C2 INTEGRATION — Worksheet A

1 Evaluate

a $\int_1^3 (4x - 1)\ dx$ **b** $\int_0^1 (3x^2 + 2)\ dx$ **c** $\int_0^3 (x - x^2)\ dx$

d $\int_2^3 (3x + 1)^2\ dx$ **e** $\int_1^2 (x^2 - 8x - 3)\ dx$ **f** $\int_{-2}^4 (8 - 4x + 3x^2)\ dx$

g $\int_1^4 (x^3 - 2x - 7)\ dx$ **h** $\int_{-2}^{-1} (5 + x^2 - 4x^3)\ dx$ **i** $\int_{-1}^2 (x^4 + 6x^2 - x)\ dx$

2 Given that $\int_1^4 (3x^2 + ax - 5)\ dx = 18$, find the value of the constant a.

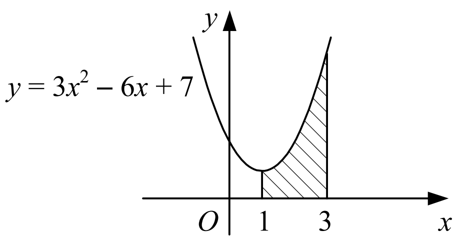
3 Given that $\int_{-1}^k (3x^2 - 12x + 9)\ dx = 16$, find the value of the non-zero constant k.

4 Evaluate

a $\int_1^3 (2 - \frac{1}{x^2})\ dx$ **b** $\int_{-2}^{-1} (6x + \frac{4}{x^3})\ dx$ **c** $\int_1^4 (3x^{\frac{1}{2}} - 4)\ dx$

d $\int_{-1}^2 \frac{4x^4 - x}{2x}\ dx$ **e** $\int_1^8 (x - x^{-\frac{1}{3}})\ dx$ **f** $\int_2^3 \frac{1 - 6x^3}{3x^2}\ dx$

5



The diagram shows the curve with the equation $y = 3x^2 - 6x + 7$.

Find the area of the shaded region enclosed by the curve, the x-axis and the lines $x = 1$ and $x = 3$.

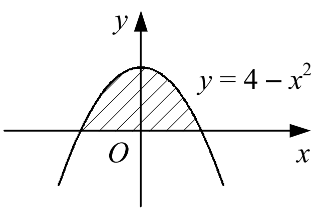
6 Find the area of the region enclosed by the curve $y = f(x)$, the x-axis and the given ordinates.
In each case, $f(x) > 0$ over the interval being considered.

a $f(x) \equiv x^2 + 2$, $x = 0$, $x = 2$ **b** $f(x) \equiv 3x^2 + 8x + 6$, $x = -2$, $x = 1$

c $f(x) \equiv 9 + 2x - x^2$, $x = 2$, $x = 4$ **d** $f(x) \equiv x^3 - 4x + 1$, $x = -1$, $x = 0$

e $f(x) \equiv 2x + 3x^{\frac{1}{2}}$, $x = 1$, $x = 4$ **f** $f(x) \equiv 3 + \frac{5}{x^2}$, $x = -5$, $x = -1$

7



The diagram shows the curve with the equation $y = 4 - x^2$.

a Find the coordinates of the points where the curve crosses the x-axis.

b Find the area of the shaded region enclosed by the curve and the x-axis.

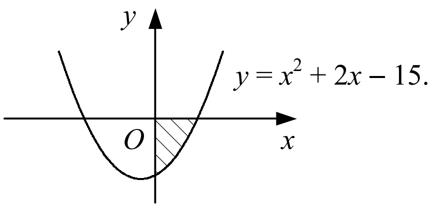
8 In each part of this question, sketch the given curve and find the area of the region enclosed by the curve and the x-axis.

a $y = 6x - 3x^2$ **b** $y = -x^2 + 4x - 3$ **c** $y = 4 - 3x - x^2$ **d** $y = 2x^{\frac{1}{2}} - x$

9 **a** Sketch the curve with the equation $y = x^2 + 4x$.

b Find the area of the region enclosed by the curve, the x-axis and the line $x = 2$.

