# C3 TRIGONOMETRY — Answers - Worksheet C

**1** **a** $\sin^2 x + \cos^2 x \equiv 1$

$\Rightarrow \quad \frac{\sin^2 x}{\cos^2 x} + 1 \equiv \frac{1}{\cos^2 x}$

$\Rightarrow \quad \tan^2 x + 1 \equiv \sec^2 x$

**b** $\sin^2 x + \cos^2 x \equiv 1$

$\Rightarrow \quad 1 + \frac{\cos^2 x}{\sin^2 x} \equiv \frac{1}{\sin^2 x}$

$\Rightarrow \quad 1 + \cot^2 x \equiv \text{cosec}^2 x$

**2** **a** $\tan^2 A = \frac{1}{9}$

$\sec^2 A = 1 + \frac{1}{9} = \frac{10}{9}$

**b** $\text{cosec}^2 B = 1 + 2\sqrt{3} + 3$

$= 4 + 2\sqrt{3}$

$\cot^2 B = (4 + 2\sqrt{3}) - 1$

$= 3 + 2\sqrt{3}$

**c** $\sec^2 C = \frac{9}{4}$

$\tan^2 C = \frac{9}{4} - 1 = \frac{5}{4}$

$\tan C = \pm\sqrt{\frac{5}{4}} = \pm\frac{1}{2}\sqrt{5}$

**3** **a** $3(1 + \tan^2 \theta) = 4\tan^2 \theta$

$\tan^2 \theta = 3$

$\tan \theta = \pm\sqrt{3}$

$\theta = \frac{\pi}{3}, \pi + \frac{\pi}{3}$ or $\pi - \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$

$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

**b** $\sec^2 \theta - 1 - 2\sec \theta + 1 = 0$

$\sec^2 \theta - 2\sec \theta = 0$

$\sec \theta(\sec \theta - 2) = 0$

$\sec \theta = 2$ or $0$ [no solutions]

$\cos \theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

**c** $\text{cosec}^2 \theta - 1 - 3\,\text{cosec}\, \theta + 3 = 0$

$\text{cosec}^2 \theta - 3\,\text{cosec}\, \theta + 2 = 0$

$(\text{cosec}\, \theta - 1)(\text{cosec}\, \theta - 2) = 0$

$\text{cosec}\, \theta = 1$ or $2$

$\sin \theta = \frac{1}{2}$ or $1$

$\theta = \frac{\pi}{6}, \pi - \frac{\pi}{6}$ or $\frac{\pi}{2}$

$\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

**d** $1 + \cot^2 \theta + \cot^2 \theta = 3$

$\cot^2 \theta = 1$

$\cot \theta = \pm 1$

$\tan \theta = \pm 1$

$\theta = \frac{\pi}{4}, \pi + \frac{\pi}{4}$ or $\pi - \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

**e** $1 + \tan^2 \theta + 2\tan \theta = 0$

$(\tan \theta + 1)^2 = 0$

$\tan \theta = -1$

$\theta = \pi - \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$

$\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

**f** $1 + \cot^2 \theta - \sqrt{3}\cot \theta - 1 = 0$

$\cot^2 \theta - \sqrt{3}\cot \theta = 0$

$\cot \theta(\cot \theta - \sqrt{3}) = 0$

$\cot \theta = 0$ or $\sqrt{3}$

$\cos \theta = 0$ or $\tan \theta = \frac{1}{\sqrt{3}}$

$\theta = \frac{\pi}{2}, 2\pi - \frac{\pi}{2}$ or $\frac{\pi}{6}, \pi + \frac{\pi}{6}$

$\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}$

© Solomon Press

**4** **a** $\sec^2 x - 1 - 2\sec x - 2 = 0$
$\sec^2 x - 2\sec x - 3 = 0$
$(\sec x + 1)(\sec x - 3) = 0$
$\sec x = -1$ or $3$
$\cos x = -1$ or $\frac{1}{3}$
$x = 180, -180$ or $70.5, -70.5$
$x = -180°, -70.5°, 70.5°, 180°$

**b** $2(1 + \cot^2 x) + 2 = 9\cot x$
$2\cot^2 x - 9\cot x + 4 = 0$
$(2\cot x - 1)(\cot x - 4) = 0$
$\cot x = \frac{1}{2}$ or $4$
$\tan x = \frac{1}{4}$ or $2$
$x = 14.0, 14.0 - 180$ or $63.4, 63.4 - 180$
$x = -166.0°, -116.6°, 14.0°, 63.4°$

**c** $\text{cosec}^2 x + 5\,\text{cosec}\, x + 2(\text{cosec}^2 x - 1) = 0$
$3\,\text{cosec}^2 x + 5\,\text{cosec}\, x - 2 = 0$
$(3\,\text{cosec}\, x - 1)(\text{cosec}\, x + 2) = 0$
$\text{cosec}\, x = -2$ or $\frac{1}{3}$ [no solutions]
$\sin x = -\frac{1}{2}$
$x = -30, 30 - 180$
$x = -150°, -30°$

**d** $3\tan^2 x - 3\tan x + 1 + \tan^2 x = 2$
$4\tan^2 x - 3\tan x - 1 = 0$
$(4\tan x + 1)(\tan x - 1) = 0$
$\tan x = -\frac{1}{4}$ or $1$
$x = 180 - 14.0, -14.0$ or $45, 45 - 180$
$x = -135°, -14.0°, 45°, 166.0°$

