C3 FUNCTIONS *Worksheet G*

1 $\mathrm{f} : x \to 2 + \log_4 x,\ x \in \mathbb{R},\ x > 0.$

a Evaluate ff(1). **(3)**

b Solve the equation $\mathrm{f}(x) = 0$. **(2)**

c Find the inverse function $\mathrm{f}^{-1}(x)$. **(3)**

2 The function f is defined by

$$\mathrm{f} : x \to |3x - a|,\ x \in \mathbb{R}.$$

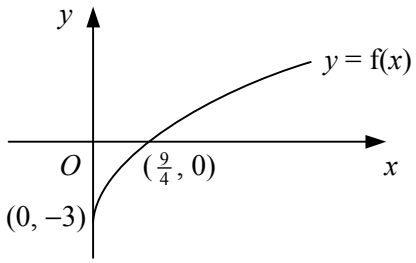
where a is a positive constant.

a Find ff($-2a$). **(2)**

b Sketch the graph $y = \mathrm{f}(x)$, showing the coordinates of any points where the graph meets the coordinate axes. **(3)**

c Solve the equation $\mathrm{f}(x) = x$, giving your answers in terms of a. **(3)**

3



The diagram shows the graph of $y = \mathrm{f}(x)$ which meets the x-axis at the point $(\frac{9}{4}, 0)$ and the y-axis at the point $(0, -3)$.

a Sketch on separate diagrams the graphs of

i $y = |\mathrm{f}(x)|$,

ii $y = \mathrm{f}^{-1}(x)$. **(4)**

Given that $\mathrm{f}(x)$ is of the form $\mathrm{f}(x) \equiv ax^{\frac{1}{2}} + b,\ x \in \mathbb{R},\ x \geq 0$,

b find the values of the constants a and b, **(3)**

c find an expression for $\mathrm{f}^{-1}(x)$. **(3)**

4 The function f is defined by

$$\mathrm{f} : x \to \frac{x+2}{x-1},\ x \in \mathbb{R},\ x \neq 1.$$

a Show that $\mathrm{ff}(x) = x$ for all $x \in \mathbb{R},\ x \neq 1$. **(3)**

b Hence, write down an expression for $\mathrm{f}^{-1}(x)$. **(1)**

The function g is defined by

$$\mathrm{g} : x \to 2x - 3,\ x \in \mathbb{R}.$$

c Solve the equation $\mathrm{gf}(x) = 0$. **(4)**

5 **a** Sketch on the same set of axes the graphs of $y = |x|$ and $y = |2x - 3|$. **(3)**

b Hence, or otherwise, solve the equation

$$|x| = |2x - 3|.$$ **(4)**

6 The function f(x) is defined for all real values of x by

$$f(x) = x + 2, \quad x < 1,$$
$$f(x) = 4 - x^2, \quad x \geq 1.$$

a Sketch the graph of f(x) showing the coordinates of any points of intersection with the coordinate axes. **(4)**

b Evaluate ff(3). **(2)**

c Solve the equation f(x) = 1. **(4)**

7 The functions f and g are defined by

$$f : x \to kx + 2, \ x \in \mathbb{R},$$
$$g : x \to x - 3k, \ x \in \mathbb{R},$$

where k is a constant.

a Find expressions in terms of k for

i $f^{-1}(x)$,

ii fg(x). **(4)**

Given that fg(7) = 4,

b find the two possible values of k. **(3)**

8 $$f(x) \equiv x^2 - 4x + 5, \ x \in \mathbb{R}, \ x \geq 2.$$

a Express f(x) in the form $a(x + b)^2 + c$. **(2)**

b State the range of f. **(1)**

c Find an expression for $f^{-1}(x)$ and state its domain. **(4)**

d Sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same diagram and state the relationship between the graphs. **(4)**

9 The functions f and g are defined by

$$f : x \to x^2 + 4, \ x \in \mathbb{R},$$
$$g : x \to 2x - \frac{1}{x}, \ x \in \mathbb{R}, \ x \neq 0.$$

a Evaluate gf(−2). **(2)**

b Find and simplify an expression for fg(x). **(3)**

c Find the values of x for which fg(x) = 5. **(4)**

10 The function f is given by

$$f : x \to e^{\frac{1}{2}x} - 3, \ x \in \mathbb{R}.$$

a Find $f^{-1}(x)$ and state its domain. **(4)**

b Sketch the curve $y = f^{-1}(x)$, showing the coordinates of any points of intersection with the coordinate axes. **(3)**

The function g is given by

$$g : x \to \ln (x + 5), \ x \in \mathbb{R}, \ x > -5.$$

c Evaluate fg(4). **(2)**

d Solve the equation $f^{-1}(x) = g(x)$. **(4)**