1 **a** $\cos x + \sin x$
$= R\cos x\cos\alpha + R\sin x\sin\alpha$
$\Rightarrow R\cos\alpha = 1,\ R\sin\alpha = 1$
$\therefore R = \sqrt{1+1} = \sqrt{2} = 1.4$
$\tan\alpha = 1,\ \alpha = 45$
$\therefore \cos x° + \sin x° = 1.4\cos(x - 45)°$

b $3\cos x + 4\sin x$
$= R\cos x\cos\alpha + R\sin x\sin\alpha$
$\Rightarrow R\cos\alpha = 3,\ R\sin\alpha = 4$
$\therefore R = \sqrt{9+16} = 5$
$\tan\alpha = \frac{4}{3},\ \alpha = 53.1$
$\therefore 3\cos x° + 4\sin x° = 5\cos(x - 53.1)°$

c $2\sin x + \cos x$
$= R\cos x\cos\alpha + R\sin x\sin\alpha$
$\Rightarrow R\cos\alpha = 1,\ R\sin\alpha = 2$
$\therefore R = \sqrt{1+4} = \sqrt{5} = 2.2$
$\tan\alpha = 2,\ \alpha = 63.4$
$\therefore 2\sin x° + \cos x° = 2.2\cos(x - 63.4)°$

d $\cos x + \sqrt{3}\sin x$
$= R\cos x\cos\alpha + R\sin x\sin\alpha$
$\Rightarrow R\cos\alpha = 1,\ R\sin\alpha = \sqrt{3}$
$\therefore R = \sqrt{1+3} = 2$
$\tan\alpha = \sqrt{3},\ \alpha = 60$
$\therefore \cos x° + \sqrt{3}\sin x° = 2\cos(x - 60)°$

2 **a** $5\cos x - 12\sin x$
$= R\cos x\cos\alpha - R\sin x\sin\alpha$
$\Rightarrow R\cos\alpha = 5,\ R\sin\alpha = 12$
$\therefore R = \sqrt{25+144} = 13$
$\tan\alpha = \frac{12}{5},\ \alpha = 67.4$
$\therefore 5\cos x° - 12\sin x° = 13\cos(x + 67.4)°$

b $4\sin x + 2\cos x$
$= R\sin x\cos\alpha + R\cos x\sin\alpha$
$\Rightarrow R\cos\alpha = 4,\ R\sin\alpha = 2$
$\therefore R = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$
$\tan\alpha = \frac{1}{2},\ \alpha = 26.6$
$\therefore 4\sin x° + 2\cos x° = 2\sqrt{5}\sin(x + 26.6)°$

c $\sin x - 7\cos x$
$= R\sin x\cos\alpha - R\cos x\sin\alpha$
$\Rightarrow R\cos\alpha = 1,\ R\sin\alpha = 7$
$\therefore R = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$
$\tan\alpha = 7,\ \alpha = 81.9$
$\therefore \sin x° - 7\cos x° = 5\sqrt{2}\sin(x - 81.9)°$

d $8\cos 2x - 15\sin 2x$
$= R\cos 2x\cos\alpha - R\sin 2x\sin\alpha$
$\Rightarrow R\cos\alpha = 8,\ R\sin\alpha = 15$
$\therefore R = \sqrt{64+225} = 17$
$\tan\alpha = \frac{15}{8},\ \alpha = 61.9$
$\therefore 8\cos 2x° - 15\sin 2x° = 17\cos(2x + 61.9)°$

3 **a** $3\sin x - 2\cos x$
$= R\sin x\cos\alpha - R\cos x\sin\alpha$
$\Rightarrow R\cos\alpha = 3,\ R\sin\alpha = 2$
$\therefore R = \sqrt{9+4} = \sqrt{13}$
$\tan\alpha = \frac{2}{3},\ \alpha = 0.59$
$\therefore 3\sin x - 2\cos x = \sqrt{13}\sin(x - 0.59)$

