C2 INTEGRATION — Answers - Worksheet A

1

a $= [2x^2 - x]_1^3$
$= (18 - 3) - (2 - 1)$
$= 14$

b $= [x^3 + 2x]_0^1$
$= (1 + 2) - (0)$
$= 3$

c $= [\frac{1}{2}x^2 - \frac{1}{3}x^3]_0^3$
$= (\frac{9}{2} - 9) - (0)$
$= -\frac{9}{2}$

d $= \int_2^3 (9x^2 + 6x + 1)\ dx$
$= [3x^3 + 3x^2 + x]_2^3$
$= (81 + 27 + 3) - (24 + 12 + 2)$
$= 73$

e $= [\frac{1}{3}x^3 - 4x^2 - 3x]_1^2$
$= (\frac{8}{3} - 16 - 6) - (\frac{1}{3} - 4 - 3)$
$= -12\frac{2}{3}$

f $= [8x - 2x^2 + x^3]_{-2}^4$
$= (32 - 32 + 64) - (-16 - 8 - 8)$
$= 96$

g $= [\frac{1}{4}x^4 - x^2 - 7x]_1^4$
$= (64 - 16 - 28) - (\frac{1}{4} - 1 - 7)$
$= 27\frac{3}{4}$

h $= [5x + \frac{1}{3}x^3 - x^4]_{-2}^{-1}$
$= (-5 - \frac{1}{3} - 1) - (-10 - \frac{8}{3} - 16)$
$= 22\frac{1}{3}$

i $= [\frac{1}{5}x^5 + 2x^3 - \frac{1}{2}x^2]_{-1}^2$
$= (\frac{32}{5} + 16 - 2) - (-\frac{1}{5} - 2 - \frac{1}{2})$
$= 23\frac{1}{10}$

2 $\int_1^4 (3x^2 + ax - 5)\ dx = [x^3 + \frac{1}{2}ax^2 - 5x]_1^4$
$= (64 + 8a - 20) - (1 + \frac{1}{2}a - 5) = 48 + \frac{15}{2}a$
$\therefore\ 48 + \frac{15}{2}a = 18$
$a = -4$

3 $\int_{-1}^k (3x^2 - 12x + 9)\ dx = [x^3 - 6x^2 + 9x]_{-1}^k$
$= (k^3 - 6k^2 + 9k) - (-1 - 6 - 9) = k^3 - 6k^2 + 9k + 16$
$\therefore\ k^3 - 6k^2 + 9k + 16 = 16$
$k(k^2 - 6k + 9) = 0$
$k(k - 3)^2 = 0$
$k \neq 0\ \therefore\ k = 3$

4

a $= \int_1^3 (2 - x^{-2})\ dx$
$= [2x + x^{-1}]_1^3$
$= (6 + \frac{1}{3}) - (2 + 1)$
$= \frac{10}{3}$

b $= \int_{-2}^{-1} (6x + 4x^{-3})\ dx$
$= [3x^2 - 2x^{-2}]_{-2}^{-1}$
$= (3 - 2) - (12 - \frac{1}{2})$
$= -10\frac{1}{2}$

c $= [2x^{\frac{3}{2}} - 4x]_1^4$
$= (16 - 16) - (2 - 4)$
$= 2$

d $= \int_{-1}^2 (2x^3 - \frac{1}{2})\ dx$
$= [\frac{1}{2}x^4 - \frac{1}{2}x]_{-1}^2$
$= (8 - 1) - (\frac{1}{2} + \frac{1}{2})$
$= 6$

e $= [\frac{1}{2}x^2 - \frac{3}{2}x^{\frac{2}{3}}]_1^8$
$= (32 - 6) - (\frac{1}{2} - \frac{3}{2})$
$= 27$

f $= \int_2^3 (\frac{1}{3}x^{-2} - 2x)\ dx$
$= [-\frac{1}{3}x^{-1} - x^2]_2^3$
$= (-\frac{1}{9} - 9) - (-\frac{1}{6} - 4)$
$= -4\frac{17}{18}$

5 $= \int_1^3 (3x^2 - 6x + 7)\ dx$
$= [x^3 - 3x^2 + 7x]_1^3$
$= (27 - 27 + 21) - (1 - 3 + 7) = 16$

