

Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Integration

Exercise A, Question 1

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$$x^5$$

Solution:

$$\frac{dy}{dx} = x^5$$

$$y = \frac{x^6}{6} + c$$

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Integration

Exercise A, Question 2

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$$10x^4$$

Solution:

$$\frac{dy}{dx} = 10x^4$$

$$y = 10 \frac{x^5}{5} + c$$

$$y = 2x^5 + c$$

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Exercise A, Question 3

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$$3x^2$$

Solution:

$$\frac{dy}{dx} = 3x^2$$

$$y = 3 \frac{x^3}{3} + c$$

$$y = x^3 + c$$

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Exercise A, Question 4

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$$-x^{-2}$$

Solution:

$$\frac{dy}{dx} = -x^{-2}$$

$$y = -\frac{x^{-1}}{-1} + c$$

$$y = x^{-1} + c \text{ or}$$

$$y = \frac{1}{x} + c$$

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Exercise A, Question 5

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$$-4x^{-3}$$

Solution:

$$\frac{dy}{dx} = -4x^{-3}$$

$$y = -4 \frac{x^{-2}}{-2} + c$$

$$y = 2x^{-2} + c \text{ or}$$

$$y = \frac{2}{x^2} + c$$

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Exercise A, Question 6

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$$x^{-\frac{2}{3}}$$

Solution:

$$\frac{dy}{dx} = x^{-\frac{2}{3}}$$

$$y = \frac{x^{-\frac{2}{3} + 1}}{-\frac{2}{3} + 1} + c$$

$$y = \frac{3}{5}x^{\frac{1}{3}} + c$$

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Exercise A, Question 7

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$$4x^{\frac{1}{2}}$$

Solution:

$$\frac{dy}{dx} = 4x^{\frac{1}{2}}$$

$$y = 4 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$y = \frac{8}{3}x^{\frac{3}{2}} + c$$

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Exercise A, Question 8

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$$-2x^6$$

Solution:

$$\frac{dy}{dx} = -2x^6$$

$$y = -2 \frac{x^7}{7} + c$$

$$y = -\frac{2}{7}x^7 + c$$

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Exercise A, Question 9

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$$3x^5$$

Solution:

$$\frac{dy}{dx} = 3x^5$$

$$y = 3 \frac{x^6}{6} + c$$

$$y = \frac{1}{2}x^6 + c$$

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Exercise A, Question 10

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$$3x^{-4}$$

Solution:

$$\frac{dy}{dx} = 3x^{-4}$$

$$y = 3 \frac{x^{-3}}{-3} + c$$

$$y = -x^{-3} + c \text{ or}$$

$$y = -\frac{1}{x^3} + c$$

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Exercise A, Question 11

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$$x^{-\frac{1}{2}}$$

Solution:

$$\frac{dy}{dx} = x^{-\frac{1}{2}}$$

$$y = \frac{x^{+\frac{1}{2}}}{\frac{1}{2}} + c$$

$$y = 2x^{\frac{1}{2}} + c \text{ or}$$

$$y = 2\sqrt{x} + c$$

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Exercise A, Question 12

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$$5x^{-\frac{3}{2}}$$

Solution:

$$\frac{dy}{dx} = 5x^{-\frac{3}{2}}$$

$$y = 5 \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + c$$

$$y = -10x^{-\frac{1}{2}} + c \text{ or}$$

$$y = \frac{-10}{\sqrt{x}} + c$$

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Exercise A, Question 13

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$$-2x^{-\frac{3}{2}}$$

Solution:

$$\frac{dy}{dx} = -2x^{-\frac{3}{2}}$$

$$y = -2 \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + c$$

$$y = 4x^{-\frac{1}{2}} + c \text{ or}$$

$$y = \frac{4}{\sqrt{x}} + c$$

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Exercise A, Question 14

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$$6x^{\frac{1}{3}}$$

Solution:

$$\frac{dy}{dx} = 6x^{\frac{1}{3}}$$

$$y = 6 \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + c$$

$$y = \frac{18}{4} x^{\frac{4}{3}} + c$$

$$y = \frac{9}{2} x^{\frac{4}{3}} + c$$

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Exercise A, Question 15

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$$36x^{11}$$

Solution:

$$\frac{dy}{dx} = 36x^{11}$$

$$y = 36 \frac{x^{12}}{12} + c$$

$$y = 3x^{12} + c$$

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Exercise A, Question 16

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$$-14x^{-8}$$

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Exercise A, Question 17

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$$-3x^{-\frac{2}{3}}$$

Solution:

$$\frac{dy}{dx} = -3x^{-\frac{2}{3}}$$

$$y = -3 \frac{x^{\frac{1}{3}}}{\frac{1}{3}} + c$$

$$y = -9x^{\frac{1}{3}} + c$$

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Exercise A, Question 18

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$$- 5$$

Solution:

$$\frac{dy}{dx} = - 5 = - 5x^0$$

$$y = - 5 \frac{x^1}{1} + c$$

$$y = - 5x + c$$

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Exercise A, Question 19

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$$6x$$

Solution:

$$\frac{dy}{dx} = 6x$$

$$y = 6 \frac{x^2}{2} + c$$

$$y = 3x^2 + c$$

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Exercise A, Question 20

Question:

Find an expression for y when $\frac{dy}{dx}$ is:

$$2x^{-0.4}$$

Solution:

$$\frac{dy}{dx} = 2x^{-0.4}$$

$$y = 2 \frac{x^{0.6}}{0.6} + c$$

$$y = \frac{20}{6} x^{0.6} + c$$

$$y = \frac{10}{3} x^{0.6} + c$$

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Exercise B, Question 1

Question:

Find y when $\frac{dy}{dx}$ is given by the following expressions. In each case simplify your answer:

(a) $4x - x^{-2} + 6x^{\frac{1}{2}}$

(b) $15x^2 + 6x^{-3} - 3x^{-\frac{5}{2}}$

(c) $x^3 - \frac{3}{2}x^{-\frac{1}{2}} - 6x^{-2}$

(d) $4x^3 + x^{-\frac{2}{3}} - x^{-2}$

(e) $4 - 12x^{-4} + 2x^{-\frac{1}{2}}$

(f) $5x^{\frac{2}{3}} - 10x^4 + x^{-3}$

(g) $-\frac{4}{3}x^{-\frac{4}{3}} - 3 + 8x$

(h) $5x^4 - x^{-\frac{3}{2}} - 12x^{-5}$

Solution:

(a) $\frac{dy}{dx} = 4x - x^{-2} + 6x^{\frac{1}{2}}$

$$y = 4 \frac{x^2}{2} - \frac{x^{-1}}{-1} + 6 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$y = 2x^2 + x^{-1} + 4x^{\frac{3}{2}} + c$$

(b) $\frac{dy}{dx} = 15x^2 + 6x^{-3} - 3x^{-\frac{5}{2}}$

$$y = 15 \frac{x^3}{3} + 6 \frac{x^{-2}}{-2} - 3 \frac{x^{-\frac{3}{2}}}{-\frac{3}{2}} + c$$

$$y = 5x^3 - 3x^{-2} + 2x^{-\frac{3}{2}} + c$$

$$(c) \frac{dy}{dx} = x^3 - \frac{3}{2}x^{-\frac{1}{2}} - 6x^{-2}$$

$$y = \frac{x^4}{4} - \frac{3}{2} \frac{x^{\frac{1}{2}}}{\frac{1}{2}} - 6 \frac{x^{-1}}{-1} + c$$

$$y = \frac{1}{4}x^4 - 3x^{\frac{1}{2}} + 6x^{-1} + c$$

$$(d) \frac{dy}{dx} = 4x^3 + x^{-\frac{2}{3}} - x^{-2}$$

$$y = 4 \frac{x^4}{4} + \frac{x^{\frac{1}{3}}}{\frac{1}{3}} - \frac{x^{-1}}{-1} + c$$

$$y = x^4 + 3x^{\frac{1}{3}} + x^{-1} + c$$

$$(e) \frac{dy}{dx} = 4 - 12x^{-4} + 2x^{-\frac{1}{2}}$$

$$y = 4x - 12 \frac{x^{-3}}{-3} + 2 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$y = 4x + 4x^{-3} + 4x^{\frac{1}{2}} + c$$

$$(f) \frac{dy}{dx} = 5x^{\frac{2}{3}} - 10x^4 + x^{-3}$$

$$y = 5 \frac{x^{\frac{5}{3}}}{\frac{5}{3}} - 10 \frac{x^5}{5} + \frac{x^{-2}}{-2} + c$$

$$y = 3x^{\frac{5}{3}} - 2x^5 - \frac{1}{2}x^{-2} + c$$

$$(g) \frac{dy}{dx} = -\frac{4}{3}x^{-\frac{4}{3}} - 3 + 8x$$

$$y = -\frac{4}{3} \frac{x^{-\frac{1}{3}}}{-\frac{1}{3}} - 3x + 8 \frac{x^2}{2} + c$$

$$y = 4x^{-\frac{1}{3}} - 3x + 4x^2 + c$$

$$(h) \frac{dy}{dx} = 5x^4 - x^{-\frac{3}{2}} - 12x^{-5}$$

$$y = 5 \frac{x^5}{5} - \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} - 12 \frac{x^{-4}}{-4} + c$$

$$y = x^5 + 2x^{-\frac{1}{2}} + 3x^{-4} + c$$

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Exercise B, Question 2

Question:

Find $f(x)$ when $f'(x)$ is given by the following expressions. In each case simplify your answer:

(a) $12x + \frac{3}{2}x^{-\frac{3}{2}} + 5$

(b) $6x^5 + 6x^{-7} - \frac{1}{6}x^{-\frac{7}{6}}$

(c) $\frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}}$

(d) $10x + 8x^{-3}$

(e) $2x^{-\frac{1}{3}} + 4x^{-\frac{5}{3}}$

(f) $9x^2 + 4x^{-3} + \frac{1}{4}x^{-\frac{1}{2}}$

(g) $x^2 + x^{-2} + x^{\frac{1}{2}}$

(h) $-2x^{-3} - 2x + 2x^{\frac{1}{2}}$

Solution:

(a) $f'(x) = 12x + \frac{3}{2}x^{-\frac{3}{2}} + 5$

$$f(x) = 12 \frac{x^2}{2} + \frac{3}{2} \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + 5x + c$$

$$f(x) = 6x^2 - 3x^{-\frac{1}{2}} + 5x + c$$

(b) $f'(x) = 6x^5 + 6x^{-7} - \frac{1}{6}x^{-\frac{7}{6}}$

$$f(x) = 6 \frac{x^6}{6} + 6 \frac{x^{-6}}{-6} - \frac{1}{6} \frac{x^{-\frac{1}{6}}}{-\frac{1}{6}} + c$$

$$f(x) = x^6 - x^{-6} + x^{-\frac{1}{6}} + c$$

$$(c) f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}}$$

$$f(x) = \frac{1}{2} \frac{x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{1}{2} \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + c$$

$$f(x) = x^{\frac{1}{2}} + x^{-\frac{1}{2}} + c$$

$$(d) f'(x) = 10x + 8x^{-3}$$

$$f(x) = 10 \frac{x^2}{2} + 8 \frac{x^{-2}}{-2} + c$$

$$f(x) = 5x^2 - 4x^{-2} + c$$

$$(e) f'(x) = 2x^{-\frac{1}{3}} + 4x^{-\frac{5}{3}}$$

$$f(x) = 2 \frac{x^{\frac{2}{3}}}{\frac{2}{3}} + 4 \frac{x^{-\frac{2}{3}}}{-\frac{2}{3}} + c$$

