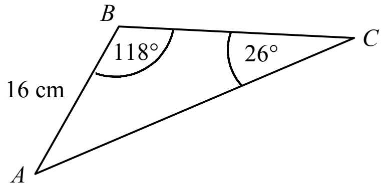
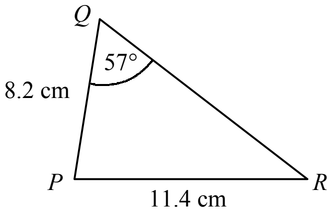
C2 TRIGONOMETRY Worksheet A

1



The diagram shows triangle *ABC* in which $AB = 16$ cm, $\angle ABC = 118°$ and $\angle ACB = 26°$.
Use the sine rule to find the length *AC* to 3 significant figures.

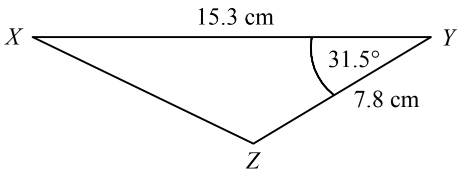
2



The diagram shows triangle *PQR* in which $PQ = 8.2$ cm, $PR = 11.4$ cm and $\angle PQR = 57°$.
Use the sine rule to find the size of $\angle PRQ$ in degrees to 1 decimal place.

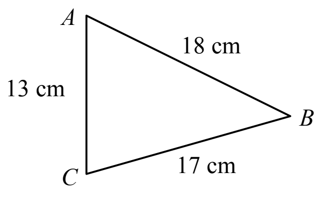
3 In triangle *ABC*, $AB = 16.2$ cm, $BC = 12.3$ cm and $\angle BAC = 37°$.
Find the two possible sizes of $\angle ACB$ and the corresponding lengths of *AC*.

4



The diagram shows triangle *XYZ* in which $XY = 15.3$ cm, $YZ = 7.8$ cm and $\angle XYZ = 31.5°$.
Use the cosine rule to find the length *XZ*.

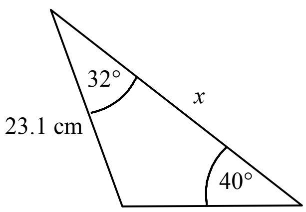
5



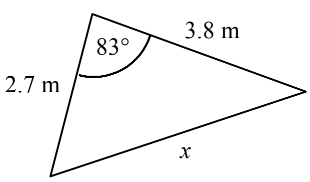
The diagram shows triangle *ABC* in which $AB = 18$ cm, $AC = 13$ cm and $BC = 17$ cm.
Use the cosine rule to find the size of $\angle ACB$.

6 Find the length x in each triangle.

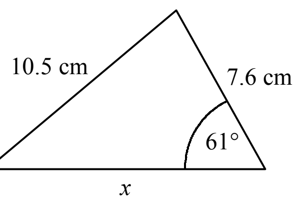
a



b

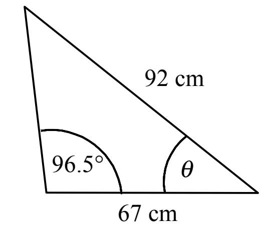


c

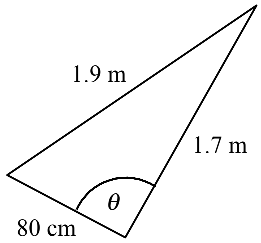


7 Find the angle θ in each triangle.

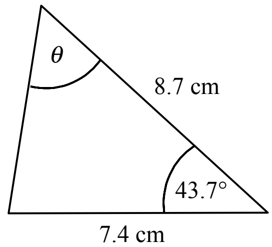
a



b

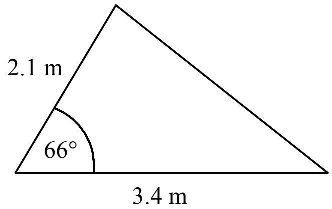


c

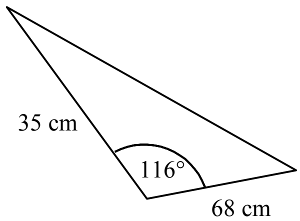


8 Find the area of each of the following triangles.

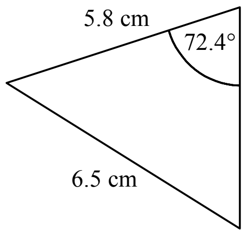
a



b



c



9 Joanne walks 4.2 miles on a bearing of 138°. She then walks 7.8 miles on a bearing of 251°.

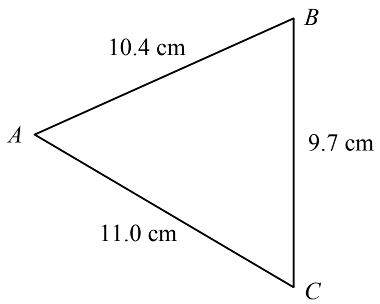
a Calculate how far Joanne is from the point where she started.

b Find, as a bearing, the direction in which Joanne would have to walk in order to return to the point where she started.

10 A ferry and a cargo ship are both approaching the same port. The ferry is 3.2 km from the port on a bearing of 076° and the cargo ship is 6.9 km from the port on a bearing of 323°.

Find the distance between the two vessels and the bearing of the cargo ship from the ferry.

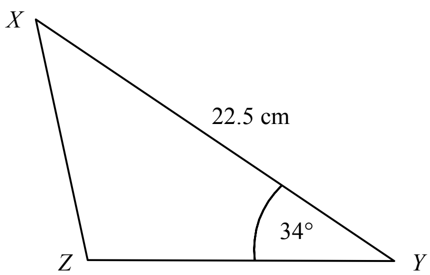
11



The diagram shows triangle ABC in which $AB = 10.4$ cm, $AC = 11.0$ cm and $BC = 9.7$ cm.

Find the area of the triangle to 3 significant figures.

12



The diagram shows triangle XYZ in which $XY = 22.5$ cm and $\angle XYZ = 34°$.

Given that the area of the triangle is 100 cm^2, find the length XZ.