6 **a** $= \int_0^2 (x^2 + 2)\ dx$
$= [\frac{1}{3}x^3 + 2x]_0^2$
$= (\frac{8}{3} + 4) - 0 = 6\frac{2}{3}$

b $= \int_{-2}^1 (3x^2 + 8x + 6)\ dx$
$= [x^3 + 4x^2 + 6x]_{-2}^1$
$= (1 + 4 + 6) - (-8 + 16 - 12) = 15$

c $= \int_2^4 (9 + 2x - x^2)\ dx$
$= [9x + x^2 - \frac{1}{3}x^3]_2^4$
$= (36 + 16 - \frac{64}{3}) - (18 + 4 - \frac{8}{3}) = 11\frac{1}{3}$

d $= \int_{-1}^0 (x^3 - 4x + 1)\ dx$
$= [\frac{1}{4}x^4 - 2x^2 + x]_{-1}^0$
$= 0 - (\frac{1}{4} - 2 - 1) = \frac{11}{4}$

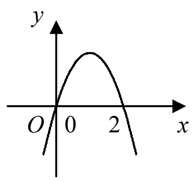
e $= \int_1^4 (2x + 3x^{\frac{1}{2}})\ dx$
$= [x^2 + 2x^{\frac{3}{2}}]_1^4$
$= (16 + 16) - (1 + 2) = 29$

f $= \int_{-5}^{-1} (3 + 5x^{-2})\ dx$
$= [3x - 5x^{-1}]_{-5}^{-1}$
$= (-3 + 5) - (-15 + 1) = 16$

7 **a** $y = 0 \Rightarrow 4 - x^2 = 0$
$x^2 = 4$
$x = \pm 2$
$\therefore$ $(-2, 0)$ and $(2, 0)$

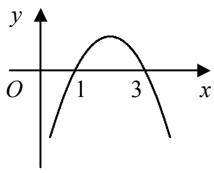
b $= \int_{-2}^2 (4 - x^2)\ dx$
$= [4x - \frac{1}{3}x^3]_{-2}^2$
$= (8 - \frac{8}{3}) - (-8 + \frac{8}{3})$
$= 10\frac{2}{3}$

8 **a** $6x - 3x^2 = 0$
$3x(2 - x) = 0$
$x = 0, 2$



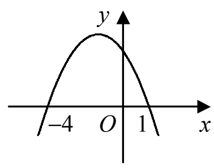
area =
$\int_0^2 (6x - 3x^2)\ dx$
$= [3x^2 - x^3]_0^2$
$= (12 - 8) - 0$
$= 4$

b $-x^2 + 4x - 3 = 0$
$-(x - 1)(x - 3) = 0$
$x = 1, 3$



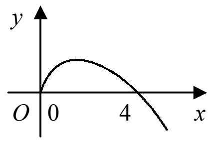
area =
$\int_1^3 (-x^2 + 4x - 3)\ dx$
$= [-\frac{1}{3}x^3 + 2x^2 - 3x]_1^3$
$= (-9 + 18 - 9)$
$- (-\frac{1}{3} + 2 - 3)$
$= \frac{4}{3}$

c $4 - 3x - x^2 = 0$
$-(x + 4)(x - 1) = 0$
$x = -4, 1$



area =
$\int_{-4}^1 (4 - 3x - x^2)\ dx$
$= [4x - \frac{3}{2}x^2 - \frac{1}{3}x^3]_{-4}^1$
$= (4 - \frac{3}{2} - \frac{1}{3})$
$- (-16 - 24 + \frac{64}{3})$
$= 20\frac{5}{6}$

d $2x^{\frac{1}{2}} - x = 0$
$x^{\frac{1}{2}}(2 - x^{\frac{1}{2}}) = 0$
$x^{\frac{1}{2}} = 0, 2$
$x = 0, 4$

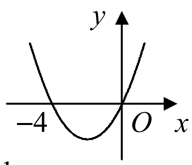


area =
$\int_0^4 (2x^{\frac{1}{2}} - x)\ dx$
$= [\frac{4}{3}x^{\frac{3}{2}} - \frac{1}{2}x^2]_0^4$
$= (\frac{32}{3} - 8) - 0$
$= \frac{8}{3}$