10



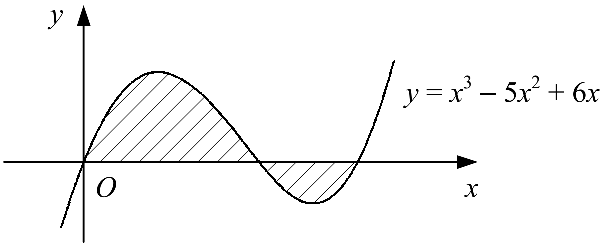
The diagram shows the curve with the equation $y = x^2 + 2x - 15$.

a Find the coordinates of the points where the curve crosses the x-axis.

b Evaluate the integral $\int_0^3 (x^2 + 2x - 15)\ \mathrm{d}x$.

c State the area of the shaded region enclosed by the curve, the y-axis and the positive x-axis.

11

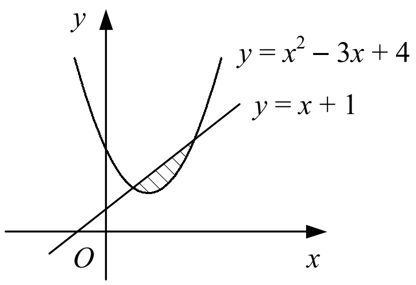


The diagram shows the curve with the equation $y = x^3 - 5x^2 + 6x$.

a Find the coordinates of the points where the curve crosses the x-axis.

b Show that the total area of the shaded regions enclosed by the curve and the x-axis is $3\frac{1}{12}$.

12



The diagram shows the curve $y = x^2 - 3x + 4$ and the straight line $y = x + 1$.

a Find the coordinates of the points where the curve and line intersect.

b Find the area of the shaded region enclosed by the curve and the line.

13 In each part of this question sketch the given curve and line on the same set of coordinate axes and find the area of the region enclosed by the curve and line.

a $y = 9 - x^2$ and $y = 6 - 2x$ **b** $y = x^2 - 4x + 4$ and $y = 16$

c $y = x^2 - 5x - 6$ and $y = x - 11$ **d** $y = \sqrt{x}$ and $x - 2y = 0$

C2 INTEGRATION Worksheet B

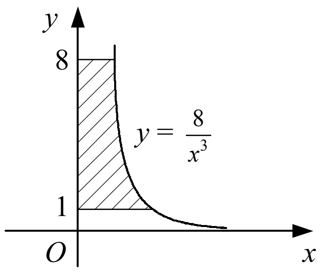
1 $$f(x) \equiv 3 + 4x - x^2.$$

a Express f(x) in the form $a(x + b)^2 + c$, stating the values of the constants a, b and c.

b State the coordinates of the turning point of the curve $y = f(x)$.

c Find the area of the region enclosed by the curve $y = f(x)$ and the line $y = 3$.

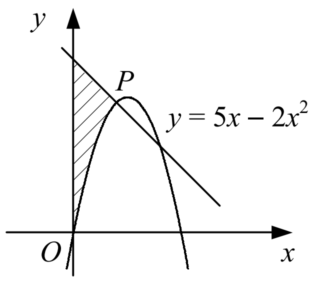
2 **a** Evaluate $\int_1^2 \frac{8}{x^3} \, dx$.



The diagram shows the curve with the equation $y = \frac{8}{x^3}$, $x > 0$.

b Using your answer to part **a**, find the area of the shaded region bounded by the curve, the lines $y = 1$ and $y = 8$ and the y-axis.

3



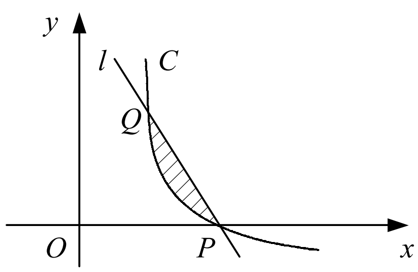
The diagram shows the curve $y = 5x - 2x^2$ and the normal to the curve at the point P (1, 3).

a Find an equation of the normal to the curve at P.