C2 TRIGONOMETRY *Worksheet B*

1 Convert each angle from degrees to radians, giving your answers in terms of π.

a 180° **b** 30° **c** 45° **d** 720° **e** 18° **f** 120°

g 15° **h** 40° **i** 270° **j** 7.5° **k** 144° **l** 220°

2 Convert each angle from degrees to radians, giving your answers to 2 decimal places.

a 10° **b** 38° **c** 291° **d** 63.8° **e** 507° **f** 126.2°

3 Convert each angle from radians to degrees.

a 2π **b** $\frac{\pi}{3}$ **c** $\frac{\pi}{2}$ **d** $\frac{3\pi}{4}$ **e** $\frac{\pi}{18}$ **f** $\frac{\pi}{30}$

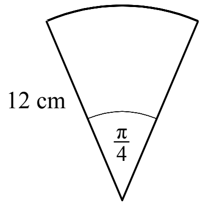
g $\frac{5\pi}{6}$ **h** $\frac{\pi}{8}$ **i** 3π **j** $\frac{2\pi}{15}$ **k** $\frac{7\pi}{3}$ **l** $\frac{9\pi}{20}$

4 Convert each angle from radians to degrees, giving your answers to 1 decimal place.

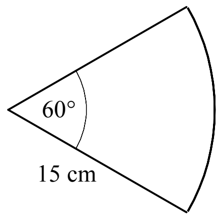
a 2^c **b** 0.5^c **c** 3.1^c **d** 1.43^c **e** 8.7^c **f** 0.742^c

5 Find, in terms of π, the length of the arc in each of the following circular sectors.

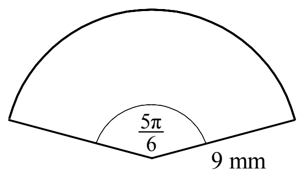
a



b

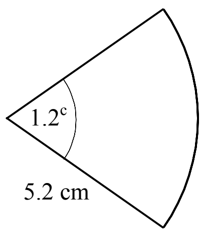


c

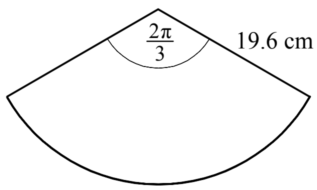


6 Find, to 3 significant figures, the perimeter of each of the following circular sectors.

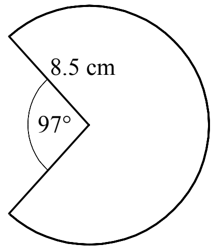
a



b

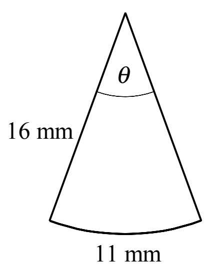


c

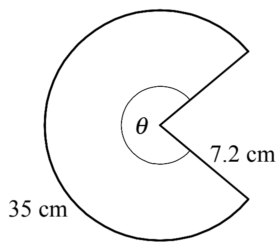


7 Find, in radians to 2 decimal places, the angle θ in each of the following circular sectors.

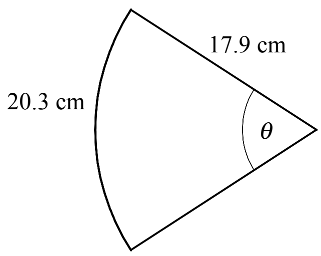
a



b



c



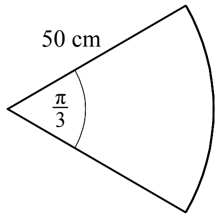
8 The minor arc AB of a circle, centre O, has length 46.2 cm.

Given that $\angle AOB = 78.5°$, find

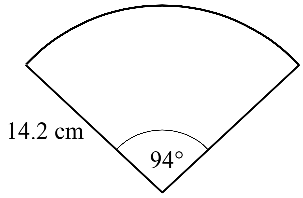
a the distance OA, **b** the perimeter of sector OAB.

9 Find, in cm^2 to 1 decimal place, the area of each of the following circular sectors.

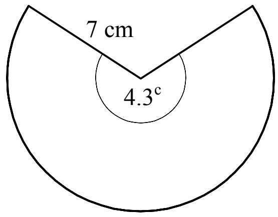
a



b



c



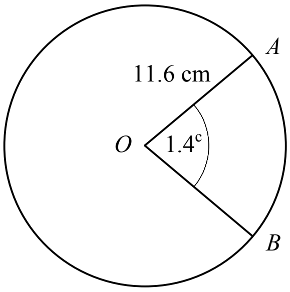
10 *PQ* is an arc of a circle of radius 8 cm, centre *O*.

Given that arc *PQ* has length 12 cm, find

a the angle, in radians, subtended by *PQ* at *O*,

b the area of sector *OPQ*.

11



The diagram shows a circle of radius 11.6 cm, centre *O*. The arc of the circle *AB* subtends an angle of 1.4 radians at *O*. Find, to 3 significant figures,

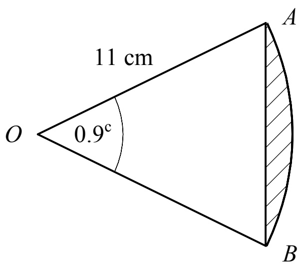
a the perimeter of the minor sector *OAB*,

b the perimeter of the major sector *OAB*,

c the area of the minor sector *OAB*,

d the area of the major sector *OAB*.

12



The diagram shows a circular sector *OAB*. Find the area of

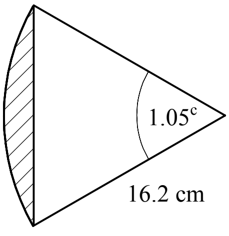
a the sector *OAB*,

b the triangle *OAB*,

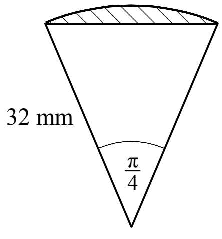
c the shaded segment.

13 Find the area of the shaded segment in each of the following circular sectors.

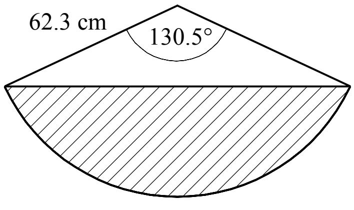
a



b

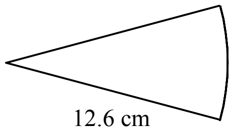


c



C2 TRIGONOMETRY Worksheet C

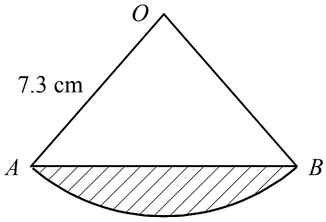
1



The diagram shows a sector of a circle of radius 12.6 cm.