9 **a** $x^2 + 4x = 0$

$x(x + 4) = 0$

$x = -4, 0$



b $= \int_0^2 (x^2 + 4x)\ dx$

$= [\tfrac{1}{3}x^3 + 2x^2]_0^2$

$= (\tfrac{8}{3} + 8) - 0 = 10\tfrac{2}{3}$

10 **a** $x^2 + 2x - 15 = 0$

$(x + 5)(x - 3) = 0$

$x = -5, 3$

$\therefore$ $(-5, 0)$ and $(3, 0)$

b $= [\tfrac{1}{3}x^3 + x^2 - 15x]_0^3$

$= (9 + 9 - 45) - 0 = -27$

c 27

11 **a** $x^3 - 5x^2 + 6x = 0$

$x(x - 2)(x - 3) = 0$

$x = 0, 2, 3$

$\therefore$ $(0, 0)$, $(2, 0)$ and $(3, 0)$

b $\int_0^2 (x^3 - 5x^2 + 6x)\ dx$

$= [\tfrac{1}{4}x^4 - \tfrac{5}{3}x^3 + 3x^2]_0^2$

$= (4 - \tfrac{40}{3} + 12) - 0 = \tfrac{8}{3}$

$\int_2^3 (x^3 - 5x^2 + 6x)\ dx$

$= [\tfrac{1}{4}x^4 - \tfrac{5}{3}x^3 + 3x^2]_2^3$

$= (\tfrac{81}{4} - 45 + 27) - \tfrac{8}{3} = -\tfrac{5}{12}$

total area $= \tfrac{8}{3} + \tfrac{5}{12} = 3\tfrac{1}{12}$

12 **a** $x^2 - 3x + 4 = x + 1$

$x^2 - 4x + 3 = 0$

$(x - 1)(x - 3) = 0$

$x = 1, 3$

$\therefore$ $(1, 2)$ and $(3, 4)$

b area below curve

$= \int_1^3 (x^2 - 3x + 4)\ dx$

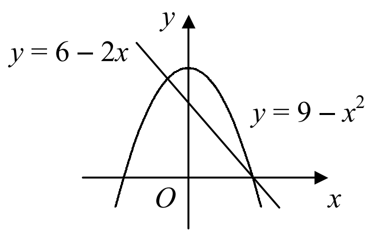
$= [\tfrac{1}{3}x^3 - \tfrac{3}{2}x^2 + 4x]_1^3$

$= (9 - \tfrac{27}{2} + 12) - (\tfrac{1}{3} - \tfrac{3}{2} + 4) = \tfrac{14}{3}$

area below line $= \tfrac{1}{2} \times 2 \times (2 + 4) = 6$

shaded area $= 6 - \tfrac{14}{3} = \tfrac{4}{3}$

13 **a**



$9 - x^2 = 6 - 2x$

$x^2 - 2x - 3 = 0$

$(x + 1)(x - 3) = 0$

$x = -1, 3$

$\therefore$ intersect at $(-1, 8)$ and $(3, 0)$

area below curve

$= \int_{-1}^3 (9 - x^2)\ dx$

$= [9x - \tfrac{1}{3}x^3]_{-1}^3$

$= (27 - 9) - (-9 + \tfrac{1}{3})$

$= 26\tfrac{2}{3}$

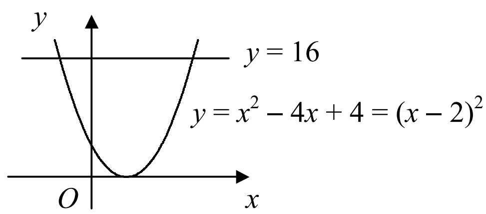
area below line

$= \tfrac{1}{2} \times 4 \times 8 = 16$

area between line and curve

$= 26\tfrac{2}{3} - 16 = 10\tfrac{2}{3}$

b



$x^2 - 4x + 4 = 16$

$x^2 - 4x - 12 = 0$

$(x + 2)(x - 6) = 0$

$x = -2, 6$

$\therefore$ intersect at $(-2, 16)$ and $(6, 16)$

area below curve

$= \int_{-2}^6 (x^2 - 4x + 4)\ dx$

$= [\tfrac{1}{3}x^3 - 2x^2 + 4x]_{-2}^6$

$= (72 - 72 + 24) - (-\tfrac{8}{3} - 8 - 8)$

$= 42\tfrac{2}{3}$

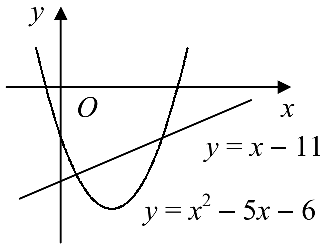
area below line

$= 8 \times 16 = 128$

area between line and curve

$= 128 - 42\tfrac{2}{3} = 85\tfrac{1}{3}$

c $y = x^2 - 5x - 6 \Rightarrow y = (x + 1)(x - 6)$



$x^2 - 5x - 6 = x - 11$

$x^2 - 6x + 5 = 0$

$(x - 1)(x - 5) = 0$
$x = 1, 5$
$\therefore$ intersect at (1, −10) and (5, −6)
area above curve