The shaded region is bounded by the curve, the normal to the curve at P and the y-axis.

b Show that the area of the shaded region is $\frac{5}{3}$.

4



The diagram shows the curve C with the equation $y = \frac{4 - x^2}{x^2}$, $x > 0$, and the straight line l.

a Find the coordinates of the point P where C crosses the x-axis.

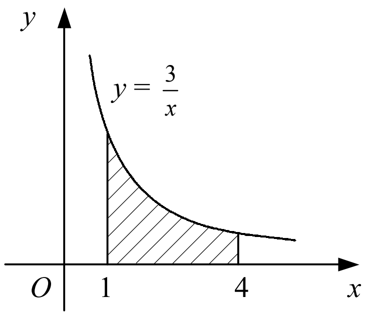
The line l has gradient -3 and intersects C at the points P and Q.

b Find the coordinates of the point Q.

c Show that the area of the shaded region enclosed by C and l is $\frac{1}{2}$.

C2 INTEGRATION Worksheet C

1



The diagram shows the curve with equation $y = \frac{3}{x}$, $x > 0$.

a Copy and complete the table below, giving the exact y-coordinate corresponding to each x-coordinate for points on the curve.

x	1	2	3	4
y				

The shaded region is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 4$.

b Use the trapezium rule with all the values in your table to show that the area of the shaded region is approximately $4\frac{3}{8}$.

c With the aid of a sketch diagram, explain whether the true area is more or less than $4\frac{3}{8}$.

2 **a** Sketch the curve $y = x(3x + 2)$ showing the coordinates of any points of intersection with the coordinate axes.

b Use the trapezium rule with 4 intervals of equal width to estimate the area bounded by the curve, the x-axis and the line $x = 2$.

c Find this area exactly using integration.

d Hence, find the percentage error in the estimate made in part **b**.

3 Use the trapezium rule with the stated number of intervals of equal width to estimate the area of the region enclosed by the given curve, the x-axis and the given ordinates.

a $y = \frac{3}{2x+1}$ $x = 4$ $x = 6$ 2 intervals

b $y = \lg (x^2 + 9)$ $x = 0$ $x = 3$ 3 intervals

c $y = x^2 \sin x$ $x = 0$ $x = \pi$ 4 intervals

d $y = \sqrt[3]{2x+5}$ $x = -2$ $x = 2$ 4 intervals

4 Use the trapezium rule with the stated number of equally-spaced ordinates to estimate the area of the region enclosed by the given curve, the x-axis and the given ordinates.

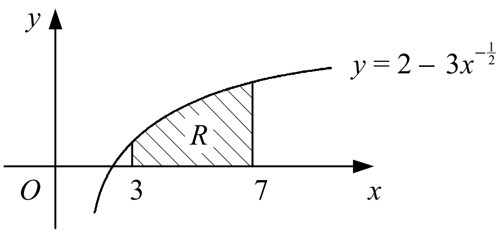
a $y = 3^x$ $x = 0$ $x = 3$ 4 ordinates

b $y = \sin (\lg x)$ $x = 2$ $x = 2.4$ 3 ordinates

c $y = \frac{x}{x^3 + 2}$ $x = 0$ $x = 0.5$ 6 ordinates

d $y = \sqrt{\cos (\frac{1}{2}x)}$ $x = 0$ $x = \frac{2\pi}{3}$ 5 ordinates

5

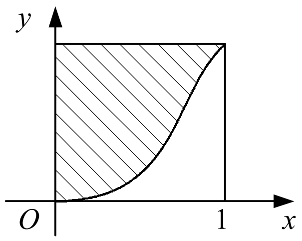


The diagram shows the finite region, R, which is bounded by the curve $y = 2 - 3x^{-\frac{1}{2}}$, the x-axis and the lines $x = 3$ and $x = 7$.

a Use the trapezium rule with 5 intervals of equal width to estimate the area of R.

b Use integration to find the exact area of R.

