

1. A sequence is defined by the recurrence relation

$$u_{n+1} = \sqrt{\left(\frac{u_n}{2} + \frac{a}{u_n}\right)}, \quad n = 1, 2, 3, \dots,$$

where a is a constant.

- (a) Given that $a = 20$ and $u_1 = 3$, find the values of u_2 , u_3 and u_4 , giving your answers to 2 decimal places.

(3)

- (b) Given instead that $u_1 = u_2 = 3$,

- (i) calculate the value of a ,

(3)

- (ii) write down the value of u_5 .

(1)

2. The equation $x^2 + 5kx + 2k = 0$, where k is a constant, has real roots.

- (a) Prove that $k(25k - 8) \geq 0$.

(2)

- (b) Hence find the set of possible values of k .

(4)

- (c) Write down the values of k for which the equation $x^2 + 5kx + 2k = 0$ has equal roots.

(1)

3. (a) Given that $3^x = 9^{y-1}$, show that $x = 2y - 2$.

(2)

- (b) Solve the simultaneous equations

$$x = 2y - 2,$$

$$x^2 = y^2 + 7.$$

(6)

4. The curve C with equation $y = f(x)$ is such that

$$\frac{dy}{dx} = 3\sqrt{x} + \frac{12}{\sqrt{x}}, \quad x > 0.$$

(a) Show that, when $x = 8$, the exact value of $\frac{dy}{dx}$ is $9\sqrt{2}$. (3)

The curve C passes through the point $(4, 30)$.

(b) Using integration, find $f(x)$. (6)

5. The points A and B have coordinates $(4, 6)$ and $(12, 2)$ respectively.

The straight line l_1 passes through A and B .

(a) Find an equation for l_1 in the form $ax + by = c$, where a , b and c are integers. (4)

The straight line l_2 passes through the origin and has gradient -4 .

(b) Write down an equation for l_2 . (1)

The lines l_1 and l_2 intersect at the point C .

(c) Find the exact coordinates of the mid-point of AC . (5)

6.
$$f(x) = 9 - (x - 2)^2$$

(a) Write down the maximum value of $f(x)$. (1)

(b) Sketch the graph of $y = f(x)$, showing the coordinates of the points at which the graph meets the coordinate axes. (5)

The points A and B on the graph of $y = f(x)$ have coordinates $(-2, -7)$ and $(3, 8)$ respectively.

(c) Find, in the form $y = mx + c$, an equation of the straight line through A and B . (4)

(d) Find the coordinates of the point at which the line AB crosses the x -axis. (2)

The mid-point of AB lies on the line with equation $y = kx$, where k is a constant.

(e) Find the value of k . (2)

7. For the curve C with equation $y = x^4 - 8x^2 + 3$,

(a) find $\frac{dy}{dx}$, (2)

The point A , on the curve C , has x -coordinate 1.

(b) Find an equation for the normal to C at A , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers. (5)

8.

$$f(x) = \frac{(x^2 - 3)^2}{x^3}, x \neq 0.$$

(a) Show that $f(x) \equiv x - 6x^{-1} + 9x^{-3}$. (2)

(b) Hence, or otherwise, differentiate $f(x)$ with respect to x . (3)

END