$= -\int_1^5 (x^2 - 5x - 6)\ dx$

$= -[\frac{1}{3}x^3 - \frac{5}{2}x^2 - 6x]_1^5$

$= -[(\frac{125}{3} - \frac{125}{2} - 30) - (\frac{1}{3} - \frac{5}{2} - 6)]$

$= 42\frac{2}{3}$

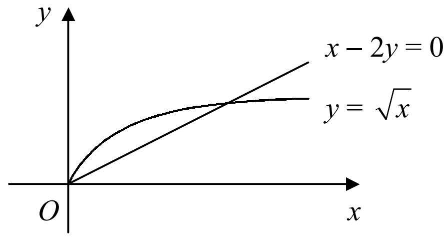
area above line

$= \frac{1}{2} \times 4 \times (10 + 6) = 32$

area between line and curve

$= 42\frac{2}{3} - 32 = 10\frac{2}{3}$

d $x - 2y = 0 \Rightarrow y = \frac{1}{2}x$



$x^{\frac{1}{2}} = \frac{1}{2}x$

$\frac{1}{2}x^{\frac{1}{2}}(2 - x^{\frac{1}{2}}) = 0$

$x^{\frac{1}{2}} = 0, 2$
$x = 0, 4$
$\therefore$ intersect at (0, 0) and (4, 2)
area below curve

$= \int_0^4 x^{\frac{1}{2}}\ dx$

$= [\frac{2}{3}x^{\frac{3}{2}}]_0^4$

$= \frac{16}{3} - 0 = \frac{16}{3}$

area below line

$= \frac{1}{2} \times 4 \times 2 = 4$

area between line and curve

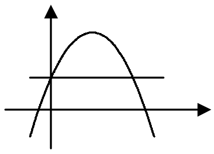
$= \frac{16}{3} - 4 = \frac{4}{3}$

C2 INTEGRATION *Answers - Worksheet B*

1 **a** $f(x) = -[x^2 - 4x] + 3$
$= -[(x-2)^2 - 4] + 3$
$= -(x-2)^2 + 7$
$\therefore\ a = -1,\ b = -2,\ c = 7$

b (2, 7)

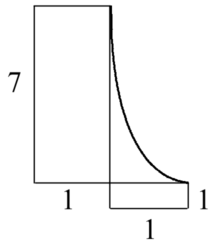
c intersect when
$3 + 4x - x^2 = 3$
$x(4 - x) = 0$
$x = 0, 4$



area below curve
$= \int_0^4 (3 + 4x - x^2)\ dx$
$= [3x + 2x^2 - \frac{1}{3}x^3]_0^4$
$= (12 + 32 - \frac{64}{3}) - 0 = \frac{68}{3}$
area below line
$= 4 \times 3 = 12$
area between line and curve
$= \frac{68}{3} - 12 = 10\frac{2}{3}$

2 **a** $= [-4x^{-2}]_1^2$
$= -1 - (-4)$
$= 3$

b $y = 1 \Rightarrow x = 2$
$y = 8 \Rightarrow x = 1$



shaded area
$= 3 - (1 \times 1) + (7 \times 1)$
$= 9$

3 **a** $\frac{dy}{dx} = 5 - 4x$
grad = 1
$\therefore$ grad of normal = -1
$\therefore\ y - 3 = -(x - 1)$
$[y = 4 - x]$

b area below curve
$= \int_0^1 (5x - 2x^2)\ dx$
$= [\frac{5}{2}x^2 - \frac{2}{3}x^3]_0^1$
$= (\frac{5}{2} - \frac{2}{3}) - 0 = \frac{11}{6}$
normal meets y-axis at (0, 4)
area below line
$= \frac{1}{2} \times 1 \times (4 + 3) = \frac{7}{2}$
shaded area
$= \frac{7}{2} - \frac{11}{6} = \frac{5}{3}$

4 **a** $\frac{4 - x^2}{x^2} = 0$
$4 - x^2 = 0$
$x^2 = 4$
$x > 0\ \therefore\ x = 2,\ P\ (2, 0)$

b l: $y - 0 = -3(x - 2)$
$y = 6 - 3x$
intersect when $\frac{4 - x^2}{x^2} = 6 - 3x$
$4 - x^2 = 6x^2 - 3x^3$
$3x^3 - 7x^2 + 4 = 0$
$x = 2$ is a solution $\therefore\ (x - 2)$ is a factor
$(x - 2)(3x^2 - x - 2) = 0$
$(x - 2)(3x + 2)(x - 1) = 0$
$x = 2$ (at P), $-\frac{2}{3}$, 1
$x > 0\ \therefore\ Q\ (1, 3)$

c area below curve
$= \int_1^2 (4x^{-2} - 1)\ dx$
$= [-4x^{-1} - x]_1^2$
$= (-2 - 2) - (-4 - 1) = 1$
area below line
$= \frac{1}{2} \times 1 \times 3 = \frac{3}{2}$
area between line and curve
$= \frac{3}{2} - 1 = \frac{1}{2}$