Given that the perimeter of the sector is 31.7 cm, find its area.

2



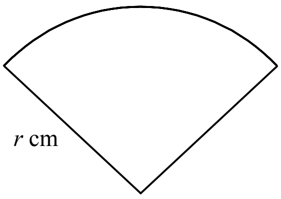
The diagram shows a sector *OAB* of a circle, centre *O* and radius 7.3 cm.

Given that the area of the sector is 38.4 cm^2, find

a the size of $\angle AOB$ in radians,

b the perimeter of the shaded segment.

3

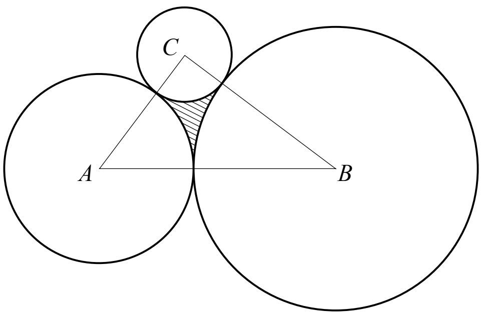


The diagram shows a sector of a circle of radius *r* cm. The area of the sector is 40 cm^2.

a Show that the perimeter of the sector is $(2r + \frac{80}{r})$ cm.

b Hence find the set of values of *r* for which the perimeter of the sector is less than 26 cm.

4



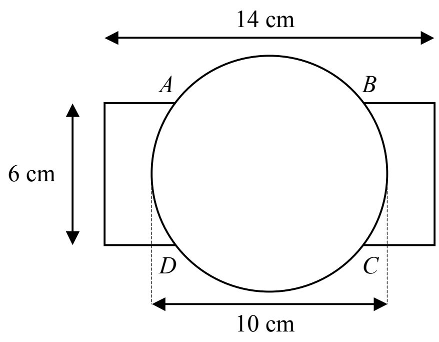
The diagram shows three circles with centres *A*, *B* and *C*, and radii 4 cm, 6 cm and 2 cm respectively. Each circle touches the other two circles.

a Prove that triangle *ABC* is a right-angled triangle.

b Find $\angle ABC$ in radians to 2 decimal places.

c Show that the area of the shaded region enclosed by the three circles is 1.86 cm^2 to 3 significant figures.

5



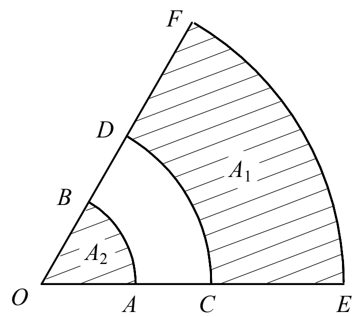
The diagram shows a company logo which consists of a circle of diameter 10 cm drawn on top of a rectangle measuring 6 cm by 14 cm. The centres of the circle and rectangle are coincident and the two shapes intersect at *A*, *B*, *C* and *D*.

a Find the length of the chord of the circle *AB*.

b Show that the perimeter of the logo is 42.5 cm to 3 significant figures.

c Find the area of the logo.

6



AB, *CD* and *EF* are arcs of concentric circles, centre *O*, such that *OACE* and *OBDF* are straight lines as shown in the diagram. The area of the shaded region *CEFD* is denoted by A_1 and the area of the shaded sector *OAB* by A_2.

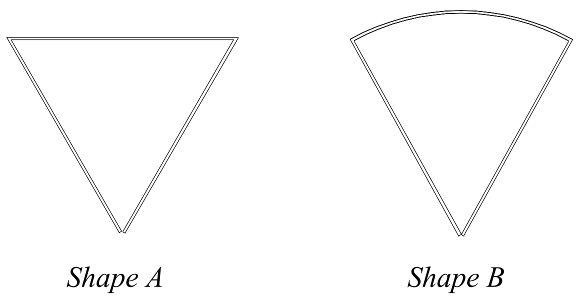
Given that $OA = r$ cm, $AC = 2$ cm, $OE = 8$ cm and $\angle AOB = \theta$ radians,

a find an expression for A_1 in terms of r and θ.

Given also that $A_1 = 7A_2$,

b show that $r = 2.5$

7



A girl is playing with a paper clip. She straightens the wire and then bends it to form an equilateral triangle, *Shape A* above. She then curves one side of the triangle to form a sector of a circle, *Shape B* above.

Find, to 1 decimal place, the percentage change in the area enclosed by the paper clip when it is changed from *Shape A* to *Shape B*, indicating whether this is an increase or decrease.

C2 TRIGONOMETRY Worksheet D

1 Find to 3 decimal places the value of

a $\sin 131°$ **b** $\tan 340.5°$ **c** $\cos 418°$ **d** $\sin (-165.2°)$

2 Give the exact value of

a $\cos 60°$ **b** $\sin 45°$ **c** $\tan 45°$ **d** $\cos 30°$

e $\sin 90°$ **f** $\tan 30°$ **g** $\cos 120°$ **h** $\sin 135°$

i $\tan 210°$ **j** $\cos 225°$ **k** $\sin 300°$ **l** $\tan 120°$

m $\cos 330°$ **n** $\tan 150°$ **o** $\cos (-60°)$ **p** $\sin 405°$

q $\tan (-45°)$ **r** $\sin (-240°)$ **s** $\tan 570°$ **t** $\cos (-150°)$

3 Find to 3 decimal places the value of

a $\cos 0.42^c$ **b** $\sin 4.16^c$ **c** $\tan (-3.1^c)$ **d** $\cos 11.25^c$

4 Give the exact value of

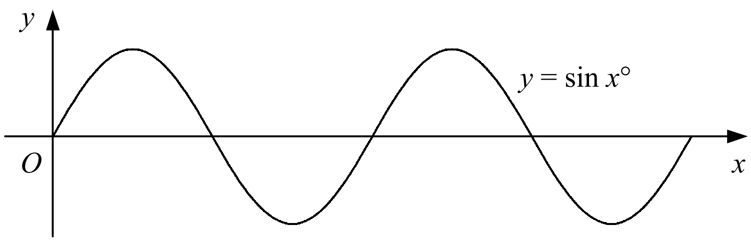
a $\sin \frac{\pi}{6}$ **b** $\cos \frac{\pi}{2}$ **c** $\sin \frac{\pi}{4}$ **d** $\tan \frac{\pi}{3}$

e $\cos \frac{\pi}{3}$ **f** $\sin \frac{2\pi}{3}$ **g** $\tan \frac{3\pi}{4}$ **h** $\cos \frac{5\pi}{6}$

i $\tan \frac{5\pi}{3}$ **j** $\cos \frac{5\pi}{4}$ **k** $\sin (-\frac{\pi}{6})$ **l** $\tan (-\frac{5\pi}{6})$

m $\sin 3\pi$ **n** $\tan (-\frac{5\pi}{4})$ **o** $\cos \frac{8\pi}{3}$ **p** $\sin (-\frac{7\pi}{3})$