b $3\cos x + \sqrt{3}\sin x$
$= R\cos x\cos\alpha + R\sin x\sin\alpha$
$\Rightarrow R\cos\alpha = 3,\ R\sin\alpha = \sqrt{3}$
$\therefore R = \sqrt{9+3} = \sqrt{12} = 2\sqrt{3}$
$\tan\alpha = \frac{1}{\sqrt{3}},\ \alpha = \frac{\pi}{6}$
$\therefore 3\cos x + \sqrt{3}\sin x = 2\sqrt{3}\cos(x - \frac{\pi}{6})$

c $8\sin 3x + 6\cos 3x$
$= R\sin 3x\cos\alpha + R\cos 3x\sin\alpha$
$\Rightarrow R\cos\alpha = 8,\ R\sin\alpha = 6$
$\therefore R = \sqrt{64+36} = 10$
$\tan\alpha = \frac{3}{4},\ \alpha = 0.64$
$\therefore 8\sin 3x + 6\cos 3x = 10\sin(3x + 0.64)$

d $\cos x + \frac{1}{2}\sin x$
$= R\cos x\cos\alpha + R\sin x\sin\alpha$
$\Rightarrow R\cos\alpha = 1,\ R\sin\alpha = \frac{1}{2}$
$\therefore R = \sqrt{1+\frac{1}{4}} = \sqrt{\frac{5}{4}} = \frac{1}{2}\sqrt{5}$
$\tan\alpha = \frac{1}{2},\ \alpha = 0.46$
$\therefore \cos x + \frac{1}{2}\sin x = \frac{1}{2}\sqrt{5}\cos(x - 0.46)$

4 **a** $24 \sin x - 7 \cos x = R \sin (x - \alpha)$
$= R \sin x \cos \alpha - R \cos x \sin \alpha$
$\Rightarrow R \cos \alpha = 24,\ R \sin \alpha = 7$
$\therefore R = \sqrt{576 + 49} = 25$
$\tan \alpha = \frac{7}{24},\ \alpha = 16.3°$
$\therefore 24 \sin x - 7 \cos x = 25 \sin (x - 16.3°)$
$\therefore$ max. = 25 when $x - 16.3 = 90$
$x = 106.3°$ (1dp)

b $4 \cos 2x + 4 \sin 2x = R \cos (2x - \alpha)$
$= R \cos 2x \cos \alpha + R \sin 2x \sin \alpha$
$\Rightarrow R \cos \alpha = 4,\ R \sin \alpha = 4$
$\therefore R = \sqrt{16 + 16} = \sqrt{32} = 4\sqrt{2}$
$\tan \alpha = 1,\ \alpha = 45°$
$\therefore 4 \cos 2x + 4 \sin 2x = 4\sqrt{2} \cos (2x - 45°)$
$\therefore$ max. = $4\sqrt{2}$ when $2x - 45 = 0$
$x = 22.5°$

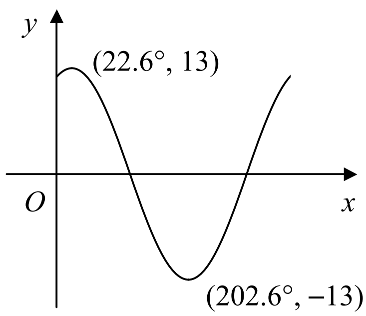
c $3 \cos x - 5 \sin x = R \cos (x + \alpha)$
$= R \cos x \cos \alpha - R \sin x \sin \alpha$
$\Rightarrow R \cos \alpha = 3,\ R \sin \alpha = 5$
$\therefore R = \sqrt{9 + 25} = \sqrt{34}$
$\tan \alpha = \frac{5}{3},\ \alpha = 59.0°$
$\therefore 3 \cos x - 5 \sin x = \sqrt{34} \cos (x + 59.0°)$
$\therefore$ max. = $\sqrt{34}$ when $x + 59.0 = 360$
$x = 301.0°$ (1dp)

d $5 \sin 3x + \cos 3x = R \sin (3x + \alpha)$
$= R \sin 3x \cos \alpha + R \cos 3x \sin \alpha$
$\Rightarrow R \cos \alpha = 5,\ R \sin \alpha = 1$
$\therefore R = \sqrt{25 + 1} = \sqrt{26}$
$\tan \alpha = \frac{1}{5},\ \alpha = 11.3°$
$\therefore 5 \sin 3x + \cos 3x = \sqrt{26} \sin (3x + 11.3°)$
$\therefore$ max. = $\sqrt{26}$ when $3x + 11.3 = 90$
$x = 26.2°$ (1dp)

