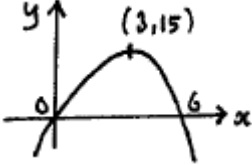
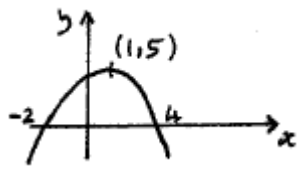
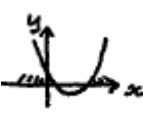


June 2005  
6663 Core Mathematics C1  
Mark Scheme

Question Number	Scheme	Marks
1. (a)	<u>2</u>	Penalise ±
(b)	$8^{-\frac{2}{3}} = \frac{1}{\sqrt[3]{64}} \text{ or } \frac{1}{(a)^2} \text{ or } \frac{1}{\sqrt[3]{8^2}} \text{ or } \frac{1}{8^{\frac{2}{3}}}$ $= \frac{1}{4} \text{ or } 0.25$	Allow ±
(b)	M1 for understanding that “-“ power means reciprocal $8^{\frac{2}{3}} = 4$ is M0A0 and $-\frac{1}{4}$ is M1A0	B1 (1)  M1 A1 (2) <b>(3)</b>
2. (a)	$\frac{dy}{dx} = 6 + 8x^{-3}$	$x^n \rightarrow x^{n-1}$  both ( $6x^0$ is OK)
(b)	$\int (6x - 4x^{-2}) dx = \frac{6x^2}{2} + 4x^{-1} + c$	M1 A1 (2)  M1 A1 A1 (3) <b>(5)</b>
(b)	In (a) and (b) M1 is for a correct power of $x$ in at least one term. This could be 6 in (a) or $+c$ in (b) 1 <sup>st</sup> A1 for one correct term in $x$ : $\frac{6x^2}{2}$ <u>or</u> $+4x^{-1}$ (or better simplified versions) 2 <sup>nd</sup> A1 for all 3 terms as printed or better in one line. N.B. M1A0A1 is not possible. SC. For integrating their answer to part (a) just allow the M1 if $+c$ is present	

Question Number	Scheme	Marks
3. (a)	$x^2 - 8x - 29 \equiv (x - 4)^2 - 45$ $(x \pm 4)^2$ $(x - 4)^2 - 16 + (-29)$ $(x \pm 4)^2 - 45$	M1 A1 A1  (3)
ALT	Compare coefficients $-8 = 2a$ equation for $a$ $a = -4$ <b>AND</b> $a^2 + b = -29$ $b = -45$	M1 A1 A1  (3)
(b)	$(x - 4)^2 = 45$ $\Rightarrow x - 4 = \pm\sqrt{45}$ $x = 4 \pm 3\sqrt{5}$ (follow through their $a$ and $b$ from (a)) $c = 4$ $d = 3 (\pm \text{OK})$	M1 A1 A1  (3)  <b>(6)</b>
(a)	M1 for $(x \pm 4)^2$ or an equation for $a$ (allow sign error $\pm 4$ or $\pm 8$ on ALT) 1stA1 for $(x - 4)^2 - 16(-29)$ can ignore -29 <u>or</u> for stating $a = -4$ and an equation for $b$ 2 <sup>nd</sup> A1 for $b = -45$  Note M1A0 A1 is possible for $(x + 4)^2 - 45$  <b>N.B. On EPEN these marks are called B1M1A1 but apply them as M1A1A1</b>	
(b)	M1 for a full method leading to $x - 4 = \dots$ or $x = \dots$ (condone $x - 4 = \sqrt{-n}$ )  N.B. $(x - 4)^2 - 45 = 0$ leading to $(x - 4) \pm \sqrt{45} = 0$ is M0A0A0  A1 for $c$ and A1 for $d$ N.B. M1 and A1 for $c$ do not need $\pm$ (so this is a special case for the formula method) but $\pm$ must be present for the $d$ mark)  <u>Note</u> Use of formula that ends with $\frac{8 \pm 6\sqrt{5}}{2}$ scores M1 A1 A0 (but must be $\sqrt{5}$ ) i.e. only penalise non-integers by one mark.	