$$f(x) = 3x^{\frac{2}{3}} - 6x^{-\frac{2}{3}} + c$$

$$(f) f'(x) = 9x^2 + 4x^{-3} + \frac{1}{4}x^{-\frac{1}{2}}$$

$$f(x) = 9 \frac{x^3}{3} + 4 \frac{x^{-2}}{-2} + \frac{1}{4} \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$f(x) = 3x^3 - 2x^{-2} + \frac{1}{2}x^{\frac{1}{2}} + c$$

$$(g) f'(x) = x^2 + x^{-2} + x^{\frac{1}{2}}$$

$$f(x) = \frac{x^3}{3} + \frac{x^{-1}}{-1} + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$f(x) = \frac{1}{3}x^3 - x^{-1} + \frac{2}{3}x^{\frac{3}{2}} + c$$

$$(h) f'(x) = -2x^{-3} - 2x + 2x^{\frac{1}{2}}$$

$$f(x) = -2 \frac{x^{-2}}{-2} - 2 \frac{x^2}{2} + 2 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$f(x) = x^{-2} - x^2 + \frac{4}{3}x^{\frac{3}{2}} + c$$

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Integration

Exercise C, Question 1

Question:

Find the following integral:

$$\int (x^3 + 2x) \, dx$$

Solution:

$$\begin{aligned}\int (x^3 + 2x) \, dx \\ &= \frac{x^4}{4} + 2 \frac{x^2}{2} + c \\ &= \frac{1}{4}x^4 + x^2 + c\end{aligned}$$

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Exercise C, Question 2

Question:

Find the following integral:

$$\int (2x^{-2} + 3) dx$$

Solution:

$$\int (2x^{-2} + 3) dx$$

$$= 2 \frac{x^{-1}}{-1} + 3x + c$$

$$= -2x^{-1} + 3x + c$$

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Exercise C, Question 3

Question:

Find the following integral:

$$\int \left(5x^{\frac{3}{2}} - 3x^2 \right) dx$$

Solution:

$$\int \left(5x^{\frac{3}{2}} - 3x^2 \right) dx$$

$$= 5 \frac{x^{\frac{5}{2}}}{\frac{5}{2}} - 3 \frac{x^3}{3} + c$$

$$= 2x^{\frac{5}{2}} - x^3 + c$$

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Exercise C, Question 4

Question:

Find the following integral:

$$\int \left(2x^{\frac{1}{2}} - 2x^{-\frac{1}{2}} + 4 \right) dx$$

Solution:

$$\int \left(2x^{\frac{1}{2}} - 2x^{-\frac{1}{2}} + 4 \right) dx$$

$$= 2 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - 2 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + 4x + c$$

$$= \frac{4}{3}x^{\frac{3}{2}} - 4x^{\frac{1}{2}} + 4x + c$$

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Exercise C, Question 5

Question:

Find the following integral:

$$\int (4x^3 - 3x^{-4} + r) dx$$

Solution:

$$\begin{aligned} \int (4x^3 - 3x^{-4} + r) dx \\ &= 4 \frac{x^4}{4} - 3 \frac{x^{-3}}{-3} + rx + c \\ &= x^4 + x^{-3} + rx + c \end{aligned}$$

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Exercise C, Question 6

Question:

Find the following integral:

$$\int (3t^2 - t^{-2}) dt$$

Solution:

$$\begin{aligned} \int (3t^2 - t^{-2}) dt \\ &= 3 \frac{t^3}{3} - \frac{t^{-1}}{-1} + c \\ &= t^3 + t^{-1} + c \end{aligned}$$

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Exercise C, Question 7

Question:

Find the following integral:

$$\int \left(2t^2 - 3t^{-\frac{3}{2}} + 1 \right) dt$$

Solution:

$$\int \left(2t^2 - 3t^{-\frac{3}{2}} + 1 \right) dt$$

$$= 2 \frac{t^3}{3} - 3 \frac{t^{-\frac{1}{2}}}{-\frac{1}{2}} + t + c$$

$$= \frac{2}{3}t^3 + 6t^{-\frac{1}{2}} + t + c$$

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Exercise C, Question 8

Question:

Find the following integral:

$$\int \left(x + x^{-\frac{1}{2}} + x^{-\frac{3}{2}} \right) dx$$

Solution:

$$\int \left(x + x^{-\frac{1}{2}} + x^{-\frac{3}{2}} \right) dx$$

$$= \frac{x^2}{2} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + c$$

$$= \frac{1}{2}x^2 + 2x^{\frac{1}{2}} - 2x^{-\frac{1}{2}} + c$$

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Exercise C, Question 9

Question:

Find the following integral:

$$\int (px^4 + 2t + 3x^{-2}) dx$$

Solution:

$$\begin{aligned} \int (px^4 + 2t + 3x^{-2}) dx \\ &= p \frac{x^5}{5} + 2tx + 3 \frac{x^{-1}}{-1} + c \\ &= \frac{p}{5}x^5 + 2tx - 3x^{-1} + c \end{aligned}$$

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Exercise C, Question 10

Question:

Find the following integral:

$$\int (pt^3 + q^2 + px^3) dt$$

Solution:

$$\begin{aligned} \int (pt^3 + q^2 + px^3) dt \\ = p \frac{t^4}{4} + q^2t + px^3t + c \end{aligned}$$

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Exercise D, Question 1

Question:

Find the following integrals:

$$(a) \int (2x + 3) x^2 dx$$

$$(b) \int \frac{(2x^2 + 3)}{x^2} dx$$

$$(c) \int (2x + 3)^2 dx$$

$$(d) \int (2x + 3)(x - 1) dx$$

$$(e) \int (2x + 3) \sqrt{x} dx$$

Solution:

$$\begin{aligned} (a) \int (2x + 3) x^2 dx &= \int (2x^3 + 3x^2) dx \\ &= 2 \frac{x^4}{4} + 3 \frac{x^3}{3} + c \\ &= \frac{1}{2} x^4 + x^3 + c \end{aligned}$$

$$\begin{aligned} (b) \int \frac{(2x^2 + 3)}{x^2} dx &= \int \left(\frac{2x^2}{x^2} + \frac{3}{x^2} \right) dx \\ &= \int (2 + 3x^{-2}) dx \\ &= 2x + 3 \frac{x^{-1}}{-1} + c \\ &= 2x - 3x^{-1} + c \\ \text{or } &= 2x - \frac{3}{x} + c \end{aligned}$$

$$\begin{aligned} (c) \int (2x + 3)^2 dx &= \int (4x^2 + 12x + 9) dx \\ &= 4 \frac{x^3}{3} + 12 \frac{x^2}{2} + 9x + c \\ &= \frac{4}{3} x^3 + 6x^2 + 9x + c \end{aligned}$$

$$\begin{aligned} (d) \int (2x + 3)(x - 1) dx &= \int (2x^2 + x - 3) dx \\ &= 2 \frac{x^3}{3} + \frac{x^2}{2} - 3x + c \end{aligned}$$

$$= \frac{2}{3}x^3 + \frac{1}{2}x^2 - 3x + c$$

$$(e) \int (2x + 3) \sqrt{x} \, dx$$

$$= \int (2x + 3) x^{\frac{1}{2}} \, dx$$

$$= \int \left(2x^{\frac{3}{2}} + 3x^{\frac{1}{2}} \right) \, dx$$

$$= 2 \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + 3 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= \frac{4}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + c$$

$$\text{or} = \frac{4}{5}\sqrt{x^5} + 2\sqrt{x^3} + c$$

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Exercise D, Question 2

Question:

Find $\int f(x)dx$ when $f(x)$ is given by the following:

(a) $(x + 2)^2$

(b) $\left(x + \frac{1}{x}\right)^2$

(c) $(\sqrt{x+2})^2$

(d) $\sqrt{x(x+2)}$

(e) $\left(\frac{x+2}{\sqrt{x}}\right)$

(f) $\left(\frac{1}{\sqrt{x}} + 2\sqrt{x}\right)$

Solution:

$$\begin{aligned} \text{(a)} \quad & \int (x + 2)^2 dx \\ &= \int (x^2 + 4x + 4) dx \\ &= \frac{1}{3}x^3 + \frac{4}{2}x^2 + 4x + c \\ &= \frac{1}{3}x^3 + 2x^2 + 4x + c \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & \int \left(x + \frac{1}{x}\right)^2 dx \\ &= \int \left(x^2 + 2 + \frac{1}{x^2}\right) dx \\ &= \int (x^2 + 2 + x^{-2}) dx \\ &= \frac{1}{3}x^3 + 2x + \frac{x^{-1}}{-1} + c \\ &= \frac{1}{3}x^3 + 2x - x^{-1} + c \\ \text{or} \quad &= \frac{1}{3}x^3 + 2x - \frac{1}{x} + c \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad & \int (\sqrt{x+2})^2 dx \\ &= \int (x + 4\sqrt{x+4}) dx \end{aligned}$$

$$= \int \left(x + 4x^{\frac{1}{2}} + 4 \right) dx$$

$$= \frac{1}{2}x^2 + 4 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 4x + c$$

$$= \frac{1}{2}x^2 + \frac{8}{3}x^{\frac{3}{2}} + 4x + c$$

(d) $\int \sqrt{x(x+2)} dx$

$$= \int \left(x^{\frac{3}{2}} + 2x^{\frac{1}{2}} \right) dx$$

$$= \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + 2 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= \frac{2}{5}x^{\frac{5}{2}} + \frac{4}{3}x^{\frac{3}{2}} + c$$

$$\text{or} = \frac{2}{5}\sqrt{x^5} + \frac{4}{3}\sqrt{x^3} + c$$

(e) $\int \left(\frac{x+2}{\sqrt{x}} \right) dx$

$$= \int \left(\frac{x}{x^{\frac{1}{2}}} + \frac{2}{x^{\frac{1}{2}}} \right) dx$$

$$= \int \left(x^{\frac{1}{2}} + 2x^{-\frac{1}{2}} \right) dx$$

$$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 2 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= \frac{2}{3}x^{\frac{3}{2}} + 4x^{\frac{1}{2}} + c$$

$$\text{or} = \frac{2}{3}\sqrt{x^3} + 4\sqrt{x} + c$$

(f) $\int \left(\frac{1}{\sqrt{x}} + 2\sqrt{x} \right) dx$

$$= \int \left(x^{-\frac{1}{2}} + 2x^{\frac{1}{2}} \right) dx$$

$$= \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + 2 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= 2x^{\frac{1}{2}} + \frac{4}{3}x^{\frac{3}{2}} + c$$

$$\text{or } = 2\sqrt{x} + \frac{4}{3}\sqrt{x^3} + c$$

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Edexcel Modular Mathematics for AS and A-Level

Integration

Exercise D, Question 3

Question:

Find the following integrals:

$$(a) \int \left(3\sqrt{x} + \frac{1}{x^2} \right) dx$$

$$(b) \int \left(\frac{2}{\sqrt{x}} + 3x^2 \right) dx$$

$$(c) \int \left(x^{\frac{2}{3}} + \frac{4}{x^3} \right) dx$$

$$(d) \int \left(\frac{2+x}{x^3} + 3 \right) dx$$

$$(e) \int (x^2 + 3)(x - 1) dx$$

$$(f) \int \left(\frac{2}{\sqrt{x}} + 3x\sqrt{x} \right) dx$$

$$(g) \int (x - 3)^2 dx$$

$$(h) \int \frac{(2x+1)^2}{\sqrt{x}} dx$$

$$(i) \int \left(3 + \frac{\sqrt{x+6x^3}}{x} \right) dx$$

$$(j) \int \sqrt{x}(\sqrt{x+3})^2 dx$$

Solution:

$$(a) \int \left(3\sqrt{x} + \frac{1}{x^2} \right) dx$$

$$= \int \left(3x^{\frac{1}{2}} + x^{-2} \right) dx$$

$$= 3 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{-1}}{-1} + c$$

$$= 2x^{\frac{3}{2}} - x^{-1} + c$$

$$\text{or } = 2\sqrt{x^3} - \frac{1}{x} + c$$

$$\text{(b) } \int \left(\frac{2}{\sqrt{x}} + 3x^2 \right) dx$$

$$= \int \left(2x^{-\frac{1}{2}} + 3x^2 \right) dx$$

$$= 2 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{3}{3}x^3 + c$$

$$= 4x^{\frac{1}{2}} + x^3 + c$$

$$\text{or } = 4\sqrt{x} + x^3 + c$$

$$\text{(c) } \int \left(x^{\frac{2}{3}} + \frac{4}{x^3} \right) dx$$

$$= \int \left(x^{\frac{2}{3}} + 4x^{-3} \right) dx$$

$$= \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + 4 \frac{x^{-2}}{-2} + c$$

$$= \frac{3}{5}x^{\frac{5}{3}} - 2x^{-2} + c$$

$$\text{or } = \frac{3}{5}x^{\frac{5}{3}} - \frac{2}{x^2} + c$$

$$\text{(d) } \int \left(\frac{2+x}{x^3} + 3 \right) dx$$

$$= \int (2x^{-3} + x^{-2} + 3) dx$$

$$= 2 \frac{x^{-2}}{-2} + \frac{x^{-1}}{-1} + 3x + c$$

$$= -x^{-2} - x^{-1} + 3x + c$$

$$\text{or } = -\frac{1}{x^2} - \frac{1}{x} + 3x + c$$

$$\text{(e) } \int (x^2 + 3)(x - 1) dx$$

$$= \int (x^3 - x^2 + 3x - 3) dx$$

$$= \frac{1}{4}x^4 - \frac{1}{3}x^3 + \frac{3}{2}x^2 - 3x + c$$

$$\text{(f) } \int \left(\frac{2}{\sqrt{x}} + 3x\sqrt{x} \right) dx$$

$$= \int \left(2x^{-\frac{1}{2}} + 3x^{\frac{3}{2}} \right) dx$$

$$= 2 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + 3 \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + c$$

$$= 4x^{\frac{1}{2}} + \frac{6}{5}x^{\frac{5}{2}} + c$$

$$\text{or } = 4\sqrt{x} + \frac{6}{5}x^2\sqrt{x} + c$$

$$\begin{aligned} \text{(g)} \int (x-3)^2 dx &= \int (x^2 - 6x + 9) dx \\ &= \frac{1}{3}x^3 - \frac{6}{2}x^2 + 9x + c \\ &= \frac{1}{3}x^3 - 3x^2 + 9x + c \end{aligned}$$

$$\begin{aligned} \text{(h)} \int \frac{(2x+1)^2}{\sqrt{x}} dx &= \int x^{-\frac{1}{2}} \left(4x^2 + 4x + 1 \right) dx \\ &= \int \left(4x^{\frac{3}{2}} + 4x^{\frac{1}{2}} + x^{-\frac{1}{2}} \right) dx \\ &= 4 \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + 4 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c \\ &= \frac{8}{5}x^{\frac{5}{2}} + \frac{8}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c \\ \text{or } &= \frac{8}{5}\sqrt{x^5} + \frac{8}{3}\sqrt{x^3} + 2\sqrt{x} + c \end{aligned}$$

$$\begin{aligned} \text{(i)} \int \left(3 + \frac{\sqrt{x+6x^3}}{x} \right) dx &= \int \left(3 + x^{-\frac{1}{2}} + 6x^2 \right) dx \\ &= 3x + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{6}{3}x^3 + c \\ &= 3x + 2x^{\frac{1}{2}} + 2x^3 + c \\ \text{or } &= 3x + 2\sqrt{x} + 2x^3 + c \end{aligned}$$

$$\text{(j)} \int \sqrt{x} (\sqrt{x+3})^2 dx$$

$$= \int x^{\frac{1}{2}} \left(x + 6x^{\frac{1}{2}} + 9 \right) dx$$

$$= \int \left(x^{\frac{3}{2}} + 6x + 9x^{\frac{1}{2}} \right) dx$$

$$= \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{6}{2}x^2 + 9 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= \frac{2}{5}x^{\frac{5}{2}} + 3x^2 + 6x^{\frac{3}{2}} + c$$

$$\text{or} = \frac{2}{5}\sqrt{x^5} + 3x^2 + 6\sqrt{x^3} + c$$

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Edexcel Modular Mathematics for AS and A-Level

Integration

Exercise E, Question 1

Question:

Find the equation of the curve with the given $\frac{dy}{dx}$ that passes through the given point:

(a) $\frac{dy}{dx} = 3x^2 + 2x$; point (2 , 10)

(b) $\frac{dy}{dx} = 4x^3 + \frac{2}{x^3} + 3$; point (1 , 4)