5

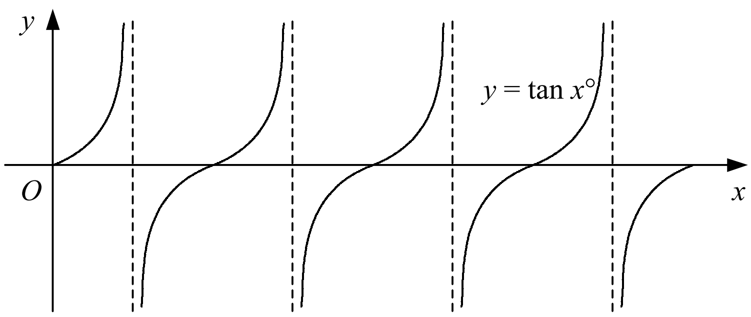


The graph shows the curve $y = \sin x°$ in the interval $0 \le x \le 720$.

a Write down the coordinates of any points where the curve intersects the coordinate axes.

b Write down the coordinates of the turning points of the curve.

6



The graph shows the curve $y = \tan x°$ in the interval $0 \le x \le 720$.

a Write down the coordinates of any points where the curve intersects the coordinate axes.

b Write down the equations of the asymptotes.

7 Describe the transformation that maps the graph of $y = \sin x°$ onto the graph of

a $y = 3 \sin x°$ **b** $y = \sin 4x°$ **c** $y = \sin (x + 60)°$ **d** $y = \sin (-x°)$

8 Sketch each of the following pairs of curves on the same set of axes in the interval $0 \le x \le 360°$.

a $y = \cos x$ and $y = 3 \cos x$ **b** $y = \sin x$ and $y = \sin (x - 30°)$

c $y = \cos x$ and $y = \cos 2x$ **d** $y = \tan x$ and $y = 2 + \tan x$

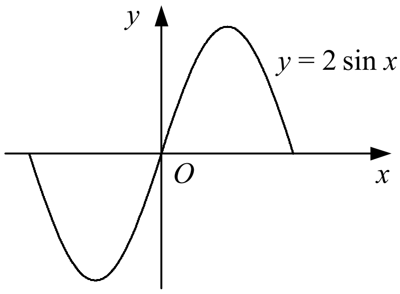
e $y = \sin x$ and $y = -\sin x$ **f** $y = \cos x$ and $y = \cos (x + 60°)$

g $y = \tan x$ and $y = \tan \frac{1}{2}x$ **h** $y = \sin x$ and $y = 1 + \sin x$

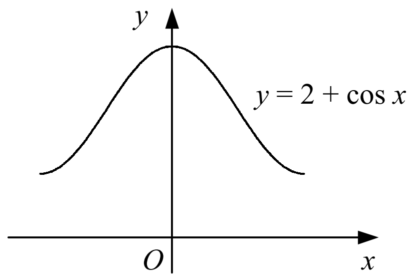
9 Each curve is shown for the interval $-180° \le x \le 180°$.

Write down the coordinates of the turning points of each curve in this interval.

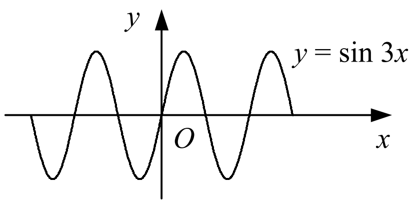
a



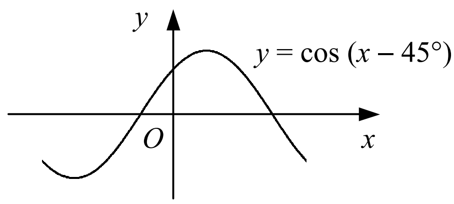
b



c



d



10 Write down the period of each of the following graphs.

a $y = \sin x°$ **b** $y = \tan x°$ **c** $y = 2 \cos x°$

d $y = \sin 2x°$ **e** $y = \tan (x + 30)°$ **f** $y = \cos \frac{1}{3}x°$

11 Sketch each of the following curves for x in the interval $0 \le x \le 360$. Show the coordinates of any points of intersection with the coordinate axes and the equations of any asymptotes.

a $y = \tan x°$ **b** $y = \cos (x + 30)°$ **c** $y = \sin 2x°$

d $y = 1 + \cos x°$ **e** $y = \sin \frac{1}{2}x°$ **f** $y = \tan (x + 90)°$

g $y = \sin (x - 45)°$ **h** $y = -\tan x°$ **i** $y = \cos (x - 120)°$

12 Sketch each of the following curves for x in the interval $0 \le x \le 2\pi$. Show the coordinates of any turning points and the equations of any asymptotes.

a $y = \cos x$ **b** $y = 3 \sin x$ **c** $y = \tan 2x$

d $y = \sin (x - \frac{\pi}{3})$ **e** $y = \cos \frac{1}{3}x$ **f** $y = \sin x - 2$

g $y = \tan (x + \frac{\pi}{4})$ **h** $y = \sin \frac{3}{4}x$ **i** $y = \cos (x - \frac{\pi}{6})$

C2 TRIGONOMETRY Worksheet E

1 Find all values of x in the interval $0 \le x \le 360°$ such that

a $\sin x = \frac{1}{2}$ **b** $\tan x = \sqrt{3}$ **c** $\cos x = 0$ **d** $\sin x = -1$

e $\cos x = \frac{\sqrt{3}}{2}$ **f** $\sin x = \frac{1}{\sqrt{2}}$ **g** $\tan x = -1$ **h** $\cos x = -\frac{1}{2}$

i $\sin x = -\frac{\sqrt{3}}{2}$ **j** $\tan x = \frac{1}{\sqrt{3}}$ **k** $\cos x = -\frac{1}{\sqrt{2}}$ **l** $\tan x = -\sqrt{3}$