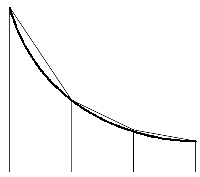
C2 INTEGRATION *Answers - Worksheet C*

1 **a**

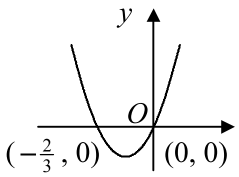
x	1	2	3	4
y	3	$\frac{3}{2}$	1	$\frac{3}{4}$

b $= \frac{1}{2} \times 1 \times [3 + \frac{3}{4} + 2(\frac{3}{2} + 1)] = 4\frac{3}{8}$

c the true area is less
the curve passes below the top of each trapezium as shown:



2 **a**



b

x	0	0.5	1	1.5	2
$x(3x + 2)$	0	1.75	5	9.75	16

area $\approx \frac{1}{2} \times 0.5 \times [0 + 16 + 2(1.75 + 5 + 9.75)] = 12.25$

c $= \int_0^2 (3x^2 + 2x)\ dx = [x^3 + x^2]_0^2 = (8 + 4) - 0 = 12$

d % error $= \frac{12.25 - 12}{12} \times 100\% = 2.08\%$ (3sf)

3 **a**

x	4	5	6
$\frac{3}{2x+1}$	$\frac{1}{3}$	$\frac{3}{11}$	$\frac{3}{13}$

$\therefore$ area $\approx \frac{1}{2} \times 1 \times [\frac{1}{3} + \frac{3}{13} + 2(\frac{3}{11})] = 0.555$ (3sf)

b

x	0	1	2	3
$\lg(x^2 + 9)$	lg 9	lg 10	lg 13	lg 18

$\therefore$ area $\approx \frac{1}{2} \times 1 \times [\lg 9 + \lg 18 + 2(\lg 10 + \lg 13)] = 3.22$ (3sf)

c

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$x^2 \sin x$	0	0.436	2.467	3.926	0

$\therefore$ area $\approx \frac{1}{2} \times \frac{\pi}{4} \times [0 + 0 + 2(0.436 + 2.467 + 3.926)] = 5.36$ (3sf)

d

x	−2	−1	0	1	2
$\sqrt[3]{2x+5}$	1	$\sqrt[3]{3}$	$\sqrt[3]{5}$	$\sqrt[3]{7}$	$\sqrt[3]{9}$

$\therefore$ area $\approx \frac{1}{2} \times 1 \times [1 + \sqrt[3]{9} + 2(\sqrt[3]{3} + \sqrt[3]{5} + \sqrt[3]{7})] = 6.61$ (3sf)

4 **a**

x	0	1	2	3
3^x	1	3	9	27

$\therefore$ area $\approx \frac{1}{2} \times 1 \times [1 + 27 + 2(3 + 9)] = 26$

b

x	2	2.2	2.4
$\sin(\lg x)$	0.2965	0.3358	0.3711

$\therefore$ area $\approx \frac{1}{2} \times 0.2 \times [0.2965 + 0.3711 + 2(0.3358)] = 0.134$ (3sf)

c

x	0	0.1	0.2	0.3	0.4	0.5
$\frac{x}{x^3+2}$	0	0.04998	0.09960	0.14800	0.19380	0.23529

$\therefore$ area $\approx \frac{1}{2} \times 0.1 \times [0 + 0.23529 + 2(0.04998 + 0.09960 + 0.14800 + 0.19380)] = 0.0609$ (3sf)

d

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$
$\sqrt{\cos(\frac{1}{2}x)}$	1	0.9828	0.9306	0.8409	0.7071

$\therefore$ area $\approx \frac{1}{2} \times \frac{\pi}{6} \times [1 + 0.7071 + 2(0.9828 + 0.9306 + 0.8409)] = 1.89$ (3sf)

