

Solutionbank C1

Edexcel Modular Mathematics for AS and A-Level

Practice paper C1

Exercise 1, Question 1

Question:

(a) Write down the value of $16^{\frac{1}{2}}$. (1)

(b) Hence find the value of $16^{\frac{3}{2}}$. (2)

Solution:

(a) $16^{\frac{1}{2}} = \sqrt{16} = 4$

(b) $16^{\frac{3}{2}} = \left(16^{\frac{1}{2}}\right)^3 = 4^3 = 64$

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Practice paper C1 Exercise 1, Question 2

Question:

Find $\int (6x^2 + \sqrt{x}) dx$. (4)

Solution:

$$\int \left(6x^2 + x^{\frac{1}{2}} \right) dx$$

$$= 6 \frac{x^3}{3} + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= 2x^3 + \frac{2}{3}x^{\frac{3}{2}} + c$$

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Exercise 1, Question 3

Question:

A sequence $a_1, a_2, a_3, \dots, a_n$ is defined by

$$a_1 = 2, a_{n+1} = 2a_n - 1.$$

(a) Write down the value of a_2 and the value of a_3 . (2)

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(b) Calculate $\sum_{r=1}^5 a_r$. (2)

$r = 1$

Solution:

$$(a) a_2 = 2a_1 - 1 = 4 - 1 = 3$$

$$a_3 = 2a_2 - 1 = 6 - 1 = 5$$

$$(b) a_4 = 2a_3 - 1 = 10 - 1 = 9$$

$$a_5 = 2a_4 - 1 = 18 - 1 = 17$$

5

$$\sum_{r=1}^5 a_r = a_1 + a_2 + a_3 + a_4 + a_5 = 2 + 3 + 5 + 9 + 17 = 36$$

$r = 1$

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Practice paper C1 Exercise 1, Question 4

Question:

(a) Express $(5 + \sqrt{2})^2$ in the form $a + b\sqrt{2}$, where a and b are integers. (3)

(b) Hence, or otherwise, simplify $(5 + \sqrt{2})^2 - (5 - \sqrt{2})^2$. (2)

Solution:

$$(a) (5 + \sqrt{2})^2 = (5 + \sqrt{2})(5 + \sqrt{2}) = 25 + 10\sqrt{2} + 2 = 27 + 10\sqrt{2}$$

$$(b) (5 - \sqrt{2})^2 = (5 - \sqrt{2})(5 - \sqrt{2}) = 25 - 10\sqrt{2} + 2 = 27 - 10\sqrt{2}$$

$$\begin{aligned} & (5 + \sqrt{2})^2 - (5 - \sqrt{2})^2 \\ &= (27 + 10\sqrt{2}) - (27 - 10\sqrt{2}) \\ &= 27 + 10\sqrt{2} - 27 + 10\sqrt{2} \\ &= 20\sqrt{2} \end{aligned}$$

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Practice paper C1 Exercise 1, Question 5

Question:

Solve the simultaneous equations:

$$x - 3y = 6$$

$$3xy + x = 24 \quad (7)$$

Solution:

$$x - 3y = 6$$

$$x = 6 + 3y$$

Substitute into $3xy + x = 24$:

$$3y(6 + 3y) + (6 + 3y) = 24$$

$$18y + 9y^2 + 6 + 3y = 24$$

$$9y^2 + 21y - 18 = 0$$

Divide by 3:

$$3y^2 + 7y - 6 = 0$$

$$(3y - 2)(y + 3) = 0$$

$$y = \frac{2}{3}, y = -3$$

Substitute into $x = 6 + 3y$:

$$y = \frac{2}{3} \Rightarrow x = 6 + 2 = 8$$

$$y = -3 \Rightarrow x = 6 - 9 = -3$$

$$x = -3, y = -3 \text{ or } x = 8, y = \frac{2}{3}$$

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Exercise 1, Question 6

Question:

The points A and B have coordinates $(-3, 8)$ and $(5, 4)$ respectively.
The straight line l_1 passes through A and B .

- (a) Find an equation for l_1 , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers. (4)
- (b) Another straight line l_2 is perpendicular to l_1 and passes through the origin. Find an equation for l_2 . (2)
- (c) The lines l_1 and l_2 intersect at the point P . Use algebra to find the coordinates of P . (3)

Solution:

$$(a) \text{ Gradient of } l_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 8}{5 - (-3)} = -\frac{4}{8} = -\frac{1}{2}$$

Equation for l_1 :