2 Solve each equation for θ in the interval $0 \le \theta \le 360°$ giving your answers to 1 decimal place.

a $\cos\theta = 0.4$ **b** $\sin\theta = 0.27$ **c** $\tan\theta = 1.6$ **d** $\sin\theta = 0.813$

e $\tan\theta = 0.1$ **f** $\cos\theta = 0.185$ **g** $\sin\theta = -0.6$ **h** $\tan\theta = -0.7$

i $\cos\theta = -0.39$ **j** $\tan\theta = -3.4$ **k** $\cos\theta = -0.636$ **l** $\sin\theta = -0.203$

3 Solve each equation for x in the interval $0 \le x \le 360$.
Give your answers to 1 decimal place where appropriate.

a $\sin (x - 60)° = 0.5$ **b** $\tan (x + 30)° = 1$ **c** $\cos (x - 45)° = 0.2$

d $\tan (x + 30)° = 0.78$ **e** $\cos (x + 45)° = -0.5$ **f** $\sin (x - 60)° = -0.89$

g $\cos (x + 45)° = 0.9$ **h** $\sin (x + 30)° = 0.14$ **i** $\cos (x - 60)° = 0.6$

j $\sin (x - 30)° = -0.3$ **k** $\tan (x - 60)° = -1.26$ **l** $\sin 2x° = 0.5$

m $\cos 2x° = 0.64$ **n** $\sin 2x° = -0.18$ **o** $\tan 2x° = -2.74$

p $\sin \frac{1}{2}x° = 0.703$ **q** $\tan 3x° = 0.591$ **r** $\cos 2x° = -0.415$

4 Solve each equation for x in the interval $0 \le x \le 2\pi$ giving your answers in terms of π.

a $\sin x = 0$ **b** $\cos x = \frac{1}{2}$ **c** $\tan x = 1$

d $\cos x = -1$ **e** $\tan x = -\frac{1}{\sqrt{3}}$ **f** $\sin x = -\frac{1}{\sqrt{2}}$

g $\tan (x + \frac{\pi}{6}) = \sqrt{3}$ **h** $\sin (x - \frac{\pi}{4}) = \frac{1}{2}$ **i** $\cos (x + \frac{\pi}{3}) = -\frac{\sqrt{3}}{2}$

j $\sin (x + \frac{\pi}{3}) = \frac{1}{\sqrt{2}}$ **k** $\cos 2x = -\frac{1}{\sqrt{2}}$ **l** $\tan 3x = \frac{1}{\sqrt{3}}$

5 Solve each equation for θ in the interval $-180° \le \theta \le 180°$.
Give your answers to 1 decimal place where appropriate.

a $\cos\theta = 0$ **b** $\tan 2\theta + 1 = 0$ **c** $\sin (\theta + 60°) = 0.291$

d $2\tan (\theta - 15°) = 3.7$ **e** $\sin 2\theta - 0.3 = 0$ **f** $4\cos 3\theta = 2$

g $1 + \sin (\theta + 110°) = 0$ **h** $5\cos (\theta - 27°) = 3$ **i** $7 - 3\tan\theta = 0$

j $3 + 8\cos 2\theta = 0$ **k** $2 + 6\tan (\theta + 92°) = 0$ **l** $1 - 4\sin \frac{1}{3}\theta = 0$

6 Solve each equation for x in the interval $0 \le x \le 180°$.
Give your answers to 1 decimal place where appropriate.

a $\tan(2x + 30°) = 1$ **b** $\sin(2x - 15°) = 0$ **c** $\cos(2x + 70°) = 0.5$

d $\sin(2x + 210°) = 0.26$ **e** $\cos(2x - 38°) = -0.64$ **f** $\tan(2x - 56°) = -0.32$

g $\cos(3x - 24°) = 0.733$ **h** $\tan(3x + 60°) = -1.9$ **i** $\sin(\frac{1}{2}x + 18°) = 0.572$

7 Solve each equation for x in the interval $0 \le x \le 2\pi$, giving your answers to 2 decimal places.

a $\tan x = 0.52$ **b** $\cos 2x = 0.315$ **c** $\sin(x + \frac{\pi}{4}) = 0.7$

d $3\cos x + 1 = 0$ **e** $\sin \frac{1}{2}x = 0.09$ **f** $\tan 2x = -0.225$

g $3 - 4\sin(x - \frac{\pi}{3}) = 0$ **h** $\tan(2x + \frac{\pi}{6}) = 2$ **i** $\cos 3x = -0.81$

j $5 + 3\tan x = 0$ **k** $\cos(2x - \frac{\pi}{2}) = -0.34$ **l** $1 + 6\sin 2x = 0$

8 **a** Solve the equation

$$2y^2 - 3y + 1 = 0.$$

b Hence, find the values of x in the interval $0 \le x \le 360°$ for which

$$2\sin^2 x - 3\sin x + 1 = 0.$$

9 Solve each equation for θ in the interval $0 \le \theta \le 360$.
Give your answers to 1 decimal place where appropriate.

a $\sin^2 \theta° = 0.75$ **b** $1 - \tan^2 \theta° = 0$

c $2\cos^2 \theta° + \cos \theta° = 0$ **d** $\sin \theta°(4\cos \theta° - 1) = 0$

e $4\sin \theta° = \sin \theta° \tan \theta°$ **f** $(2\cos \theta° - 1)(\cos \theta° + 1) = 0$

g $\tan^2 \theta° - 3\tan \theta° + 2 = 0$ **h** $3\sin^2 \theta° - 7\sin \theta° + 2 = 0$

i $\tan^2 \theta° - \tan \theta° = 6$ **j** $6\cos^2 \theta° - \cos \theta° - 2 = 0$

k $4\sin^2 \theta° + 3 = 8\sin \theta°$ **l** $\cos^2 \theta° + 2\cos \theta° - 1 = 0$

m $\tan^2 \theta° + 3\tan \theta° - 1 = 0$ **n** $3\sin^2 \theta° + \sin \theta° = 1$

10 **a** Sketch the curve $y = \cos x°$ for x in the interval $0 \le x \le 360$.

b Sketch on the same diagram the curve $y = \cos(x + 90)°$ for x in the interval $0 \le x \le 360$.

c Using your diagram, find all values of x in the interval $0 \le x \le 360$ for which

$$\cos x° = \cos(x + 90)°.$$

11 **a** Sketch the curves $y = \cos x°$ and $y = \cos 3x°$ on the same set of axes for x in the interval $0 \le x \le 360$.

b Solve, for x in the interval $0 \le x \le 360$, the equation

$$\cos x° = \cos 3x°.$$

c Hence solve, for x in the interval $0 \le x \le 180$, the equation

$$\cos 2x° = \cos 6x°.$$

C2 TRIGONOMETRY Worksheet F

1 **a** Given that $4 \sin x + \cos x = 0$, show that $\tan x = -\frac{1}{4}$.

b Hence, find the values of x in the interval $0 \le x \le 360°$ for which

$$4 \sin x + \cos x = 0,$$

giving your answers to 1 decimal place.

