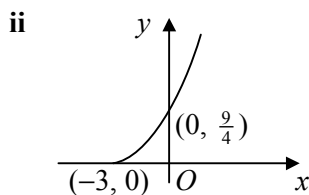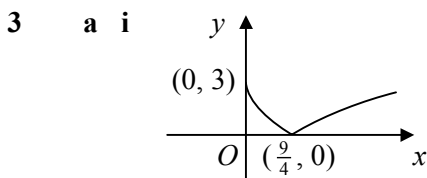**1** **a** $f(1) = 2 + \log_4 1 = 2$
$ff(1) = f(2) = 2 + \log_4 2$
$= 2 + \frac{1}{2} = \frac{5}{2}$
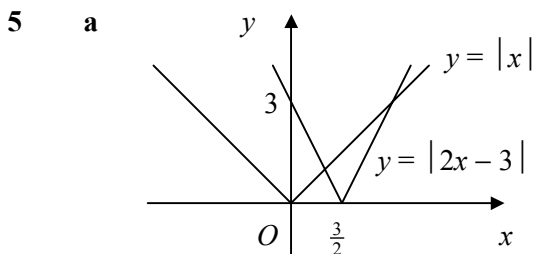
b $2 + \log_4 x = 0$
$\log_4 x = -2$
$x = 4^{-2} = \frac{1}{16}$

c $y = 2 + \log_4 x$
swap $x = 2 + \log_4 y$
$\log_4 y = x - 2$
$y = 4^{x-2}$
$f^{-1}(x) = 4^{x-2}, \ x \in \mathbb{R}$

2 **a** $f(-2a) = \left| -7a \right| = 7a$
$ff(-2a) = f(7a) = \left| 20a \right| = 20a$

b

(0, a)

O $(\frac{1}{3}a, 0)$ x

c $3x - a = x \quad \Rightarrow \quad x = \frac{1}{2}a$
$-(3x - a) = x \quad \Rightarrow \quad x = \frac{1}{4}a$
$\therefore \ x = \frac{1}{4}a, \ \frac{1}{2}a$

3 **a** **i**

(0, 3)

O $(\frac{9}{4}, 0)$ x

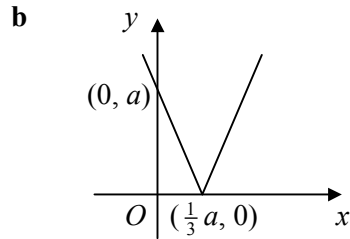**ii**

$(0, \frac{9}{4})$
$(-3, 0)$ O x

b $f(0) = b = -3$
$f(\frac{9}{4}) = \frac{3}{2}a - 3 = 0$
$\therefore \ a = 2, b = -3$

c $y = 2x^{\frac{1}{2}} - 3$
swap $x = 2y^{\frac{1}{2}} - 3$
$y^{\frac{1}{2}} = \frac{1}{2}(x + 3)$
$f^{-1}(x) = \frac{1}{4}(x + 3)^2, \ x \in \mathbb{R}, \ x \geq -3$

4 **a** $ff(x) = f(\frac{x+2}{x-1}) = \dfrac{\frac{x+2}{x-1} + 2}{\frac{x+2}{x-1} - 1}$
$= \dfrac{x + 2 + 2(x - 1)}{x + 2 - (x - 1)}$
$= \frac{3x}{3} = x$

b $f^{-1}(x) = \frac{x+2}{x-1}, \ x \in \mathbb{R}, \ x \neq 1$

c $gf(x) = 2f(x) - 3 = 0$
$\therefore \ f(x) = \frac{x+2}{x-1} = \frac{3}{2}$
$2(x + 2) = 3(x - 1)$
$2x + 4 = 3x - 3$
$x = 7$

5 **a**

$y = \left| x \right|$
3
$y = \left| 2x - 3 \right|$
O $\frac{3}{2}$ x

b $x = 2x - 3 \ \Rightarrow \ x = 3$
$x = -(2x - 3) \ \Rightarrow x = 1$
$\therefore \ x = 1, 3$