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -\frac{1}{2} \left(x - 5 \right)$$

$$y - 4 = -\frac{1}{2}x + \frac{5}{2}$$

$$\frac{1}{2}x + y - \frac{13}{2} = 0$$

$$x + 2y - 13 = 0$$

(b) For perpendicular lines, $m_1 m_2 = -1$

$$m_1 = -\frac{1}{2}, \text{ so } m_2 = 2$$

Equation for l_2 is $y = 2x$

(c) Substitute $y = 2x$ into $x + 2y - 13 = 0$:

$$x + 4x - 13 = 0$$

$$5x = 13$$

$$x = 2\frac{3}{5}$$

$$y = 2x = 5\frac{1}{5}$$

Coordinates of P are $\left(2\frac{3}{5}, 5\frac{1}{5} \right)$

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Exercise 1, Question 7

Question:

On separate diagrams, sketch the curves with equations:

(a) $y = \frac{2}{x}$, $-2 \leq x \leq 2$, $x \neq 0$ (2)

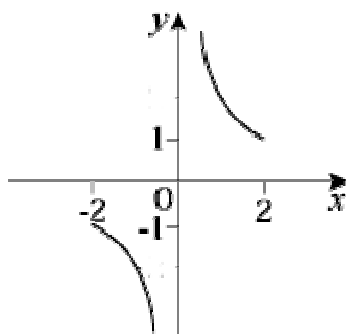
(b) $y = \frac{2}{x} - 4$, $-2 \leq x \leq 2$, $x \neq 0$ (3)

(c) $y = \frac{2}{x+1}$, $-2 \leq x \leq 2$, $x \neq -1$ (3)

In each part, show clearly the coordinates of any point at which the curve meets the x -axis or the y -axis.

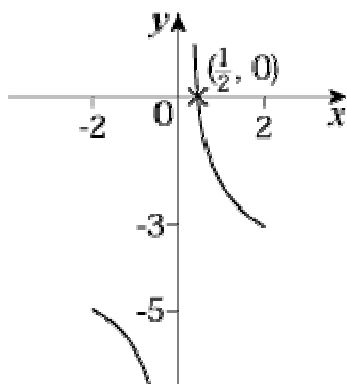
Solution:

(a)



$$y = \frac{2}{x}$$

(b) Translation of -4 units parallel to the y -axis.



$$y = \frac{2}{x} - 4$$

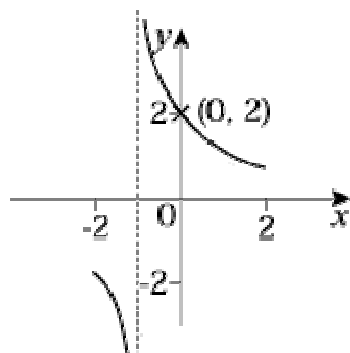
Curve crosses the x -axis where $y = 0$:

$$\frac{2}{x} - 4 = 0$$

$$\frac{2}{x} = 4$$

$$x = \frac{1}{2}$$

(c) Translation of -1 unit parallel to the x -axis.



$$y = \frac{2}{x+1}$$

The line $x = -1$ is an asymptote.

Curve crosses the y -axis where $x = 0$:

$$y = \frac{2}{0+1} = 2$$

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Exercise 1, Question 8

Question:

In the year 2007, a car dealer sold 400 new cars. A model for future sales assumes that sales will increase by x cars per year for the next 10 years, so that $(400 + x)$ cars are sold in 2008, $(400 + 2x)$ cars are sold in 2009, and so on. Using this model with $x = 30$, calculate:

- (a) The number of cars sold in the year 2016. (2)
- (b) The total number of cars sold over the 10 years from 2007 to 2016. (3)
The dealer wants to sell at least 6000 cars over the 10-year period.
Using the same model:
- (c) Find the least value of x required to achieve this target. (4)

Solution:

(a) $a = 400$, $d = x = 30$
 $T_{10} = a + 9d = 400 + 270 = 670$
 670 cars sold in 2016

$$(b) S_n = \frac{1}{2}n \left[2a + \left(n - 1 \right) d \right]$$

So $S_{10} = 5 [(2 \times 400) + (9 \times 30)] = 5 \times 1070 = 5350$
 5350 cars sold from 2007 to 2016

(c) S_{10} required to be at least 6000:

$$\frac{1}{2}n \left[2a + \left(n - 1 \right) d \right] \geq 6000$$

$$5(800 + 9x) \geq 6000$$

$$4000 + 45x \geq 6000$$

$$45x \geq 2000$$

$$x \geq 44 \frac{4}{9}$$

To achieve the target, $x = 45$.