2 **a** Show that

$$5 \sin^2 x + 5 \sin x + 4 \cos^2 x \equiv \sin^2 x + 5 \sin x + 4.$$

b Hence, find the values of x in the interval $0 \le x \le 360°$ for which

$$5 \sin^2 x + 5 \sin x + 4 \cos^2 x = 0$$

3 Solve each equation for x in the interval $0 \le x \le 360°$.
Give your answers to 1 decimal place where appropriate.

a $2 \sin x - \cos x = 0$

b $3 \sin x = 4 \cos x$

c $\cos^2 x + 3 \sin x - 3 = 0$

d $3 \cos^2 x - \sin^2 x = 2$

e $2 \sin^2 x + 3 \cos x = 3$

f $3 \cos^2 x = 5(1 - \sin x)$

g $3 \sin x \tan x = 8$

h $\cos x = 3 \tan x$

i $3 \sin^2 x - 5 \cos x + 2 \cos^2 x = 0$

j $2 \sin^2 x + 7 \sin x - 2 \cos^2 x = 0$

k $3 \sin x - 2 \tan x = 0$

l $\sin^2 x - 9 \cos x - \cos^2 x = 5$

4 Solve each equation for θ in the interval $-\pi \le \theta \le \pi$ giving your answers in terms of π.

a $4 \cos^2 \theta = 1$

b $4 \sin^2 \theta + 4 \sin \theta + 1 = 0$

c $\cos^2 \theta + 2 \cos \theta - 3 = 0$

d $3 \sin^2 \theta - \cos^2 \theta = 0$

e $4 \sin^2 \theta - 5 \sin \theta + 2 \cos^2 \theta = 0$

f $\sin^2 \theta - 3 \cos \theta - \cos^2 \theta = 2$

5 Prove that

a $(\sin x + \cos x)^2 \equiv 1 + 2 \sin x \cos x$

b $\frac{1}{\cos x} - \cos x \equiv \sin x \tan x, \quad \cos x \ne 0$

c $\frac{\cos^2 x}{1 - \sin x} \equiv 1 + \sin x, \quad \sin x \ne 1$

d $\frac{1 + \sin x}{\cos x} \equiv \frac{\cos x}{1 - \sin x}, \quad \cos x \ne 0$

6 **a** Prove the identity

$$(\cos x - \tan x)^2 + (\sin x + 1)^2 \equiv 2 + \tan^2 x.$$

b Hence find, in terms of π, the values of x in the interval $0 \le x \le 2\pi$ such that

$$(\cos x - \tan x)^2 + (\sin x + 1)^2 = 3.$$

7 $$f(x) \equiv \cos^2 x + 2 \sin x, \quad 0 \le x \le 2\pi.$$

a Prove that f(x) can be expressed in the form

$$f(x) = 2 - (\sin x - 1)^2.$$

b Hence deduce the maximum value of f(x) and the value of x for which this occurs.

C2 TRIGONOMETRY Worksheet G

1 Find, in terms of π, the values of x in the interval $0 \leq x \leq 2\pi$ for which

a $3 \tan x - \sqrt{3} = 0$,

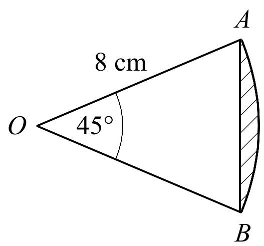
b $2 \cos (x + \frac{\pi}{3}) + \sqrt{3} = 0$.

2 Given that $\cos A = \sqrt{3} - 1$,

a find the value of $\sin^2 A$ in the form $p\sqrt{3} + q$ where p and q are integers,

b show that $\tan^2 A = \frac{\sqrt{3}}{2}$.

3



The diagram shows sector OAB of a circle, centre O, radius 8 cm, in which $\angle AOB = 45°$.

a Find the perimeter of the sector in centimetres to 1 decimal place.

b Show that the area of the shaded segment is $8(\pi - 2\sqrt{2})$ cm^2.

4 Find, to 1 decimal place, the values of θ in the interval $0 \leq \theta \leq 360°$ for which

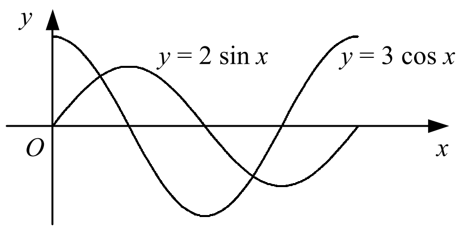
$$2 \sin^2 \theta + \sin \theta - \cos^2 \theta = 2.$$

5 Solve, for x in the interval $-\pi \leq x \leq \pi$, the equation

$$3 \sin^2 x = 4(1 - \sin x),$$

giving your answers to 2 decimal places.

6



The diagram shows the curves $y = 2 \sin x$ and $y = 3 \cos x$ for x in the interval $0 \leq x \leq 2\pi$.
Find, to 2 decimal places, the coordinates of the points where the curves intersect in this interval.

7 **a** Sketch the curve $y = \cos 2x°$ for x in the interval $0 \leq x \leq 360$.

b Find the values of x in the interval $0 \leq x \leq 360$ for which

$$\cos 2x° = -\tfrac{1}{2}.$$

8 Solve, for θ in the interval $0 \leq \theta \leq 360$, the equation

$$12 \cos \theta° = 7 \tan \theta°,$$

giving your answers to 1 decimal place.