5 **a**

x	3	3.8	4.6	5.4	6.2	7
$2 - 3x^{-\frac{1}{2}}$	0.2679	0.4610	0.6012	0.7090	0.7952	0.8661

area $\approx \frac{1}{2} \times 0.8 \times [0.2679 + 0.8661 + 2(0.4610 + 0.6012 + 0.7090 + 0.7952)] = 2.51$ (3sf)

b $= \int_3^7 (2 - 3x^{-\frac{1}{2}})\ dx$

$= [2x - 6x^{\frac{1}{2}}]_3^7 = (14 - 6\sqrt{7}) - (6 - 6\sqrt{3}) = 8 + 6(\sqrt{3} - \sqrt{7})$

6 **a**

x	0	0.2	0.4	0.6	0.8	1
$\sin x^2$	0	0.03999	0.15932	0.35227	0.59720	0.84147

area $\approx \frac{1}{2} \times 0.2 \times [0 + 0.84147 + 2(0.03999 + 0.15932 + 0.35227 + 0.59720)] = 0.3139$

b area of rectangle $= 1 \times 0.8415 = 0.8415$
$\therefore$ area of flower bed $\approx 0.8415 - 0.3139 = 0.53\ m^2$

7 **a** $= 1 + 6(\frac{x}{2}) + \frac{6\times5}{2}(\frac{x}{2})^2 + \frac{6\times5\times4}{3\times2}(\frac{x}{2})^3 + \ldots$

$= 1 + 3x + \frac{15}{4}x^2 + \frac{5}{2}x^3 + \ldots$

b area $\approx \int_0^{0.5} (1 + 3x + \frac{15}{4}x^2 + \frac{5}{2}x^3)\ dx$

$= [x + \frac{3}{2}x^2 + \frac{5}{4}x^3 + \frac{5}{8}x^4]_0^{0.5}$

$= (\frac{1}{2} + \frac{3}{8} + \frac{5}{32} + \frac{5}{128}) - 0 = 1.07$ (3sf)

c

x	0	0.1	0.2	0.3	0.4	0.5
$(1 + \frac{x}{2})^6$	1	1.3401	1.7716	2.3131	2.9860	3.8147

area $\approx \frac{1}{2} \times 0.1 \times [1 + 3.8147 + 2(1.3401 + 1.7716 + 2.3131 + 2.9860)] = 1.08$ (3sf)

8 **a** $\frac{dy}{dx} = 2x - 16x^{-2}$

SP: $2x - 16x^{-2} = 0$

$x^3 = 8$

$x = 2$

when $x = 2$, $y = 4 + 8 = 12$ $\therefore$ SP (2, 12)

b

x	2	2.5	3	3.5	4
$x^2 + \frac{16}{x}$	12	12.65	14.333	16.821	20

area $\approx \frac{1}{2} \times 0.5 \times [12 + 20 + 2(12.65 + 14.333 + 16.821)] = 29.9$ (3sf)

c over-estimate

C2 INTEGRATION *Answers - Worksheet D*

1 **a** $= [-2x^{-1}]_1^4$

$= -\frac{1}{2} - (-2)$

$= \frac{3}{2}$

b $= \int_0^2 (x^2 - 6x + 9)\ dx$

$= [\frac{1}{3}x^3 - 3x^2 + 9x]_0^2$

$= (\frac{8}{3} - 12 + 18) - 0$

$= 8\frac{2}{3}$

2 **a**

x	0	2	4	6
$\sqrt{x^2+4}$	2	$\sqrt{8}$	$\sqrt{20}$	$\sqrt{40}$

area $\approx \frac{1}{2} \times 2 \times [2 + \sqrt{40} + 2(\sqrt{8} + \sqrt{20})]$

$= 22.9$ (3sf)

