C3 FUNCTIONS *Worksheet H*

1 The function f is defined by

$$f : x \to 3 + \ln (x + 2),\ x \in \mathbb{R},\ x \geq k,$$

where k is a constant.

Given that the range of f is $f(x) \geq 3$,

a find the value of k, **(3)**

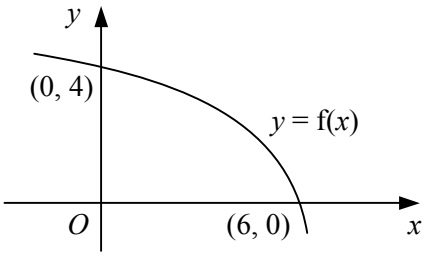
b find $f^{-1}(x)$, stating its domain clearly. **(4)**

The function g is defined by

$$g : x \to 4 + \ln (x - 1),\ x \in \mathbb{R},\ x > 1.$$

c Find, in terms of e, the value of x such that $f(x) = g(x)$. **(4)**

2



The diagram shows the curve with equation $y = f(x)$ which crosses the coordinate axes at the points (0, 4) and (6, 0).

Showing the coordinates of any points of intersection with the axes, sketch on separate diagrams the curves

a $y = f(|x|)$, **(2)**

b $y = 4 - f(x)$, **(2)**

c $y = 2f(3x)$. **(3)**

3 The functions f and g are given by

$$f(x) \equiv \frac{x}{x+2},\ x \in \mathbb{R},\ x \neq -2,$$

$$g(x) \equiv \frac{3}{x},\ x \in \mathbb{R},\ x \neq 0$$

a Solve the equation $fg(x) = 4$. **(4)**

b Find $f^{-1}(x)$, stating its domain clearly. **(4)**

c Solve the equation $f(x) = f^{-1}(x)$. **(3)**

4 The function f is defined by

$$f(x) \equiv x^2 - 2x - 9,\ x \in \mathbb{R},\ x \geq k.$$

a Find the minimum value of the constant k for which $f^{-1}(x)$ exists. **(3)**

Given that k takes the value found in part **a**,

b solve the equation $f^{-1}(x) = 4$, **(2)**

c sketch the curve $y = |f(x)|$, **(3)**

d find the values of x for which $|f(x)| = 6$. **(5)**

5 The function f is defined by

$$\mathrm{f} : x \to 2 - \frac{3}{x},\ x \in \mathbb{R},\ x \neq 0.$$

a Find the value of ff(1). **(2)**

b Find $\mathrm{f}^{-1}(x)$ and state its domain. **(4)**

The function g is defined by

$$\mathrm{g} : x \to x^2,\ x \in \mathbb{R}.$$

c Solve the equation $\mathrm{gf}(x) = 1$. **(4)**

6 The function f is defined by

$$\mathrm{f} : x \to \mathrm{e}^{\frac{1}{2}x} - 2,\ x \in \mathbb{R}.$$

a Evaluate f(ln 9). **(2)**

b State the range of f. **(1)**

c Find $\mathrm{f}^{-1}(x)$ and state its domain. **(4)**

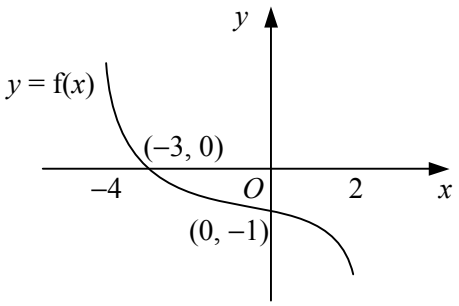
The function g is defined by

$$\mathrm{g} : x \to x^2 + 4x,\ x \in \mathbb{R}.$$

d Find and simplify an expression for $\mathrm{gf}(x)$. **(3)**

e Solve the equation $\mathrm{gf}(x) + 1 = 0$. **(2)**

7



The diagram shows the curve $y = \mathrm{f}(x)$. The domain of f is $-4 \leq x \leq 2$ and the curve intersects the coordinate axes at the points (–3, 0) and (0, –1).

a Explain how the graph shows that f is one-one. **(1)**

b Showing the coordinates of any points of intersection with the axes, sketch on separate diagrams the graphs of

i $y = |\mathrm{f}(x)|$,

ii $y = \mathrm{f}^{-1}(x)$. **(5)**

8

$$\mathrm{f}(x) \equiv \frac{5}{(x+1)(2x-3)} + \frac{1}{x+1},\ x \in \mathbb{R},\ x \geq 2.$$

a Show that $\mathrm{f}(x) = \frac{2}{2x-3}$. **(4)**

b Find the range of f. **(2)**

c Find an expression for $\mathrm{f}^{-1}(x)$. **(3)**

$$\mathrm{g}(x) \equiv \frac{1}{x-2},\ x \in \mathbb{R},\ x \neq 2.$$

d Solve the equation $\mathrm{fg}(x) = \frac{2}{3}$. **(4)**