(c) $\frac{dy}{dx} = \sqrt{x} + \frac{1}{4}x^2$; point (4 , 11)

(d) $\frac{dy}{dx} = \frac{3}{\sqrt{x}} - x$; point (4 , 0)

(e) $\frac{dy}{dx} = (x + 2)^2$; point (1 , 7)

(f) $\frac{dy}{dx} = \frac{x^2 + 3}{\sqrt{x}}$; point (0 , 1)

Solution:

(a) $\frac{dy}{dx} = 3x^2 + 2x$

$$\Rightarrow y = \frac{3}{3}x^3 + \frac{2}{2}x^2 + c$$

So $y = x^3 + x^2 + c$

$x = 2, y = 10 \Rightarrow 10 = 8 + 4 + c$

So $c = -2$

So equation is $y = x^3 + x^2 - 2$

(b) $\frac{dy}{dx} = 4x^3 + \frac{2}{x^3} + 3$

$$\Rightarrow y = \frac{4}{4}x^4 - \frac{2}{2}x^{-2} + 3x + c$$

So $y = x^4 - x^{-2} + 3x + c$

$x = 1, y = 4 \Rightarrow 4 = 1 - 1 + 3 + c$

So $c = 1$

So equation is $y = x^4 - x^{-2} + 3x + 1$

(c) $\frac{dy}{dx} = \sqrt{x} + \frac{1}{4}x^2$

$$\Rightarrow y = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{1}{4} \frac{x^3}{3} + c$$

$$\text{So } y = \frac{2}{3}x^{\frac{3}{2}} + \frac{1}{12}x^3 + c$$

$$x = 4, y = 11 \Rightarrow 11 = \frac{2}{3} \times 2^3 + \frac{1}{12} \times 4^3 + c$$

$$\text{So } c = \frac{33}{3} - \frac{32}{3} = \frac{1}{3}$$

$$\text{So equation is } y = \frac{2}{3}x^{\frac{3}{2}} + \frac{1}{12}x^3 + \frac{1}{3}$$

$$(d) \frac{dy}{dx} = \frac{3}{\sqrt{x}} - x$$

$$\Rightarrow y = 3 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{1}{2}x^2 + c$$

$$\text{So } y = 6\sqrt{x} - \frac{1}{2}x^2 + c$$

$$x = 4, y = 0 \Rightarrow 0 = 6 \times 2 - \frac{1}{2} \times 16 + c$$

$$\text{So } c = -4$$

$$\text{So equation is } y = 6\sqrt{x} - \frac{1}{2}x^2 - 4$$

$$(e) \frac{dy}{dx} = (x+2)^2 = x^2 + 4x + 4$$

$$\Rightarrow y = \frac{1}{3}x^3 + 2x^2 + 4x + c$$

$$x = 1, y = 7 \Rightarrow 7 = \frac{1}{3} + 2 + 4 + c$$

$$\text{So } c = \frac{2}{3}$$

$$\text{So equation is } y = \frac{1}{3}x^3 + 2x^2 + 4x + \frac{2}{3}$$

$$(f) \frac{dy}{dx} = \frac{x^2+3}{\sqrt{x}} = x^{\frac{3}{2}} + 3x^{-\frac{1}{2}}$$

$$\Rightarrow y = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + 3 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$\text{So } y = \frac{2}{5}x^{\frac{5}{2}} + 6x^{\frac{1}{2}} + c$$

$$x = 0, y = 1 \Rightarrow 1 = \frac{2}{5} \times 0 + 6 \times 0 + c$$

$$\text{So } c = 1$$

$$\text{So equation of curve is } y = \frac{2}{5}x^{\frac{5}{2}} + 6x^{\frac{1}{2}} + 1$$

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Integration

Exercise E, Question 2

Question:

The curve C , with equation $y = f(x)$, passes through the point $(1, 2)$ and $f'(x) = 2x^3 - \frac{1}{x^2}$. Find the equation of C in the form $y = f(x)$.

Solution:

$$f'(x) = 2x^3 - \frac{1}{x^2} = 2x^3 - x^{-2}$$

$$\text{So } f(x) = \frac{2}{4}x^4 - \frac{x^{-1}}{-1} + c = \frac{1}{2}x^4 + \frac{1}{x} + c$$

$$\text{But } f(1) = 2$$

$$\text{So } 2 = \frac{1}{2} + 1 + c$$

$$\Rightarrow c = \frac{1}{2}$$

$$\text{So } f(x) = \frac{1}{2}x^4 + \frac{1}{x} + \frac{1}{2}$$

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Integration

Exercise E, Question 3

Question:

The gradient of a particular curve is given by $\frac{dy}{dx} = \frac{\sqrt{x+3}}{x^2}$. Given that the curve passes through the point $(9, 0)$, find an equation of the curve.

Solution:

$$\frac{dy}{dx} = \frac{\sqrt{x+3}}{x^2} = x^{-\frac{3}{2}} + 3x^{-2}$$

$$\Rightarrow y = \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + 3 \frac{x^{-1}}{-1} + c$$

$$\text{So } y = -2x^{-\frac{1}{2}} - 3x^{-1} + c = -\frac{2}{\sqrt{x}} - \frac{3}{x} + c$$

$$x = 9, y = 0 \Rightarrow 0 = -\frac{2}{3} - \frac{3}{9} + c$$

$$\text{So } c = \frac{2}{3} + \frac{1}{3} = 1$$

$$\text{So equation is } y = 1 - \frac{2}{\sqrt{x}} - \frac{3}{x}$$

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Integration

Exercise E, Question 4

Question:

A set of curves, that each pass through the origin, have equations $y = f_1(x)$, $y = f_2(x)$, $y = f_3(x)$... where $f'_n(x) = f_{n-1}(x)$ and $f_1(x) = x^2$.

