - 2x - 3 = 0$   
 $(x+1)(x-3) = 0$   
 $x = -1, 3$   
 $\log_5 x$  not real for  $x = -1 \therefore x = 3$
- f  $\log_7 4x - \log_7 \frac{1}{x-6} = 1$   
 $\log_7 4x(x-6) = 1$   
 $4x(x-6) = 7$   
 $4x^2 - 24x - 7 = 0$   
 $x = \frac{24 \pm \sqrt{576 + 112}}{8} = 3 \pm \frac{1}{2}\sqrt{43}$   
 $\log_7 4x$  not real for  $x = 3 - \frac{1}{2}\sqrt{43}$   
 $\therefore x = 3 + \frac{1}{2}\sqrt{43} \quad [= 6.28 \text{ (3sf)}]$
- b  $\log_5 x - 2 \log_5 y = \log_5 2 \Rightarrow \frac{x}{y^2} = 2$   
 $\Rightarrow x = 2y^2$   
sub.  $3y^2 = 12$   
 $y^2 = 4$   
for real  $\log_5 y, y > 0 \therefore y = 2$   
 $\therefore x = 8, y = 2$
- d  $\log_y x = \frac{3}{2} \Rightarrow y^{\frac{3}{2}} = x$   
 $\Rightarrow y^{\frac{1}{2}} = x^{\frac{1}{3}}$   
sub.  $4x^{\frac{1}{3}} = 20$   
 $x^{\frac{1}{3}} = 5$   
 $x = 5^3 = 125$   
 $\therefore x = 125, y = 25$
- f  $\log_{10} y + 2 \log_{10} x = 3 \Rightarrow x^2 y = 10^3$   
 $\log_2 y - \log_2 x = 3 \Rightarrow \frac{y}{x} = 2^3$   
 $\Rightarrow y = 8x$   
sub.  $8x^3 = 1000$   
 $x^3 = 125$   
 $x = 5$   
 $\therefore x = 5, y = 40$