b over-estimate

curve passes below top of each trapezium

3 **a** $= 3\sqrt{2} - \frac{1}{\sqrt{2}}$

$= 3\sqrt{2} - \frac{1}{2}\sqrt{2}$

$= \frac{5}{2}\sqrt{2}$

b $\int_3^4 (3x^{\frac{1}{2}} - x^{-\frac{1}{2}})\ dx$

$= [2x^{\frac{3}{2}} - 2x^{\frac{1}{2}}]_3^4$

$= [16 - 4] - [(2 \times 3\sqrt{3}) - 2\sqrt{3}]$

$= 12 - 4\sqrt{3}$

4 **a** $4x^{\frac{1}{2}} - x^{\frac{3}{2}} = 0$

$x^{\frac{1}{2}}(4 - x) = 0$

$x^{\frac{1}{2}} = 0$ [$\Rightarrow x = 0$, at O] or $x = 4$

$\therefore A(4, 0)$

b $\frac{dy}{dx} = 2x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}}$

SP: $2x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} = 0$

$\frac{1}{2}x^{-\frac{1}{2}}(4 - 3x) = 0$

$x^{-\frac{1}{2}} = 0 \Rightarrow$ no solutions

$\therefore x = \frac{4}{3}$ at B

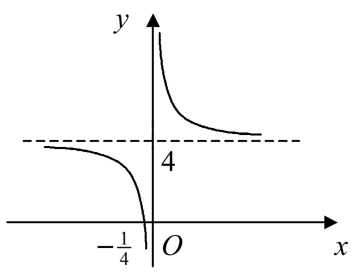
c $= \int_0^4 (4x^{\frac{1}{2}} - x^{\frac{3}{2}})\ dx$

$= [\frac{8}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}}]_0^4$

$= (\frac{64}{3} - \frac{64}{5}) - 0 = 8\frac{8}{15}$

5 **a** $p = -\frac{1}{4}$, $q = 4$

b



c

x	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3
$4 + \frac{1}{x}$	5	$4\frac{2}{3}$	$4\frac{1}{2}$	$4\frac{2}{5}$	$4\frac{1}{3}$

area $\approx \frac{1}{2} \times \frac{1}{2} \times [5 + 4\frac{1}{3} + 2(4\frac{2}{3} + 4\frac{1}{2} + 4\frac{2}{5})]$

$= 9\frac{7}{60}$ or 9.12 (3sf)

6 **a** $4x - y + 11 = 0 \Rightarrow y = 4x + 11$

intersect when $2x^2 + 6x + 7 = 4x + 11$

$x^2 + x - 2 = 0$

$(x + 2)(x - 1) = 0$

$x = -2, 1$

$\therefore (-2, 3)$ and $(1, 15)$

b area below curve

$= \int_{-2}^1 (2x^2 + 6x + 7)\ dx$

$= [\frac{2}{3}x^3 + 3x^2 + 7x]_{-2}^1$

$= (\frac{2}{3} + 3 + 7) - (-\frac{16}{3} + 12 - 14) = 18$

area below line

$= \frac{1}{2} \times 3 \times (3 + 15) = 27$

area between line and curve

$= 27 - 18 = 9$

7 **a** minimum when $\sin x = 1$

$\therefore\ x = \frac{\pi}{2}$

$\therefore\ (\frac{\pi}{2}, \frac{1}{2})$

b

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$
$\frac{1}{1+\sin x}$	1	0.6667	0.5359

$\therefore$ area $\approx \frac{1}{2} \times \frac{\pi}{6} \times [1 + 0.5359 + 2(0.6667)] = 0.751$ (3sf)

8 **a** $= 1 + 12(\frac{x}{10}) + \frac{12\times11}{2}(\frac{x}{10})^2 + \frac{12\times11\times10}{3\times2}(\frac{x}{10})^3 + \ldots$

$= 1 + \frac{6}{5}x + \frac{33}{50}x^2 + \frac{11}{50}x^3 + \ldots$

b $\approx \int_0^1 (1 + \frac{6}{5}x + \frac{33}{50}x^2 + \frac{11}{50}x^3)\ dx$