Question Number	Scheme	Marks
4. (a)		Shape Points B1 B1 (2)
(b)		M1  -2 and 4 max A1 A1 (3) <b>(5)</b>
(a)	Marks for shape: graphs must have curved sides and round top. Don't penalise twice. (If both graphs are really straight lines then penalise B0 in part (a) only) 1 <sup>st</sup> B1 for $\cap$ shape through (0, 0) and (k, 0) where $k > 0$ 2 <sup>nd</sup> B1 for max at (3, 15) and 6 labelled or (6, 0) seen Condone (15, 3) if 3 and 15 are correct on axes. Similarly (5, 1) in (b)	
(b)	M1 for $\cap$ shape <u>NOT</u> through (0, 0) but must cut x-axis twice. 1 <sup>st</sup> A1 for -2 and 4 labelled or (-2, 0) and (4, 0) seen 2 <sup>nd</sup> A1 for max at (1, 5). Must be clearly in 1 <sup>st</sup> quadrant	
5.	$x = 1 + 2y$ and sub $\rightarrow (1 + 2y)^2 + y^2 = 29$ $\Rightarrow 5y^2 + 4y - 28 (= 0)$ i.e. $(5y + 14)(y - 2) = 0$ $(y = 2)$ or $-\frac{14}{5}$ (o.e.)  $y = 2 \Rightarrow x = 1 + 4 = 5$ ; $y = -\frac{14}{5} \Rightarrow x = -\frac{23}{5}$ (o.e.)	M1 A1 M1  (both) A1  M1A1 f.t. <b>(6)</b>
	1 <sup>st</sup> M1 Attempt to sub leading to equation in 1 variable Condone sign error such as $1 - 2y$ , $x = -(1 + 2y)$ penalise 1 <sup>st</sup> A1 only 1 <sup>st</sup> A1 Correct 3TQ (condone = 0 missing) 2 <sup>nd</sup> M1 Attempt to solve 3TQ leading to 2 values for y. 2 <sup>nd</sup> A1 Condone mislabelling $x =$ for $y = \dots$ but then M0A0 in part (c). 3 <sup>rd</sup> M1 Attempt to find at least one $x$ value (must use a correct equation) 3 <sup>rd</sup> A1 f.t. f.t. only in $x = 1 + 2y$ (3sf if not exact) Both values.  N.B False squaring. (e.g. $x^2 + 4y^2 = 1$ ) can only score the last 2 marks.	

Question Number	Scheme	Marks		
6. (a)	$6x + 3 > 5 - 2x \Rightarrow 8x > 2$ $x > \frac{1}{4} \text{ or } 0.25 \text{ or } \frac{2}{8}$	M1 A1  (2)		
6. (b)	$(2x - 1)(x - 3) (> 0)$ <p>Critical values <math>x = \frac{1}{2}, 3</math></p>  <p>Choosing "outside" region</p> $x > 3 \text{ or } x < \frac{1}{2}$	M1 A1 (both)  M1 A1 f.t.  (4)		
(c)	$x > 3 \text{ or } \frac{1}{4} < x < \frac{1}{2}$ <p style="text-align: right;">[(3, ∞) or (1/4, 1/2) is OK]</p>	B1f.t. B1f.t. (2)  <b>(8)</b>		
(a)	M1 Multiply out and collect terms (allow one slip and allow use of = here)			
(b)	1 <sup>st</sup> M1 Attempting to factorise 3TQ → x = ...			
	2 <sup>nd</sup> M1 Choosing the outside region			
	2 <sup>nd</sup> A1 f.t. f.t. their critical values N.B.(x>3, x > 1/2 is M0A0)			
(c)	<b>f.t. their answers to (a) and (b)</b> 1 <sup>st</sup> B1 a correct f.t. leading to an <u>infinite</u> region 2 <sup>nd</sup> B1 a correct f.t. leading to a <u>finite</u> region  Penalise ≤ or ≥ once only at first offence. For $p < x < q$ where $p > q$ penalise the final A1 in (b) .			
e.g.	(a) $x > \frac{1}{4}$ $x > \frac{1}{4}$	(b) $\frac{1}{2} < x < 3$ $x > 3, x > \frac{1}{2}$	(c) $\frac{1}{2} < x < 3$ $x > 3$	Mark  B0 B1 B1 B0