- 1 a 1.78                      b 0.778                      c 2.40                      d -0.398
- 2 a  $x = \lg 14 = 1.15$                       b  $10^x = 4$   
 $x = \lg 4 = 0.60$                       c  $3x = \lg 49$   
 $x = \frac{1}{3} \lg 49 = 0.56$
- d  $x - 4 = \lg 23$                       e  $2x + 1 = \lg 130$                       f  $(10^2)^x = 10^{2x} = 5$   
 $x = 4 + \lg 23 = 5.36$                        $x = \frac{1}{2}(\lg 130 - 1) = 0.56$                        $2x = \lg 5$   
 $x = \frac{1}{2} \lg 5 = 0.35$
- 3 let  $y = \log_a b \Rightarrow a^y = b$   
 $y \log_c a = \log_c b$   
 $y = \frac{\log_c b}{\log_c a}$   
 $\therefore \log_a b = \frac{\log_c b}{\log_c a}$
- 4 a  $= \frac{\lg 7}{\lg 2} = 2.81$                       b  $= \frac{\lg 172}{\lg 20} = 1.72$                       c  $= \frac{\lg 49}{\lg 5} = 2.42$                       d  $= \frac{\lg 4}{\lg 9} = 0.631$
- 5 a  $x \lg 3 = \lg 12$                       b  $x \lg 2 = \lg 0.7$                       c  $-y \lg 8 = \lg 3$                       d  $\frac{1}{2}x \lg 4 = \lg 0.3$   
 $x = \frac{\lg 12}{\lg 3}$                        $x = \frac{\lg 0.7}{\lg 2}$                        $y = -\frac{\lg 3}{\lg 8}$                        $x = \frac{2 \lg 0.3}{\lg 4}$   
 $x = 2.26$                        $x = -0.515$                        $y = -0.528$                        $x = -1.74$
- e  $(t + 3) \lg 5 = \lg 24$                       f  $(4 + x) \lg 3 = \lg 16$                       g  $(2x + 4) \lg 7 = \lg 12$                       h  $2^{3x+1} = 12.4$   
 $t = \frac{\lg 24}{\lg 5} - 3$                        $x = \frac{\lg 16}{\lg 3} - 4$                        $x = \frac{1}{2} \left( \frac{\lg 12}{\lg 7} - 4 \right)$                        $(3x + 1) \lg 2 = \lg 12.4$   
 $t = -1.03$                        $x = -1.48$                        $x = -1.36$                        $x = \frac{1}{3} \left( \frac{\lg 12.4}{\lg 2} - 1 \right)$   
 $x = 0.877$
- i  $(2 - 3x) \lg 4 = \lg 32.7$                       j  $x \lg 5 = (x - 1) \lg 6$   
 $x = \frac{1}{3} \left( 2 - \frac{\lg 32.7}{\lg 4} \right)$                        $x(\lg 6 - \lg 5) = \lg 6$   
 $x = -0.172$                        $x = \frac{\lg 6}{\lg 6 - \lg 5} = 9.83$
- k  $(y + 2) \lg 7 = (y + 1) \lg 9$                       l  $(5 - x) \lg 4 = (2x - 1) \lg 11$   
 $y(\lg 9 - \lg 7) = 2 \lg 7 - \lg 9$                        $x(2 \lg 11 + \lg 4) = 5 \lg 4 + \lg 11$   
 $y = \frac{2 \lg 7 - \lg 9}{\lg 9 - \lg 7} = 6.74$                        $x = \frac{5 \lg 4 + \lg 11}{2 \lg 11 + \lg 4} = 1.51$
- m  $(\frac{1}{2}x + 3) \lg 4 = (1 - 2x) \lg 5$                       n  $(3y - 2) \lg 2 = (2y + 5) \lg 3$   
 $x(\frac{1}{2} \lg 4 + 2 \lg 5) = \lg 5 - 3 \lg 4$                        $y(3 \lg 2 - 2 \lg 3) = 5 \lg 3 + 2 \lg 2$   
 $x = \frac{\lg 5 - 3 \lg 4}{\frac{1}{2} \lg 4 + 2 \lg 5} = -0.652$                        $y = \frac{5 \lg 3 + 2 \lg 2}{3 \lg 2 - 2 \lg 3} = -58.4$
- o  $7^{2x+4} = 11^{3x-4}$                       p  $3^{x+1} = 2^{4+x}$   
 $(2x + 4) \lg 7 = (3x - 4) \lg 11$                        $(x + 1) \lg 3 = (4 + x) \lg 2$   
 $x(3 \lg 11 - 2 \lg 7) = 4 \lg 7 + 4 \lg 11$                        $x(\lg 3 - \lg 2) = 4 \lg 2 - \lg 3$   
 $x = \frac{4 \lg 7 + 4 \lg 11}{3 \lg 11 - 2 \lg 7} = 5.26$                        $x = \frac{4 \lg 2 - \lg 3}{\lg 3 - \lg 2} = 4.13$

**6 a**  $(2^x + 3)(2^x - 2) = 0$   
 $2^x = -3$  [no sols], 2  
 $x = 1$

**b**  $(3^x - 1)(3^x - 4) = 0$   
 $3^x = 1, 4$   
 $x = 0, \frac{\lg 4}{\lg 3} = 0, 1.26$

**c**  $5^{2x} - 8(5^x) + 12 = 0$   
 $(5^x - 2)(5^x - 6) = 0$   
 $5^x = 2, 6$   
 $x = \frac{\lg 2}{\lg 5}, \frac{\lg 6}{\lg 5} = 0.43, 1.11$

**d**  $2(4^{2x}) - 7(4^x) + 3 = 0$   
 $(2(4^x) - 1)(4^x - 3) = 0$   
 $4^x = \frac{1}{2}, 3$   
 $x = -\frac{1}{2}, \frac{\lg 3}{\lg 4} = -\frac{1}{2}, 0.79$