(a) Find $f_2(x)$, $f_3(x)$.

(b) Suggest an expression for $f_n(x)$.

Solution:

(a) $f'_2(x) = f_1(x) = x^2$

So $f_2(x) = \frac{1}{3}x^3 + c$

The curve passes through $(0, 0)$ so $f_2(0) = 0 \Rightarrow c = 0$.

So $f_2(x) = \frac{1}{3}x^3$

$f'_3(x) = \frac{1}{3}x^3$

$f_3(x) = \frac{1}{12}x^4 + c$, but $c = 0$ since $f_3(0) = 0$.

So $f_3(x) = \frac{1}{12}x^4$

(b) $f_2(x) = \frac{x^3}{3}$, $f_3(x) = \frac{x^4}{3 \times 4}$

So power of x is $n + 1$ for $f_n(x)$, denominator is $3 \times 4 \times \dots$ up to $n + 1$:

$$f_n(x) = \frac{x^{n+1}}{3 \times 4 \times 5 \times \dots \times (n+1)}$$

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Integration

Exercise E, Question 5

Question:

A set of curves, with equations $y = f_1(x)$, $y = f_2(x)$, $y = f_3(x)$... all pass through the point $(0, 1)$ and they are related by the property $f_n'(x) = f_{n-1}(x)$ and $f_1(x) = 1$.

Find $f_2(x)$, $f_3(x)$, $f_4(x)$.

Solution:

$$f_2'(x) = f_1(x) = 1$$

$$\Rightarrow f_2(x) = x + c$$

$$\text{But } f_2(0) = 1 \Rightarrow 1 = 0 + c \Rightarrow c = 1$$

$$\text{So } f_2(x) = x + 1$$

$$f_3'(x) = f_2(x) = x + 1$$

$$\Rightarrow f_3(x) = \frac{1}{2}x^2 + x + c$$

$$\text{But } f_3(0) = 1 \Rightarrow 1 = 0 + c \Rightarrow c = 1$$

$$\text{So } f_3(x) = \frac{1}{2}x^2 + x + 1$$

$$f_4'(x) = f_3(x) = \frac{1}{2}x^2 + x + 1$$

$$\Rightarrow f_4(x) = \frac{1}{6}x^3 + \frac{1}{2}x^2 + x + c$$

$$\text{But } f_4(0) = 1 \Rightarrow 1 = 0 + c \Rightarrow c = 1$$

$$\text{So } f_4(x) = \frac{1}{6}x^3 + \frac{1}{2}x^2 + x + 1$$

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Integration

Exercise F, Question 1

Question:

Find:

$$(a) \int (x + 1)(2x - 5) dx$$

$$(b) \int \left(x^{\frac{1}{3}} + x^{-\frac{1}{3}} \right) dx.$$

Solution:

$$\begin{aligned} (a) \int (x + 1)(2x - 5) dx &= \int (2x^2 - 3x - 5) dx \\ &= 2 \frac{x^3}{3} - 3 \frac{x^2}{2} - 5x + c \\ &= \frac{2}{3}x^3 - \frac{3}{2}x^2 - 5x + c \end{aligned}$$

$$\begin{aligned} (b) \int \left(x^{\frac{1}{3}} + x^{-\frac{1}{3}} \right) dx &= \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + \frac{x^{\frac{2}{3}}}{\frac{2}{3}} + c \\ &= \frac{3}{4}x^{\frac{4}{3}} + \frac{3}{2}x^{\frac{2}{3}} + c \end{aligned}$$

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Edexcel Modular Mathematics for AS and A-Level

Integration

Exercise F, Question 2

Question:

The gradient of a curve is given by $f'(x) = x^2 - 3x - \frac{2}{x^2}$. Given that the curve passes through the point $(1, 1)$, find the equation of the curve in the form $y = f(x)$.

Solution:

$$f'(x) = x^2 - 3x - \frac{2}{x^2} = x^2 - 3x - 2x^{-2}$$

$$\text{So } f(x) = \frac{x^3}{3} - 3 \frac{x^2}{2} - 2 \frac{x^{-1}}{-1} + c$$

$$\text{So } f(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 + \frac{2}{x} + c$$

$$\text{But } f\left(\begin{matrix} 1 \\ 1 \end{matrix}\right) = 1 \Rightarrow \frac{1}{3} - \frac{3}{2} + 2 + c = 1$$

$$\text{So } c = \frac{1}{6}$$

$$\text{So the equation is } y = \frac{1}{3}x^3 - \frac{3}{2}x^2 + \frac{2}{x} + \frac{1}{6}$$

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Integration

Exercise F, Question 3

Question:

Find

$$(a) \int (8x^3 - 6x^2 + 5) dx$$

$$(b) \int \left(5x + 2 \right) x^{\frac{1}{2}} dx.$$

Solution:

$$(a) \int (8x^3 - 6x^2 + 5) dx$$

$$= 8 \frac{x^4}{4} - 6 \frac{x^3}{3} + 5x + c$$

$$= 2x^4 - 2x^3 + 5x + c$$

$$(b) \int \left(5x + 2 \right) x^{\frac{1}{2}} dx$$

$$= \int \left(5x^{\frac{3}{2}} + 2x^{\frac{1}{2}} \right) dx$$

$$= 5 \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + 2 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= 2x^{\frac{5}{2}} + \frac{4}{3}x^{\frac{3}{2}} + c$$

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Edexcel Modular Mathematics for AS and A-Level

Integration

Exercise F, Question 4

Question:

Given $y = \frac{(x+1)(2x-3)}{\sqrt{x}}$, find $\int y dx$.