Question Number	Scheme	Marks
7. (a)	$(3 - \sqrt{x})^2 = 9 - 6\sqrt{x} + x$ $\div by \sqrt{x} \rightarrow 9x^{-\frac{1}{2}} - 6 + x^{\frac{1}{2}}$	M1 A1 c.s.o. (2)
(b)	$\int (9x^{-\frac{1}{2}} - 6 + x^{\frac{1}{2}}) dx = \frac{9x^{\frac{1}{2}}}{\frac{1}{2}} - 6x + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} (+c)$ <p>use <math>y = \frac{2}{3}</math> and <math>x = 1</math>: <math>\frac{2}{3} = 18 - 6 + \frac{2}{3} + c</math></p> <p style="text-align: right;"><math>c = -12</math></p> <p>So <math>y = 18x^{\frac{1}{2}} - 6x + \frac{2}{3}x^{\frac{3}{2}} - 12</math></p> <hr/> <p>(a) M1 Attempt to multiply out <math>(3 - \sqrt{x})^2</math>. Must have 3 or 4 terms, allow one sign error  A1 cso Fully correct solution to printed answer. Penalise invisible brackets or wrong working</p> <p>(b) 1<sup>st</sup> M1 Some correct integration: <math>x^n \rightarrow x^{n+1}</math>  A1 At least 2 correct unsimplified terms  Ignore + c  A2 All 3 terms correct (unsimplified)</p> <p>2<sup>nd</sup> M1 Use of <math>y = \frac{2}{3}</math> and <math>x = 1</math> to find <math>c</math>. No + c is M0.  A1c.s.o. for -12. (o.e.) Award this mark if “ <math>c = -12</math> ” stated i.e. not as part of an expression for y  A1f.t. for 3 simplified x terms with <math>y = \dots</math> and a numerical value for <math>c</math>. Follow through their value of <math>c</math> but it must be a number.</p>	M1 A2/1/0 M1 A1 c.s.o. A1f.t. (6) (8)
Question	Scheme	Marks

Number		
<p>8. (a)</p> <p>8. (b)</p> <p>8. (c)</p> <p>ALT</p>	$y - (-4) = \frac{1}{3}(x - 9) \quad \text{or} \quad \frac{y - (-4)}{x - 9} = \frac{1}{3}$ $3y - x + 21 = 0 \quad (\text{o.e.}) \quad (\text{condone 3 terms with integer coefficients e.g. } 3y+21=x)$ <p>Equation of <math>l_2</math> is: <math>y = -2x</math> (o.e.)  Solving <math>l_1</math> and <math>l_2</math>: <math>-6x - x + 21 = 0</math>  <math>p</math> is point where <math>x_p = 3</math>, <math>y_p = -6</math></p> <p><math>(l_1</math> is <math>y = \frac{1}{3}x - 7</math>)    <math>C</math> is <math>(0, -7)</math> or <math>OC = 7</math>  Area of <math>\Delta OCP = \frac{1}{2}OC \times x_p</math>, <math>= \frac{1}{2} \times 7 \times 3 = 10.5</math> or <math>\frac{21}{2}</math></p> <p>By Integration: M1 for <math>\pm \int_0^{x_p} (l_1 - l_2) dx</math>,  B1 ft for correct integration (follow through their <math>l_1</math>), then A1 cao.</p>	<p>M1 A1</p> <p>A1</p> <p>(3)</p> <p>B1 M1</p> <p>A1 A1 f.t. (<math>-2x</math>)</p> <p>(4)</p> <p>B1 f.t.</p> <p>M1 A1 c.a.o.</p> <p>(3)</p> <p>(10)</p>
<p>(a)</p> <p>(b)</p> <p>(c)</p> <p>MR</p>	<p>M1 for full method to find equation of <math>l_1</math>  1st A1 any unsimplified form</p> <p>M1 Attempt to solve two linear equations leading to linear equation in one variable  2<sup>nd</sup> A1 f.t. only f.t. their <math>x_p</math> or <math>y_p</math> in <math>y = -2x</math>  N.B. A fully correct solution by drawing, or correct answer with no working can score all the marks in part (b), but a partially correct solution by drawing only scores the first B1.</p> <p>B1 f.t. Either a correct <math>OC</math> or f.t. from their <math>l_1</math>  M1 for correct attempt in letters or symbols for <math>\Delta OCP</math>  A1 c.a.o.  <math>-\frac{1}{2} \times 7 \times 3</math> scores M1 A0</p> <p>(<math>x</math>-axis for <math>y</math>-axis)  Get <math>C = (21, 0)</math> Area of <math>\Delta OCP = \frac{1}{2}OC \times y_p = \frac{1}{2} \times 21 \times 6 = 63</math> (B0M1A0)</p>	