$= [x + \frac{3}{5}x^2 + \frac{11}{50}x^3 + \frac{11}{200}x^4]_0^1$

$= (1 + \frac{3}{5} + \frac{11}{50} + \frac{11}{200}) - 0 = 1\frac{7}{8}$

9 **a** at A, $x = 0 \Rightarrow (0, 2)$

$\frac{dy}{dx} = -1 - 2x$

grad at $A = -1$

$\therefore\ y = 2 - x$

b curve cuts x-axis when $y = 0$

$2 - x - x^2 = 0$

$(2 + x)(1 - x) = 0$

$x = -2, 1$

area below curve

$= \int_0^1 (2 - x - x^2)\ dx$

$= [2x - \frac{1}{2}x^2 - \frac{1}{3}x^3]_0^1$

$= (2 - \frac{1}{2} - \frac{1}{3}) - 0 = \frac{7}{6}$

tangent cuts x-axis when $y = 0$

$x = 2$

area below line

$= \frac{1}{2} \times 2 \times 2 = 2$

shaded area

$= 2 - \frac{7}{6}$

$= \frac{5}{6}$

C2 INTEGRATION *Answers - Worksheet E*

1 **a** at A, $x = 0$ $\therefore$ $A\ (0, 4)$

at B, $y = 0$

$(x^{\frac{1}{2}} - 2)^2 = 0$

$x^{\frac{1}{2}} = 2$

$x = 4$ $\therefore$ $B\ (4, 0)$

b $= \int_0^4 (x - 4x^{\frac{1}{2}} + 4)\ dx$

$= [\frac{1}{2}x^2 - \frac{8}{3}x^{\frac{3}{2}} + 4x]_0^4$

$= (8 - \frac{64}{3} + 16) - 0$

$= \frac{8}{3}$

2 $= \int_1^2 (\frac{3}{2}x + \frac{1}{2}x^{-2})\ dx$

$= [\frac{3}{4}x^2 - \frac{1}{2}x^{-1}]_1^2$

$= (3 - \frac{1}{4}) - (\frac{3}{4} - \frac{1}{2})$

$= \frac{5}{2}$

3 **a** $4^{x+1} = 32$

$(2^2)^{x+1} = 2^5$

$2x + 2 = 5$

$x = \frac{3}{2}$

b

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$
4^{x+1}	4	8	16	32

$\therefore$ area $\approx \frac{1}{2} \times \frac{1}{2} \times [4 + 32 + 2(8 + 16)]$

$= 21$

4 **a** at A, $x^2 - 2x = 0$

$x(x - 2) = 0$

$x = 0$ (at O) or 2 $\therefore$ $A\ (2, 0)$

at B, $x^2 - 2x = x$

$x(x - 3) = 0$

$x = 0$ (at O) or 3 $\therefore$ $B\ (3, 3)$

b $\int_0^2 (x^2 - 2x)\ dx$

$= [\frac{1}{3}x^3 - x^2]_0^2$

$= (\frac{8}{3} - 4) - 0 = -\frac{4}{3}$

$\therefore$ area $= \frac{4}{3}$

c area below curve between A and B

$= \int_2^3 (x^2 - 2x)\ dx$

$= [\frac{1}{3}x^3 - x^2]_2^3$

$= (9 - 9) - (-\frac{4}{3}) = \frac{4}{3}$

area below straight line OB

$= \frac{1}{2} \times 3 \times 3 = \frac{9}{2}$

area between curve and line

$= \frac{9}{2} - \frac{4}{3} + \frac{4}{3}$

$= \frac{9}{2}$

5 **a**

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
y	1	1.319	1.024	0

b $\approx \frac{1}{2}\times\frac{\pi}{6}\times[1 + 0 + 2(1.319 + 1.024)]$

$= 1.49$ (3sf)

c under-estimate

curve passes above top of each trapezium

6 $\int_1^k (3 - 4x^{-2})\ dx$

$= [3x + 4x^{-1}]_1^k$

$= (3k + \frac{4}{k}) - (3 + 4)$

$\therefore\ 3k + \frac{4}{k} - 7 = 6$

$3k^2 - 13k + 4 = 0$

$(3k - 1)(k - 4) = 0$

$k > 1\ \therefore\ k = 4$

7 **a** $\frac{dy}{dx} = 3x^2 - 6x$

SP: $3x^2 - 6x = 0$

$3x(x - 2) = 0$

$x = 0$ (at P) or 2

$\therefore\ Q\,(2, 1)$

b $x^3 - 3x^2 + 5 = 5$

$x^2(x - 3) = 0$

$x = 0$ (at P) or 3

$\therefore\ R\,(3, 5)$

c area below curve

$= \int_0^3 (x^3 - 3x^2 + 5)\ dx$

$= [\frac{1}{4}x^4 - x^3 + 5x]_0^3$

$= (\frac{81}{4} - 27 + 15) - 0 = \frac{33}{4}$

area below line

$= 3 \times 5 = 15$

shaded area

$= 15 - \frac{33}{4}$

$= 6\frac{3}{4}$

8 **a** (2, 0)

b

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$(2 - x)^3$	8	$\frac{27}{8}$	1	$\frac{1}{8}$	0

area $\approx \frac{1}{2}\times\frac{1}{2}\times[8 + 0 + 2(\frac{27}{8} + 1 + \frac{1}{8})]$

$= 4\frac{1}{4}$

c $= 2^3 + 3(2^2)(-x) + 3(2)(-x)^2 + (-x)^3$

$= 8 - 12x + 6x^2 - x^3$

d area $= \int_0^2 (8 - 12x + 6x^2 - x^3)\ dx$

$= [8x - 6x^2 + 2x^3 - \frac{1}{4}x^4]_0^2$

$= (16 - 24 + 16 - 4) - 0$

$= 4$

$\therefore$ % error $= \frac{4\frac{1}{4} - 4}{4} \times 100\% = 6.25\%$