Solution:

$$y = \frac{(x+1)(2x-3)}{\sqrt{x}}$$

$$y = \left(2x^2 - x - 3 \right) x^{-\frac{1}{2}}$$

$$y = 2x^{\frac{3}{2}} - x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$$

$$\int y dx = \int \left(2x^{\frac{3}{2}} - x^{\frac{1}{2}} - 3x^{-\frac{1}{2}} \right) dx$$

$$= 2 \frac{x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - 3 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= \frac{4}{5}x^{\frac{5}{2}} - \frac{2}{3}x^{\frac{3}{2}} - 6x^{\frac{1}{2}} + c$$

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Integration

Exercise F, Question 5

Question:

Given that $\frac{dx}{dt} = 3t^2 - 2t + 1$ and that $x = 2$ when $t = 1$, find the value of x when $t = 2$.

Solution:

$$\frac{dx}{dt} = 3t^2 - 2t + 1$$

$$\Rightarrow x = 3 \frac{t^3}{3} - 2 \frac{t^2}{2} + t + c$$

$$\text{So } x = t^3 - t^2 + t + c$$

But when $t = 1$, $x = 2$.

$$\text{So } 2 = 1 - 1 + 1 + c$$

$$\Rightarrow c = 1$$

$$\text{So } x = t^3 - t^2 + t + 1$$

$$\text{When } t = 2, x = 8 - 4 + 2 + 1$$

$$\text{So } x = 7$$

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Integration

Exercise F, Question 6

Question:

Given $y = 3x^{\frac{1}{2}} + 2x^{-\frac{1}{2}}$, $x > 0$, find $\int y dx$.

Solution:

$$\int y dx = \int \left(3x^{\frac{1}{2}} + 2x^{-\frac{1}{2}} \right) dx$$

$$= 3 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 2 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= 2x^{\frac{3}{2}} + 4x^{\frac{1}{2}} + c$$

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Integration

Exercise F, Question 7

Question:

Given that $\frac{dx}{dt} = (t + 1)^2$ and that $x = 0$ when $t = 2$, find the value of x when $t = 3$.

Solution:

$$\frac{dx}{dt} = (t + 1)^2 = t^2 + 2t + 1$$

$$\Rightarrow x = \frac{t^3}{3} + 2 \frac{t^2}{2} + t + c$$

But $x = 0$ when $t = 2$.

$$\text{So } 0 = \frac{8}{3} + 4 + 2 + c$$

$$\Rightarrow c = -\frac{26}{3}$$

$$\text{So } x = \frac{1}{3}t^3 + t^2 + t - \frac{26}{3}$$

$$\text{When } t = 3, x = \frac{27}{3} + 9 + 3 - \frac{26}{3}$$

$$\text{So } x = 12 \frac{1}{3} \text{ or } \frac{37}{3}$$

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Integration

Exercise F, Question 8

Question:

Given that $y^{\frac{1}{2}} = x^{\frac{1}{3}} + 3$:

(a) Show that $y = x^{\frac{2}{3}} + Ax^{\frac{1}{3}} + B$, where A and B are constants to be found.

(b) Hence find $\int y dx$. **[E]**

Solution:

$$(a) y^{\frac{1}{2}} = x^{\frac{1}{3}} + 3$$

$$\text{So } y = \left(x^{\frac{1}{3}} + 3 \right)^2$$

$$\text{So } y = \left(x^{\frac{1}{3}} \right)^2 + 6x^{\frac{1}{3}} + 9$$

$$\text{So } y = x^{\frac{2}{3}} + 6x^{\frac{1}{3}} + 9$$

$$(A = 6, B = 9)$$

$$(b) \int y dx = \int \left(x^{\frac{2}{3}} + 6x^{\frac{1}{3}} + 9 \right) dx$$

$$= \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + 6 \frac{x^{\frac{4}{3}}}{\frac{4}{3}} + 9x + c$$

$$= \frac{3}{5}x^{\frac{5}{3}} + \frac{9}{2}x^{\frac{4}{3}} + 9x + c$$

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Integration

Exercise F, Question 9

Question:

Given that $y = 3x^{\frac{1}{2}} - 4x^{-\frac{1}{2}}$ ($x > 0$):

(a) Find $\frac{dy}{dx}$.

(b) Find $\int y dx$. [E]

Solution:

$$y = 3x^{\frac{1}{2}} - 4x^{-\frac{1}{2}}$$

$$(a) \frac{dy}{dx} = \frac{3}{2}x^{-\frac{1}{2}} - 4 \times \left(-\frac{1}{2}\right)x^{-\frac{3}{2}}$$

$$\text{So } \frac{dy}{dx} = \frac{3}{2}x^{-\frac{1}{2}} + 2x^{-\frac{3}{2}}$$

$$(b) \int y dx = \int \left(3x^{\frac{1}{2}} - 4x^{-\frac{1}{2}}\right) dx$$

$$= 3 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - 4 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= 2x^{\frac{3}{2}} - 8x^{\frac{1}{2}} + c$$

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Integration

Exercise F, Question 10

Question:

$$\text{Find } \int \left(x^{\frac{1}{2}} - 4 \right) \left(x^{-\frac{1}{2}} - 1 \right) dx. \text{[E]}$$

Solution:

$$\int \left(x^{\frac{1}{2}} - 4 \right) \left(x^{-\frac{1}{2}} - 1 \right) dx$$

$$= \int \left(1 - 4x^{-\frac{1}{2}} - x^{\frac{1}{2}} + 4 \right) dx$$

$$= \int \left(5 - 4x^{-\frac{1}{2}} - x^{\frac{1}{2}} \right) dx$$

$$= 5x - 4 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= 5x - 8x^{\frac{1}{2}} - \frac{2}{3}x^{\frac{3}{2}} + c$$