Question Number	Scheme	Marks
<p>9 (a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p> <p>(e)</p>	$(S \Rightarrow) a + (a + d) + \dots \dots + [a + (n - 1)d]$ $(S \Rightarrow) [a + (n - 1)d] + \dots \dots + a$ $2S = [2a + (n - 1)d] + \dots \dots + [2a + (n - 1)d] \quad \} \text{ either}$ $2S = n[2a + (n - 1)d]$ $S = \frac{n}{2}[2a + (n - 1)d]$ <p>(a = 149, d = -2)</p> $u_{21} = 149 + 20(-2) = \text{£}109$ $S_n = \frac{n}{2}[2 \times 149 + (n - 1)(-2)] \quad (= n(150 - n))$ $S_n = 5000 \Rightarrow n^2 - 150n + 5000 = 0 \quad (*)$ $(n - 100)(n - 50) = 0$ $n = 50 \text{ or } 100$ $u_{100} < 0 \quad \therefore n = 100 \text{ not sensible}$	<p>B1 M1 dM1</p> <p>A1 c.s.o (4)</p> <p>M1 A1 (2)</p> <p>M1 A1 A1 c.s.o (3)</p> <p>M1 A2/1/0 (3)</p> <p>B1 f.t. (1) <b>(13)</b></p>
<p>(a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p> <p>(e)</p>	<p>B1 requires at least 3 terms, must include first and last terms, an adjacent term and dots! There must be + signs for the B1 (or at least implied see snippet 9D)</p> <p>1<sup>st</sup> M1 for reversing series. Must be arithmetic with <math>a</math>, <math>n</math> and <math>d</math> or <math>l</math>. (+ signs not essential here)</p> <p>2<sup>nd</sup> dM1 for adding, must have <math>2S</math> and be a genuine attempt. Either line is sufficient. Dependent on 1<sup>st</sup> M1</p> <p>(NB Allow first 3 marks for use of <math>l</math> for last term but as given for final mark )</p> <p>M1 for using <math>a = 149</math> and <math>d = \pm 2</math> in <math>a + (n - 1)d</math> formula.</p> <p>M1 for using their <math>a, d</math> in <math>S_n</math> A1 any correct expression</p> <p>A1cso for putting <math>S_n = 5000</math> and simplifying to given expression. No wrong work</p> <p><b>NB EPEN has B1M1A1 here but apply marks as M1A1A1 as in scheme</b></p> <p>M1 Attempt to solve leading to <math>n = \dots</math></p> <p>A2/1/0 Give A1A0 for 1 correct value and A1A1 for both correct</p> <p>B1 f.t. Must mention 100 and state <math>u_{100} &lt; 0</math> (or loan paid or equivalent)</p> <p>If giving f.t. then must have <math>n \geq 76</math>.</p>	

Question Number	Scheme	Marks
10 (a)	$x = 3, \quad y = 9 - 36 + 24 + 3 = 0$	B1 (1)
(b)	$\frac{dy}{dx} = \frac{3}{3}x^2 - 2 \times 4 \times x + 8 \quad (x^2 - 8x + 8)$ <p>When <math>x = 3, \quad \frac{dy}{dx} = 9 - 24 + 8 \Rightarrow m = -7</math></p> <p>Equation of tangent: <math>y - 0 = -7(x - 3)</math>  <math>y = -7x + 21</math></p>	M1 A1  M1  M1 A1 c.a.o (5)
(c)	$\frac{dy}{dx} = m \quad \text{gives} \quad x^2 - 8x + 8 = -7$ $(x^2 - 8x + 15 = 0)$ $(x - 5)(x - 3) = 0$ $x = (3) \quad \text{or} \quad 5$ $\therefore y = \frac{1}{3}5^3 - 4 \times 5^2 + 8 \times 5 + 3$ $y = -15\frac{1}{3} \quad \text{or} \quad -\frac{46}{3}$	M1   M1 A1  M1  A1  (5)
(b)	<p>1<sup>st</sup> M1 some correct differentiation (<math>x^n \rightarrow x^{n-1}</math> for one term)</p> <p>1<sup>st</sup> A1 correct unsimplified (all 3 terms)</p> <p>2<sup>nd</sup> M1 substituting <math>x_p (= 3)</math> in their <math>\frac{dy}{dx}</math> clear evidence</p> <p>3<sup>rd</sup> M1 using their <math>m</math> to find tangent at <math>p</math>. The <math>m</math> must be from their <math>\frac{dy}{dx}</math> at <math>x_p (= 3)</math></p> <p>Use of <math>\frac{1}{7}</math> here scores M0A0 but Could get all 3 Ms in Part (c).</p>	(11)
(c)	<p>1<sup>st</sup> M1 forming a correct equation “ their <math>\frac{dy}{dx} = \text{gradient of their tangent}</math>”</p> <p>2<sup>nd</sup> M1 for solving a quadratic based on their <math>\frac{dy}{dx}</math> leading to <math>x = \dots</math> The quadratic could be simply <math>\frac{dy}{dx} = 0</math>.</p> <p>3<sup>rd</sup> M1 for using their <math>x</math> value (obtained from their quadratic) in <math>y</math> to obtain <math>y</math> coordinate. Must have one of the other two M marks to score this.</p>	
MR	<p>For misreading (0, 3) for (3, 0) award B0 and then M1A1 as in scheme. Then allow all M marks but no A ft. (Max 7)</p>	