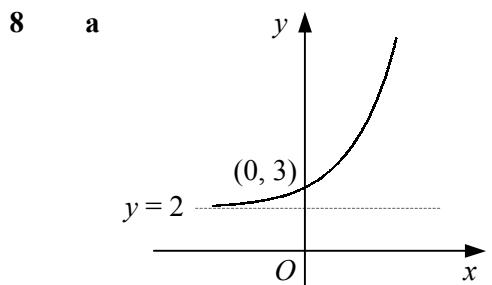
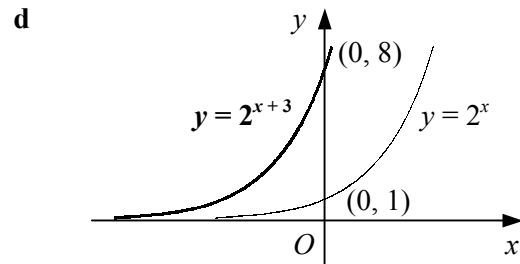
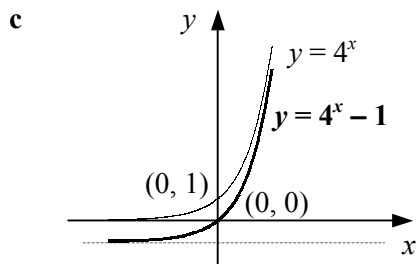
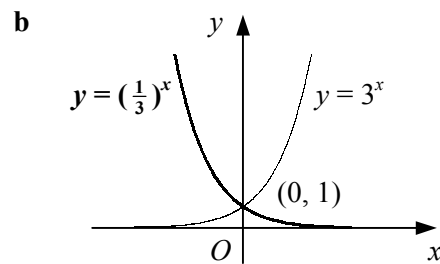
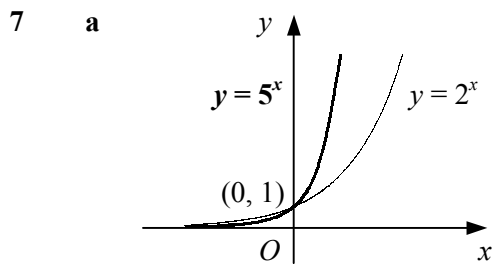
**e**  $2(2^{2y}) + 7(2^y) - 15 = 0$   
 $(2(2^y) - 3)(2^y + 5) = 0$   
 $2^y = -5$  [no sols],  $\frac{3}{2}$   
 $y = \frac{\lg \frac{3}{2}}{\lg 2} = 0.58$

**f**  $3(3^{2x}) - 17(3^x) + 10 = 0$   
 $(3(3^x) - 2)(3^x - 5) = 0$   
 $3^x = \frac{2}{3}, 5$   
 $x = \frac{\lg \frac{2}{3}}{\lg 3}, \frac{\lg 5}{\lg 3} = -0.37, 1.46$

**g**  $5^{2t} + 5(5^t) - 24 = 0$   
 $(5^t + 8)(5^t - 3) = 0$   
 $5^t = -8$  [no sols], 3  
 $t = \frac{\lg 3}{\lg 5} = 0.68$

**h**  $3(3^{2x}) - 18(3^x) + 15 = 0$   
 $3(3^x - 1)(3^x - 5) = 0$   
 $3^x = 1, 5$   
 $x = 0, \frac{\lg 5}{\lg 3} = 0, 1.46$

**i**  $3(4^{2x}) - 16(4^x) + 5 = 0$   
 $(3(4^x) - 1)(4^x - 5) = 0$   
 $4^x = \frac{1}{3}, 5$   
 $x = \frac{\lg \frac{1}{3}}{\lg 4}, \frac{\lg 5}{\lg 4} = -0.79, 1.16$