**e** $\sec^2 x - 1 + 4\sec x - 2 = 0$
$\sec^2 x + 4\sec x - 3 = 0$
$\sec x = \frac{-4 \pm \sqrt{16+12}}{2} = -2 \pm \sqrt{7}$
$\cos x = \frac{1}{-2 \pm \sqrt{7}}$
$\cos x = -0.2153$ or $1.5486$ [no solutions]
$x = 180 - 77.6, 77.6 - 180$
$x = -102.4°, 102.4°$

**f** $2\cot^2 x + 3(1 + \cot^2 x) = 4\cot x + 3$
$5\cot^2 x - 4\cot x = 0$
$\cot x\,(5\cot x - 4) = 0$
$\cot x = 0$ or $\frac{4}{5}$
$\cos x = 0$ or $\tan x = \frac{5}{4}$
$x = 90, -90$ or $51.3, 51.3 - 180$
$x = -128.7°, -90°, 51.3°, 90°$

**5** **a** $\text{cosec}^2 2x - 1 + \text{cosec}\, 2x - 1 = 0$
$\text{cosec}^2 2x + \text{cosec}\, 2x - 2 = 0$
$(\text{cosec}\, 2x + 2)(\text{cosec}\, 2x - 1) = 0$
$\text{cosec}\, 2x = -2$ or $1$
$\sin 2x = -\frac{1}{2}$ or $1$
$2x = 180 + 30, 360 - 30, 540 + 30,$
$720 - 30$ or $90, 360 + 90$
$= 90, 210, 330, 450, 570, 690$
$x = 45°, 105°, 165°, 225°, 285°, 345°$

**b** $8(1 - \cos^2 x) + \sec x = 8$
$8\cos^2 x = \sec x$
$\cos^3 x = \frac{1}{8}$
$\cos x = \frac{1}{2}$
$x = 60, 360 - 60$
$x = 60°, 300°$

**c** $\frac{3}{\sin^2 x} - 4\sin^2 x = 1$
$4\sin^4 x + \sin^2 x - 3 = 0$
$(4\sin^2 x - 3)(\sin^2 x + 1) = 0$
$\sin^2 x = \frac{3}{4}$ or $-1$ [no solutions]
$\sin x = \pm\frac{\sqrt{3}}{2}$
$x = 60, 180 - 60$ or $180 + 60, 360 - 60$
$x = 60°, 120°, 240°, 300°$

**d** $9(1 + \tan^2 x) - 8 = 1 + \cot^2 x$
$9\tan^2 x = \cot^2 x$
$\tan^4 x = \frac{1}{9}$
$\tan^2 x = \frac{1}{3}$ or $-\frac{1}{3}$ [no solutions]
$\tan x = \pm\frac{1}{\sqrt{3}}$
$x = 30, 180 + 30$ or $180 - 30, 360 - 30$
$x = 30°, 150°, 210°, 330°$

**6** **a** $\text{LHS} = 1 + \cot^2 x - (1 + \tan^2 x)$
$= \cot^2 x - \tan^2 x$
$= \text{RHS}$

**b** $\text{LHS} = \cot^2 x - 2\cot x + 1$
$= \text{cosec}^2 x - 2\cot x$
$= \text{RHS}$

**c** $\text{LHS} = \cos^2 x - 4 + 4\sec^2 x$
$= \cos^2 x - 4 + 4(1 + \tan^2 x)$
$= \cos^2 x + 4\tan^2 x$
$= \text{RHS}$

**d** $\text{LHS} = 1 + \tan^2 x - (1 - \cos^2 x)$
$= \tan^2 x + \cos^2 x$
$= \text{RHS}$

**e** $\text{LHS} = \tan^2 x + 2 + \cot^2 x$
$= \sec^2 - 1 + 2 + \text{cosec}^2 x - 1$
$= \sec^2 x + \text{cosec}^2 x$
$= \text{RHS}$

**f** $\text{LHS} = \sin^2 x - 2\sin x \sec x + \sec^2 x$
$= \sin^2 x - 2\tan x + 1 + \tan^2 x$
$= \sin^2 x + (\tan x - 1)^2$
$= \text{RHS}$

**g** $\text{LHS} = \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}$
$= \frac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x}$
$= \frac{1}{\cos^2 x \sin^2 x}$
$= \frac{1}{\cos^2 x} \times \frac{1}{\sin^2 x}$
$= \sec^2 x\, \text{cosec}^2 x$
$= \text{RHS}$

**h** $\text{LHS} = \sec^2 x\,(1 + \tan^2 x) + \tan^2 x\,(\sec^2 x - 1)$
$= \sec^2 x + \sec^2 x \tan^2 x + \sec^2 x \tan^2 x - \tan^2 x$
$= 1 + \tan^2 x + 2\sec^2 x \tan^2 x - \tan^2 x$
$= 2\sec^2 x \tan^2 x + 1$
$= \text{RHS}$

**7** $4\sec^2 x - \sec x + 2\tan^2 x = 0 \Rightarrow 4\sec^2 x - \sec x + 2(\sec^2 x - 1) = 0$
$6\sec^2 x - \sec x - 2 = 0$
$(3\sec x - 2)(2\sec x + 1) = 0$
$\sec x = -\tfrac{1}{2}, \tfrac{2}{3}$
for real values of $x$, $|\sec x| > 1$ $\therefore$ no real solutions

**8** **a** $\text{LHS} = \frac{1}{\sin x} \times \frac{1}{\cos x} - \frac{\cos x}{\sin x}$
$= \frac{1 - \cos^2 x}{\sin x \cos x}$
$= \frac{\sin^2 x}{\sin x \cos x}$
$= \frac{\sin x}{\cos x}$
$= \tan x$
$= \text{RHS}$

**b** $\text{cosec}\, x \sec x - \cot x = 3$
$\tan x = 3$
$x = 71.6, 180 + 71.6$
$x = 71.6°, 251.6°$