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Exercise 1, Question 9

Question:

(a) Given that

$$x^2 + 4x + c = (x + a)^2 + b$$

where a , b and c are constants:

(i) Find the value of a . (1)

(ii) Find b in terms of c . (2)

Given also that the equation $x^2 + 4x + c = 0$ has unequal real roots:

(iii) Find the range of possible values of c . (2)

(b) Find the set of values of x for which:

(i) $3x < 20 - x$, (2)

(ii) $x^2 + 4x - 21 > 0$, (4)

(iii) both $3x < 20 - x$ and $x^2 + 4x - 21 > 0$. (2)

Solution:

(a) (i) $x^2 + 4x + c = (x + 2)^2 - 4 + c = (x + 2)^2 + (c - 4)$
So $a = 2$

(ii) $b = c - 4$

(iii) For unequal real roots:

$$(x + 2)^2 - 4 + c = 0$$

$$(x + 2)^2 = 4 - c$$

$$4 - c > 0$$

$$c < 4$$

(b) (i) $3x < 20 - x$

$$3x + x < 20$$

$$4x < 20$$

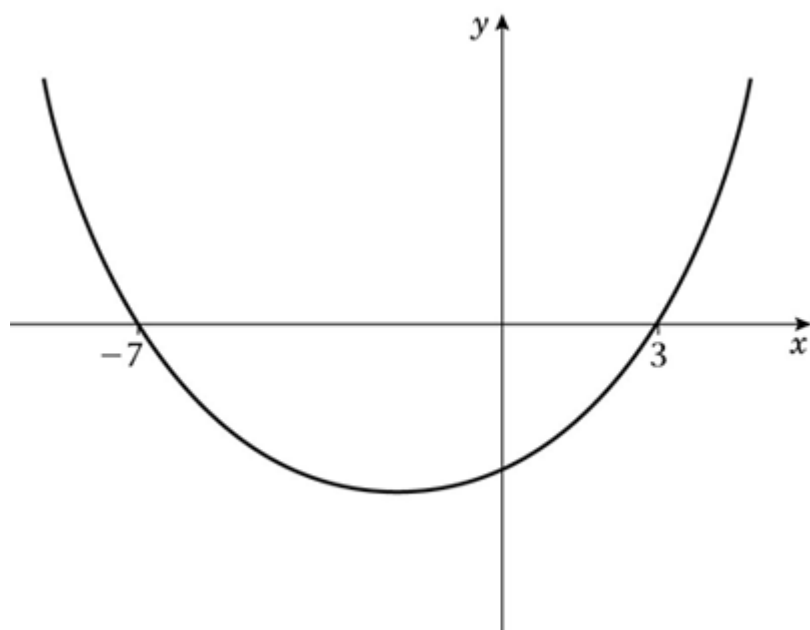
$$x < 5$$

(ii) Solve $x^2 + 4x - 21 = 0$:

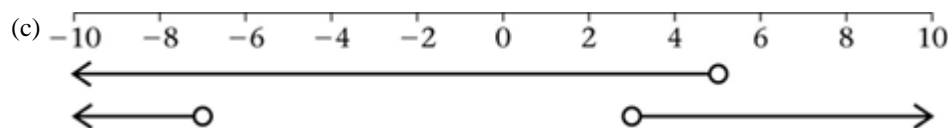
$$(x + 7)(x - 3) = 0$$

$$x = -7, x = 3$$

Sketch of $y = x^2 + 4x - 21$:



$$x^2 + 4x - 21 > 0 \text{ when } x < -7 \text{ or } x > 3$$



Both inequalities are true when
 $x < -7$ or $3 < x < 5$

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Exercise 1, Question 10

Question:

(a) Show that $\frac{(3x-4)^2}{x^2}$ may be written as $P + \frac{Q}{x} + \frac{R}{x^2}$ where P , Q and R are constants to be found. (3)

(b) The curve C has equation $y = \frac{(3x-4)^2}{x^2}$, $x \neq 0$. Find the gradient of the tangent to C at the point on C where $x = -2$. (5)

(c) Find the equation of the normal to C at the point on C where $x = -2$, giving your answer in the form $ax + by + c = 0$, where a , b and c are integers. (5)

Solution:

$$(a) (3x-4)^2 = (3x-4)(3x-4) = 9x^2 - 24x + 16$$

$$\frac{(3x-4)^2}{x^2} = \frac{9x^2 - 24x + 16}{x^2} = 9 - \frac{24}{x} + \frac{16}{x^2}$$

$$P = 9, Q = -24, R = 16$$

$$(b) y = 9 - 24x^{-1} + 16x^{-2}$$

$$\frac{dy}{dx} = 24x^{-2} - 32x^{-3}$$

$$\text{Where } x = -2, \frac{dy}{dx} = \frac{24}{(-2)^2} - \frac{32}{(-2)^3} = \frac{24}{4} + \frac{32}{8} = 10$$

Gradient of the tangent is 10.

$$(c) \text{ Where } x = -2, y = 9 - \frac{24}{(-2)} + \frac{16}{(-2)^2} = 9 + 12 + 4 = 25$$

$$\text{Gradient of the normal} = \frac{-1}{\text{Gradient of tangent}} = -\frac{1}{10}$$

The equation of the normal at $(-2, 25)$ is

$$y - 25 = -\frac{1}{10} \left[x - \begin{pmatrix} -2 \\ \end{pmatrix} \right]$$

Multiply by 10:

$$10y - 250 = -x - 2$$

$$x + 10y - 248 = 0$$