6



The diagram shows the curve $y = \sin x^2$, $0 \leq x \leq 1$ and the lines $x = 1$ and $y = \sin 1$.

a Use the trapezium rule with 5 strips of equal width to estimate the area bounded by the curve $y = \sin x^2$, the x-axis and the line $x = 1$, giving your answer to 4 decimal places.

The shaded region on the diagram is bounded by the curve, the y-axis and the line $y = \sin 1$. A flower bed is modelled by the shaded region, with the units on the axes in metres.

b Calculate an estimate for the area of the flower bed, correct to 2 significant figures.

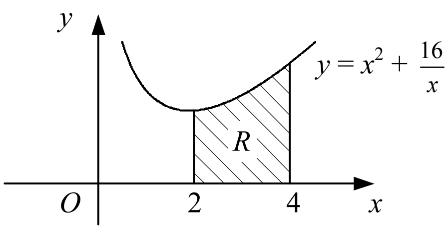
7 **a** Use the binomial theorem to expand $(1 + \frac{x}{2})^6$ in ascending powers of x up to and including the term in x^3.

The finite region R is bounded by the curve $y = (1 + \frac{x}{2})^6$, the coordinate axes and the line $x = 0.5$

b Use your expression in **a** and integration to find an estimate for the area of R.

c Use the trapezium rule with 6 equally-spaced ordinates to find another estimate for the area of R.

8



The diagram shows the curve $y = x^2 + \frac{16}{x}$ for $x > 0$.

a Show that the stationary point on the curve has coordinates (2, 12).

The region R is bounded by the curve $y = x^2 + \frac{16}{x}$, the x-axis and the lines $x = 2$ and $x = 4$.

b Use the trapezium rule with 4 strips of equal width to estimate the area of R.

c State whether your answer to **b** is an under-estimate or an over-estimate of the area of R.

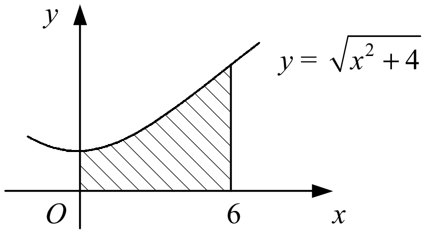
C2 INTEGRATION Worksheet D

1 Evaluate

a $\int_1^4 \frac{2}{x^2}\ dx$, **(3)**

b $\int_0^2 (x-3)^2\ dx$. **(4)**

2



The shaded region in the diagram is bounded by the curve $y = \sqrt{x^2+4}$, the x-axis and the lines $x = 0$ and $x = 6$.

a Use the trapezium rule with three intervals of equal width to estimate the area of the shaded region. **(5)**

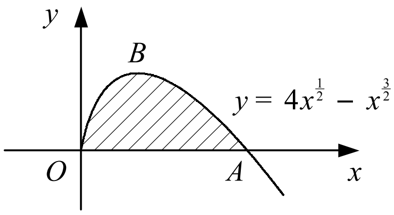
b State, with a reason, whether your answer to part **a** is an under-estimate or an over-estimate of the true area. **(2)**

3 $$\mathrm{f}(x) \equiv 3x^{\frac{1}{2}} - x^{-\frac{1}{2}}.$$

a Find the value of f(2), giving your answer in the form $k\sqrt{2}$ where k is an exact fraction. **(2)**

b Show that $\int_3^4 \mathrm{f}(x)\ dx = 12 - 4\sqrt{3}$. **(4)**

4



The diagram shows the curve with the equation $y = 4x^{\frac{1}{2}} - x^{\frac{3}{2}}$.