9 Given that $\tan 15° = \dfrac{\tan 60° - \tan 45°}{1 + (\tan 60° \times \tan 45°)}$,

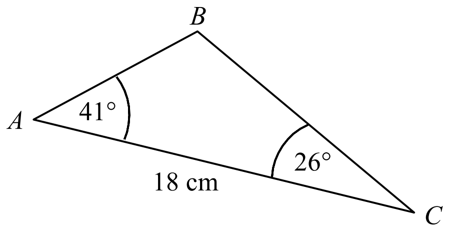
a show that $\tan 15° = 2 - \sqrt{3}$,

b find the exact value of $\tan 345°$.

10 Find, to an appropriate degree of accuracy, the values of x in the interval $0 \leq x \leq 360°$ for which

$$\sin^2 x + 5 \cos x - 3 \cos^2 x = 2.$$

11



The diagram shows triangle ABC in which $AC = 18$ cm, $\angle BAC = 41°$ and $\angle ACB = 26°$.
Find to 3 significant figures

a the length BC,

b the area of triangle ABC.

12 Solve, for θ in the interval $0 \leq \theta \leq 360°$, the equation

$$(6 \cos \theta - 1)(\cos \theta + 1) = 3.$$

13 Find, in degrees to 1 decimal place, the values of x in the interval $-180° \leq x \leq 180°$ for which

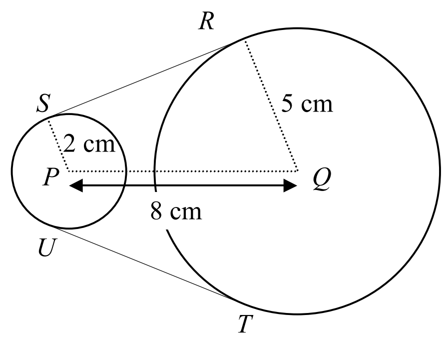
$$\sin^2 x + 5 \sin x = 2 \cos^2 x.$$

14 Prove that

a $\sin^4 \theta - 2 \sin^2 \theta \equiv \cos^4 \theta - 1$,

b $\dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta} \equiv \dfrac{2}{\sin\theta}$, for $\sin \theta \neq 0$.

15



The gears in a toy are shown in the diagram above.

A thin rubber band passes around two circular discs. The centres of the discs are at P and Q where $PQ = 8$ cm and their radii are 2 cm and 5 cm respectively. The sections of the rubber band not in contact with the discs, RS and TU, are assumed to be taught.

a Show that $\angle PQR = 1.186$ radians to 3 decimal places.

b Find the length RS.

c Find the length of the rubber band in this situation.

C2 TRIGONOMETRY Worksheet H

1 Find, in radians to 2 decimal places, the values of θ in the interval $0 \le \theta \le 2\pi$ for which

a $\sin(\theta + \frac{\pi}{4}) = 0.4$, **(3)**

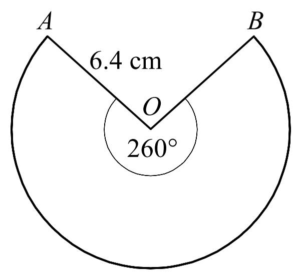
b $1 - 3\cos 2\theta = 0$. **(5)**

2 **a** Sketch the curve $y = \sin 3x$ for x in the interval $0 \le x \le 180°$, showing the coordinates of the turning points of the curve. **(3)**

b Solve, for θ in the interval $0 \le \theta \le 360°$, the equation

$$\tan^2\theta - 2\tan\theta - 3 = 0.$$ **(6)**

3



The diagram shows the major sector OAB of a circle, centre O, radius 6.4 cm.
The reflex angle subtended by the major arc AB at O is 260°.

a Express 260° in radians, correct to 3 decimal places. **(1)**

b Find the perimeter of the major sector OAB. **(3)**

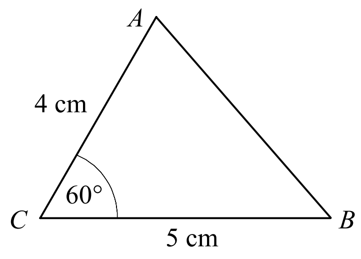
c Find the area of the major sector OAB. **(2)**

4 Solve, for θ in the interval $0 \le \theta \le 360°$, the equation

$$3\cos^2\theta + 6\cos\theta = 2\sin^2\theta + 6,$$

giving your answers to 1 decimal place. **(7)**

5



The diagram shows triangle ABC in which $AC = 4$ cm, $BC = 5$ cm and $\angle ACB = 60°$.

a Find the exact area of triangle ABC. **(2)**

b Show that $AB = \sqrt{21}$ cm. **(3)**

c Find the value of $\sin(\angle ABC)$ in the form $k\sqrt{7}$ where k is an exact fraction. **(3)**

6 Find, to 1 decimal place, the values of x in the interval $0 \le x \le 360$ for which

$$\tan(2x + 15)° = 2.$$ **(6)**

7 Find the values of θ in the interval $0 \le \theta \le 360°$ for which

$$\sin\theta\tan\theta - \cos\theta = 1.$$ **(8)**

8 The line with equation $y = 6$ intersects the circle with equation $x^2 + y^2 - 10x - 2y - 3 = 0$ at the points P and Q.

a Find the coordinates of the centre and the radius of the circle. **(3)**

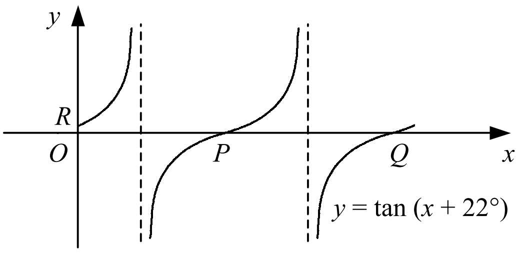
b Find the coordinates of the points P and Q. **(3)**

c Find the area of the minor segment enclosed by the chord PQ and the circle. **(6)**

9 Find the values of θ in the interval $0 \le \theta \le 360°$ for which

$$5 \sin^2 \theta + 5 \sin \theta + 2 \cos^2 \theta = 0.$$ **(8)**

10



The diagram shows the curve $y = \tan (x + 22°)$ for x in the interval $0 \le x \le 360°$.

a Write down the coordinates of the points P and Q where the curve crosses the x-axis. **(2)**

b Find the coordinates of the point R where the curve meets the y-axis. **(1)**

c Write down the equations of the curve's asymptotes. **(2)**

11 a Find, to 1 decimal place, the values of x in the interval $0 \le x \le 360°$, for which

$$5 \sin x = 2 \cos x.$$ **(4)**

b Solve, for y in the interval $0 \le y \le 2\pi$, the equation

$$2 \sin^2 y - \sin y = 1,$$

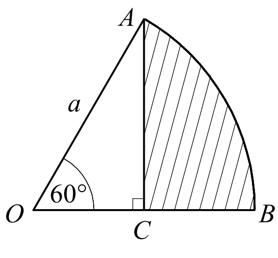
giving your answers in terms of π. **(6)**

12 Solve, for θ in the interval $-180° \le \theta \le 180°$, the equation

$$3 \cos^2 \theta - 5 \cos \theta + 2 \sin^2 \theta = 0,$$

giving your answers to 1 decimal place. **(7)**

13



The diagram shows the circular sector OAB, centre O. The point C lies on OB such that AC is perpendicular to OB.