**9**  $x = 0 \Rightarrow y = -4$   
 $y = 0 \Rightarrow 2^x = 5$   
 $x = \frac{\lg 5}{\lg 2}$   
 $AB^2 = 4^2 + \left(\frac{\lg 5}{\lg 2}\right)^2 = 21.391$   
 $AB = 4.63$

**b**  $(3, 29) \Rightarrow 29 = 2 + a^3$   
 $a^3 = 27$   
 $a = 3$

- 1 a  $= \log_{10} \frac{3}{2}$   
 $= \log_{10} 3 - \log_{10} 2$   
 $= b - a$   
 b  $= \log_{10} (2^3 \times 3)$   
 $= 3 \log_{10} 2 + \log_{10} 3$   
 $= 3a + b$   
 c  $= \log_{10} (1.5 \times 100)$   
 $= \log_{10} 1.5 + \log_{10} 100$   
 $= b - a + 2$
- 2 a  $\log_3 x = \frac{5}{4}$   
 $x = 3^{\frac{5}{4}} = 3.95$  (3sf)  
 b  $3 \log_3 x - 5 \log_3 x = 4$   
 $\log_3 x = -2$   
 $x = 3^{-2} = \frac{1}{9}$
- 3 a i  $= \log_2 q^{\frac{1}{2}} = \frac{1}{2} \log_2 q = \frac{1}{2} p$   
 ii  $= \log_2 8 + \log_2 q = 3 + p$   
 b  $3 + p - \frac{1}{2} p = 2$   
 $p = \log_2 q = -2$   
 $\therefore q = 2^{-2} = \frac{1}{4}$
- 4  $2000 = 1000 \times 1.022^{4t}$   
 $2 = 1.022^{4t}$   
 $4t \lg 1.022 = \lg 2$   
 $t = \frac{\lg 2}{4 \lg 1.022} = 7.96$   
 $\therefore 8$  years
- 5 a  $(0, -3)$   
 b  $k = -4$   
 c  $(\frac{1}{3})^x - 4 = 0$   
 $(\frac{1}{3})^x = 4$   
 $x = \frac{\lg 4}{\lg \frac{1}{3}} = -1.26$  (3sf)
- 6 a  $\log_3 \frac{x+1}{x-2} = 1$   
 $\frac{x+1}{x-2} = 3$   
 $x+1 = 3x-6$   
 $x = \frac{7}{2}$   
 b  $(2x+1) \lg 3 = (x-4) \lg 2$   
 $x(\lg 2 - 2 \lg 3) = \lg 3 + 4 \lg 2$   
 $x = \frac{\lg 3 + 4 \lg 2}{\lg 2 - 2 \lg 3}$
- 7 a i  $= 2^{-1}(2^x) = \frac{1}{2} t$   
 ii  $= 2(2^{2x}) = 2(2^x)^2 = 2t^2$   
 b  $2t^2 - 7t + 6 = 0$   
 $(2t-3)(t-2) = 0$   
 $t = 2^x = \frac{3}{2}, 2$   
 $x = \frac{\lg \frac{3}{2}}{\lg 2}, 1 = 0.585$  (3sf), 1
- 8 a  $\log_2 (3x+5) + 3 = 7$   
 $3x+5 = 2^4 = 16$   
 $x = \frac{11}{3}$   
 b  $\log_2 (x+1) + \log_2 (3x-1) = 5$   
 $(x+1)(3x-1) = 2^5 = 32$   
 $3x^2 + 2x - 33 = 0$   
 $(3x+11)(x-3) = 0$   
 $x = -\frac{11}{3}, 3$   
 for real  $\log_2 (3x-1), x > \frac{1}{3} \therefore x = 3$