The curve meets the x-axis at the origin, O, and at the point A.

a Find the coordinates of the point A. **(2)**

The curve has a maximum at the point B.

b Find the x-coordinate of the point B. **(5)**

c Find the area of the shaded region enclosed by the curve and the x-axis. **(4)**

5 The curve $y = 4 + \frac{1}{x}$ crosses the x-axis at the point $(p, 0)$ and has an asymptote $y = q$.

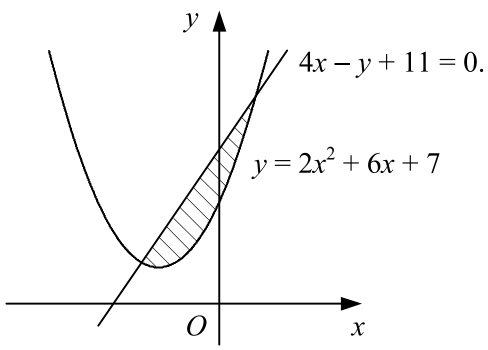
a Write down the values of p and q. **(2)**

b Sketch the curve. **(2)**

The region R is bounded by the curve $y = 4 + \frac{1}{x}$, the x-axis and the lines $x = 1$ and $x = 3$.

c Use the trapezium rule with 5 equally-spaced ordinates to estimate the area of R. **(5)**

6

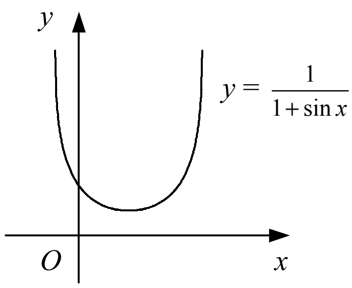


The diagram shows the curve with the equation $y = 2x^2 + 6x + 7$ and the straight line with the equation $4x - y + 11 = 0$.

a Find the coordinates of the points where the curve and line intersect. **(5)**

b Find the area of the shaded region enclosed by the curve and the line. **(6)**

7



The diagram shows the curve with equation $y = \frac{1}{1+\sin x}$, $-\frac{\pi}{2} < x < \frac{3\pi}{2}$.

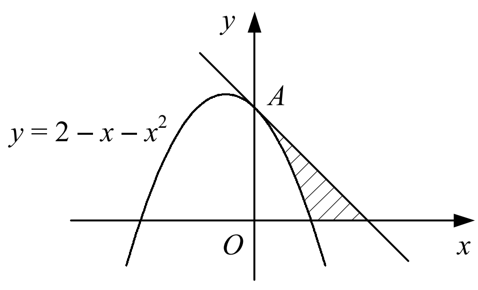
a Find the coordinates of the minimum point of the curve. **(3)**

b Use the trapezium rule with 2 intervals of equal width to estimate the area of the region bounded by the curve, the coordinate axes and the line $x = \frac{\pi}{3}$. **(5)**

8 **a** Expand $(1 + \frac{x}{10})^{12}$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient in the expansion. **(4)**

b Using your series expansion from part **a**, find an estimate for $\int_0^1 (1 + \frac{x}{10})^{12} \, dx$. **(5)**

9



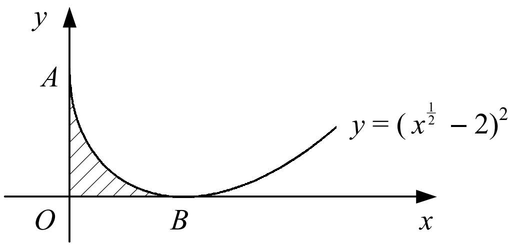
The diagram shows the curve with the equation $y = 2 - x - x^2$ and the tangent to the curve at the point A where it crosses the y-axis.

a Find an equation of the tangent to the curve at A. **(4)**

b Show that the area of the shaded region enclosed by the curve, the tangent to the curve at A and the x-axis is $\frac{5}{6}$. **(9)**

C2 INTEGRATION Worksheet E

1



The diagram shows the curve with the equation $y = (x^{\frac{1}{2}} - 2)^2$. The curve meets the y-axis at the point A and the x-axis at the point B.