Given that $OA = a$, and that $\angle AOB = 60°$,

a find the area of sector OAB in terms of a and π, **(3)**

b find the length OC in terms of a, **(1)**

c show that the area of the shaded region bounded by the arc AB and the straight lines AC and BC is given by $\frac{1}{24} a^2(4\pi - 3\sqrt{3})$. **(5)**

C2 TRIGONOMETRY Worksheet I

1 Find, to 1 decimal place, the values of x in the interval $-180° \leq x \leq 180°$ for which

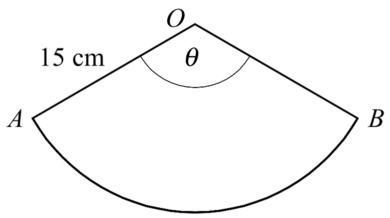
a $\cos(x + 40°) = 0.3$, **(3)**

b $2 + \tan 2x = 0$. **(5)**

2 Find, to 1 decimal place, the values of x in the interval $0 \leq x \leq 360$ for which

$$2\tan^2 x° - 4\tan x° + 1 = 0.$$ **(6)**

3



The diagram shows sector OAB of a circle, centre O, radius 15 cm.

Given that $\angle AOB = \theta$ radians and that the length of the arc AB is 32.1 cm,

a find the value of θ, **(2)**

b find the area of sector OAB. **(2)**

4 Solve, for x in the interval $0 \leq x \leq \pi$, the equation

$$\sin(2x - \tfrac{\pi}{3}) = \tfrac{1}{2},$$

giving your answers in terms of π. **(6)**

5 **a** Given that $\sin A = 1 - \sqrt{2}$, show that $\cos^2 A + 2\sin A = 0$. **(4)**

b Sketch the curve $y = \sin(x + \tfrac{\pi}{3})$ for x in the interval $0 \leq x \leq 2\pi$.

Label on your sketch

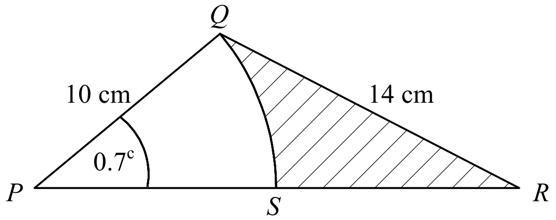
i the value of x at each point where the curve intersects the x-axis,

ii the coordinates of the maximum and minimum points of the curve. **(5)**

6 Find the values of x in the interval $0 \leq x \leq 360°$ for which

$$2\sin^2 x + \sin x + 1 = \cos^2 x.$$ **(8)**

7



The diagram shows triangle PQR in which $PQ = 10$ cm, $QR = 14$ cm and $\angle QPR = 0.7$ radians.

a Find the size of $\angle PRQ$ in radians to 2 decimal places. **(3)**

The point S lies on PR such that $PS = 10$ cm. The shaded region is bounded by the straight lines QR and RS and the arc QS of a circle, centre P.

b Find the area of the shaded region. **(6)**

8 **a** Given that $0 < A < 90°$, and that $\sin A = \frac{\sqrt{5}}{3}$,

i show that $\cos A = \frac{2}{3}$,

ii find the exact value of $\tan A$. **(5)**

b Find the values of x in the interval $0 \leq x \leq 360°$ for which

$$5 \sin x \cos x + \cos x = 0.$$ **(6)**

9 Find the values of θ in the interval $0 \leq \theta \leq 180$ for which

$$\cos (2\theta + 30)° = -\tfrac{1}{2}.$$ **(6)**

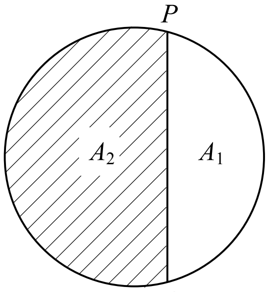
10 **a** Sketch the curve $y = \cos (x - 30)°$ for x in the interval $-180 \leq x \leq 180$, showing the coordinates of any maximum or minimum points on the curve. **(4)**

b Find the x-coordinates of the points where the curve intersects the line $y = 0.2$ in this interval, giving your answers to 1 decimal place. **(3)**

11 Find the values of x in the interval $0 \leq x \leq 360°$ for which

$$4 \cos^2 x - \cos x - 2 \sin^2 x = 0.$$ **(8)**

12



The diagram shows a circle of radius r cm. The chord PQ divides the circle into the unshaded minor segment of area A_1 and the shaded major segment of area A_2.

Given that PQ subtends an angle of θ radians at the centre of the circle,

a find an expression for A_1 in terms of r and θ. **(3)**

Given also that $\theta = \frac{5\pi}{6}$,

b show that $A_1 : A_2 = (5\pi - 3) : (7\pi + 3)$. **(6)**

13 Find, in terms of π, the values of x in the interval $0 \leq x \leq 2\pi$ for which

$$3 \tan x - 2 \cos x = 0.$$ **(7)**

14 In triangle ABC, $AB = 5$ cm, $AC = 7$ cm and $BC = 8$ cm.

a Find the value of $\cos (\angle ABC)$. **(3)**

b Show that the area of triangle ABC is $10\sqrt{3}$ cm^2. **(5)**

15 **a** Show that

$$(2 + \cos^2 \theta)(1 + \tan^2 \theta) \equiv 3 + 2 \tan^2 \theta.$$ **(3)**

b Hence find the values of θ in the interval $0 \leq \theta \leq 360°$ for which

$$(2 + \cos^2 \theta)(1 + \tan^2 \theta) = 7.$$ **(5)**