6 **a**

(0, 2)
$(-2, 0)$ O $(2, 0)$ x

b $f(3) = 4 - 9 = -5$
$ff(3) = f(-5) = -5 + 2 = -3$

c $x + 2 = 1 \ \Rightarrow \ x = -1$
$4 - x^2 = 1 \ \Rightarrow \ x^2 = 3$
$x \geq 1 \ \therefore \ x = \sqrt{3}$
$\therefore \ x = -1, \sqrt{3}$

7   **a**   **i** $y = kx + 2$

           swap $x = ky + 2$

$$y = \frac{x-2}{k}$$

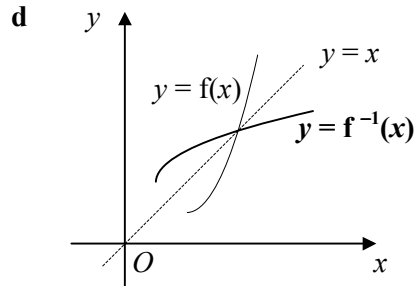$$f^{-1}(x) = \frac{x-2}{k}, \ x \in \mathbb{R}$$

       ii $fg(x) = f(x - 3k)$

$$= k(x - 3k) + 2$$

$$fg(x) = kx - 3k^2 + 2, \ x \in \mathbb{R}$$

   b $4 = 7k - 3k^2 + 2$

$$3k^2 - 7k + 2 = 0$$

$$(3k - 1)(k - 2) = 0$$

$$k = \tfrac{1}{3}, \ 2$$

8   **a** $f(x) = (x - 2)^2 - 4 + 5 = (x - 2)^2 + 1$

   b $f(x) \geq 1$

   c $y = (x - 2)^2 + 1$

       swap $x = (y - 2)^2 + 1$

$$y = 2 \pm \sqrt{x - 1}$$

       (domain of f $\Rightarrow$ +)

$$f^{-1}(x) = 2 + \sqrt{x - 1}, \ x \in \mathbb{R}, \ x \geq 1$$
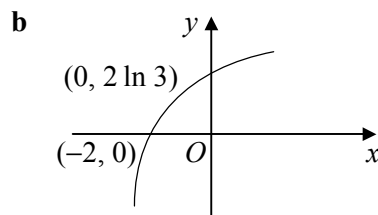
   d

      the graphs are reflections of each other in the line $y = x$

9   **a** $f(-2) = 8$

$$gf(-2) = g(8) = 15\tfrac{7}{8}$$

   b $fg(x) = f(2x - \frac{1}{x}) = (2x - \frac{1}{x})^2 + 4$

$$= 4x^2 - 4 + \frac{1}{x^2} + 4$$

$$fg(x) = 4x^2 + \frac{1}{x^2}$$

   c $4x^2 + \dfrac{1}{x^2} = 5$

$$4x^4 - 5x^2 + 1 = 0$$

$$(4x^2 - 1)(x^2 - 1) = 0$$

$$x^2 = \tfrac{1}{4}, \ 1$$

$$x = \pm\tfrac{1}{2}, \ \pm 1$$

10   **a** $y = e^{\frac{1}{2}x} - 3$

       swap $x = e^{\frac{1}{2}y} - 3$

$$\tfrac{1}{2}y = \ln(x + 3)$$

$$y = 2\ln(x + 3)$$

$$f^{-1}(x) = 2\ln(x + 3), \ x \in \mathbb{R}, \ x > -3$$

   b

   c $g(4) = \ln 9 = 2\ln 3$

       $fg(4) = f(2\ln 3) = 0$

   d $2\ln(x + 3) = \ln(x + 5)$

$$(x + 3)^2 = x + 5$$

$$x^2 + 5x + 4 = 0$$

$$(x + 4)(x + 1) = 0$$

$$x > -3 \ \therefore \ x = -1$$