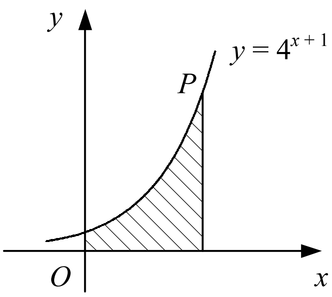
a Find the coordinates of the points A and B. **(3)**

b Find the area of the shaded region enclosed by the curve and the coordinate axes. **(6)**

2 Evaluate

$$\int_1^2 \frac{3x^3 + 1}{2x^2} \, dx.$$ **(5)**

3



The diagram shows the curve with equation $y = 4^{x+1}$.

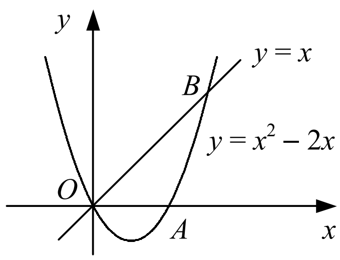
The point P on the curve has y-coordinate 32.

a Find the x-coordinate of P. **(3)**

The shaded region is bounded by the curve, the coordinate axes and the line through P parallel to the y-axis.

b Use the trapezium rule with 4 equally-spaced ordinates to estimate the area of the shaded region. **(5)**

4



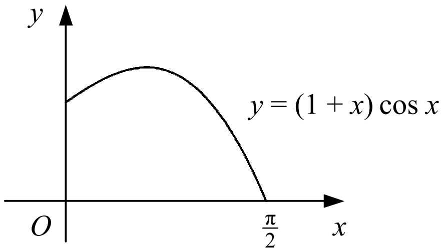
The diagram shows the curve $y = x^2 - 2x$ and the line $y = x$. The curve crosses the x-axis at the origin, O, and at the point A. The line intersects the curve at O and at the point B.

a Find the coordinates of the points A and B. **(4)**

b Find the area of the region enclosed by the curve and the x-axis. **(5)**

c Show that the area of the region enclosed by the curve and the line $y = x$ is $\frac{9}{2}$. **(5)**

5



The diagram shows the curve with equation $y = (1 + x)\cos x,\ 0 \leq x \leq \frac{\pi}{2}$.

a Copy and complete the table below for points on the curve, giving the y values correct to 3 decimal places where appropriate.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
y				

(3)

b Use the trapezium rule with the values in your table to estimate the area of the region bounded by the curve and the coordinate axes. **(4)**

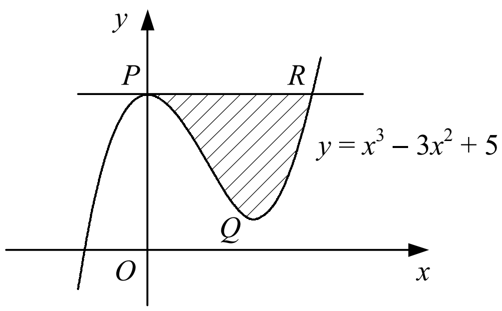
c State, with a reason, whether your answer to part **b** is an under-estimate or an over-estimate of the true area. **(2)**

6 Given that

$$\int_1^k \left(3 - \frac{4}{x^2}\right)\, dx = 6,$$

and that $k > 1$, find the value of the constant k. **(7)**

7



The diagram shows the curve with the equation $y = x^3 - 3x^2 + 5$. The curve is stationary at the point P (0, 5) and at the point Q.

a Find the coordinates of the point Q. **(5)**

The straight line passing through the point P parallel to the x-axis intersects the curve again at the point R.

b Find the coordinates of the point R. **(2)**

c Find the area of the shaded region enclosed by the curve and the straight line PR. **(7)**

8 The finite region R is bounded by the curve $y = (2 - x)^3$ and the coordinate axes.

a State the coordinates of the point where the curve crosses the x-axis. **(1)**

b Use the trapezium rule with 4 intervals of equal width to estimate the area of R. **(5)**

c Expand $(2 - x)^3$ in ascending powers of x. **(2)**

d Hence, using integration, find the percentage error in the estimate for the area of R found in part **b**. **(6)**