9 a  $x + 4 = \frac{5}{4}x$

$x = 16$

b  $y + 2 = \frac{12}{y+1}$

$(y + 2)(y + 1) = 12$

$y^2 + 3y - 10 = 0$

$(y + 5)(y - 2) = 0$

$y > 0 \therefore y = 2$

c  $\log_y x = \log_2 16 = 4$

10 a  $t = 0 \Rightarrow n = 2000$

b  $3600 = \frac{18000}{1+8c^{-3}}$

$1 + 8c^{-3} = 5$

$c^{-3} = \frac{1}{2}$

$c^3 = 2$

$c = \sqrt[3]{2}$

c  $4000 = \frac{18000}{1+8c^{-t}}$

$1 + 8c^{-t} = \frac{9}{2}$

$c^{-t} = \frac{7}{16}$

$-t = \frac{\lg \frac{7}{16}}{\lg \sqrt[3]{2}}$

$t = 3.578 \text{ weeks} = 25 \text{ days}$

11 a i  $\log_8 x^2 = 2 \log_8 x = 2y$

ii  $y = \log_8 x \Rightarrow x = 8^y = 2^{3y}$

$\therefore \log_2 x = 3y$

b  $3(2y) + 3y = 6$

$y = \log_8 x = \frac{2}{3}$

$\therefore x = 8^{\frac{2}{3}} = 4$

12  $\log_2 y - \log_2 (3 - 2x) = 1 \Rightarrow \frac{y}{3-2x} = 2$

$\Rightarrow y = 6 - 4x$

$\log_4 xy = \frac{1}{2} \Rightarrow xy = 4^{\frac{1}{2}} = 2$

sub.  $x(6 - 4x) = 2$

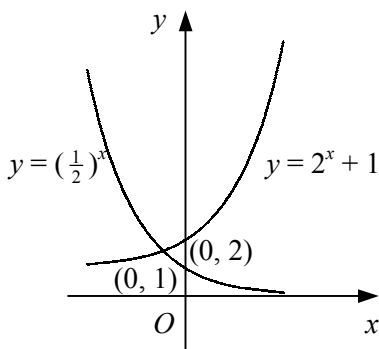
$2x^2 - 3x + 1 = 0$

$(2x - 1)(x - 1) = 0$

$x = \frac{1}{2}, 1$

$\therefore x = \frac{1}{2}, y = 4 \text{ or } x = 1, y = 2$

13 a



b at A,  $2^x + 1 = (\frac{1}{2})^x$

$(2^x)^2 + 2^x = 1$

$2^{2x} + 2^x - 1 = 0$

c  $2^x = \frac{-1 \pm \sqrt{1+4}}{2}$

$2^x = \frac{-1 - \sqrt{5}}{2}$  [no sols] or  $\frac{-1 + \sqrt{5}}{2}$

$\therefore 2^x = \frac{1}{2}\sqrt{5} - \frac{1}{2}$

$\therefore y = (\frac{1}{2}\sqrt{5} - \frac{1}{2}) + 1 = \frac{1}{2}(\sqrt{5} + 1)$

14 a when  $x = 1$ ,

LHS =  $8 - 4(4) + 2 + 6 = 0$

$\therefore x = 1$  is a solution

b  $2^{3x} = (2^x)^3 = u^3$

$2^{2x} = (2^x)^2 = u^2$

$\therefore$  (I)  $\Rightarrow u^3 - 4u^2 + u + 6 = 0$

c  $x = 1 \Rightarrow u = 2 \therefore (u - 2)$  is a factor

$$\begin{array}{r} u^2 - 2u - 3 \\ u - 2 \overline{) u^3 - 4u^2 + u + 6} \\ \underline{u^3 - 2u^2} \phantom{+} \\ -2u^2 + u \phantom{+} \\ \underline{-2u^2 + 4u} \phantom{+} \\ -3u + 6 \\ \underline{-3u + 6} \\ 0 \end{array}$$

$(u - 2)(u^2 - 2u - 3) = 0$

$(u - 2)(u - 3)(u + 1) = 0$

$u = 2^x = -1$  [no sols], 2 or 3

$x = 1$  (given) or  $\frac{\lg 3}{\lg 2} = 1.58$