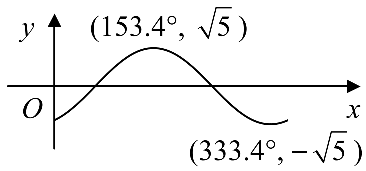
5 **a** $3 \sin x - 3 \cos x = R \sin x \cos \alpha - R \cos x \sin \alpha$
$\Rightarrow R \cos \alpha = 3,\ R \sin \alpha = 3$
$\therefore R = \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}$
$\tan \alpha = 1,\ \alpha = 45°$
$\therefore 3 \sin x° - 3 \cos x° = 3\sqrt{2} \sin (x - 45)°$

b translation by 45 units in positive x-direction and stretch by a factor of $3\sqrt{2}$ in y-direction

6 **a** $12 \cos x + 5 \sin x = R \cos (x - \alpha)$
$= R \cos x \cos \alpha + R \sin x \sin \alpha$
$\Rightarrow R \cos \alpha = 12,\ R \sin \alpha = 5$
$\therefore R = \sqrt{144 + 25} = 13$
$\tan \alpha = \frac{5}{12},\ \alpha = 22.6°$ (1dp)
$\therefore y = 13 \cos (x - 22.6°)$



b $\sin x - 2 \cos x = R \sin (x - \alpha)$
$= R \sin x \cos \alpha - R \cos x \sin \alpha$
$\Rightarrow R \cos \alpha = 1,\ R \sin \alpha = 2$
$\therefore R = \sqrt{1 + 4} = \sqrt{5}$
$\tan \alpha = 2,\ \alpha = 63.4°$ (1dp)
$\therefore y = \sqrt{5} \sin (x - 63.4°)$



c $2\sqrt{3}\cos x - 6\sin x = R\cos(x + \alpha)$

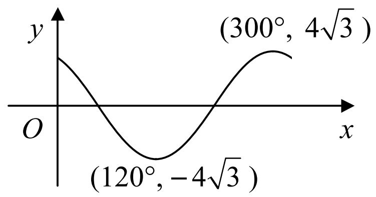
$= R\cos x\cos\alpha - R\sin x\sin\alpha$

$\Rightarrow R\cos\alpha = 2\sqrt{3},\ R\sin\alpha = 6$

$\therefore R = \sqrt{12+36} = \sqrt{48} = 4\sqrt{3}$

$\tan\alpha = \sqrt{3},\ \alpha = 60°$

$\therefore y = 4\sqrt{3}\cos(x + 60°)$



d $9\sin x + 4\cos x = R\sin(x + \alpha)$

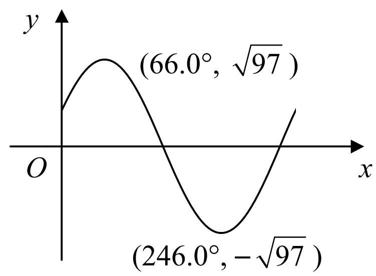
$= R\sin x\cos\alpha + R\cos x\sin\alpha$

$\Rightarrow R\cos\alpha = 9,\ R\sin\alpha = 4$

$\therefore R = \sqrt{81+16} = \sqrt{97}$

$\tan\alpha = \frac{4}{9},\ \alpha = 24.0°$ (1dp)

$\therefore y = \sqrt{97}\sin(x + 24.0°)$



7 **a** $\sqrt{3}\cos x - \sin x = R\cos x\cos\alpha - R\sin x\sin\alpha$

$\Rightarrow R\cos\alpha = \sqrt{3},\ R\sin\alpha = 1$

$\therefore R = \sqrt{3+1} = 2$

$\tan\alpha = \frac{1}{\sqrt{3}},\ \alpha = \frac{\pi}{6}$

$\therefore \sqrt{3}\cos x - \sin x = 2\cos(x + \frac{\pi}{6})$

b $2\cos(x + \frac{\pi}{6}) = 1$

$\cos(x + \frac{\pi}{6}) = \frac{1}{2}$

$x + \frac{\pi}{6} = \frac{\pi}{3}, 2\pi - \frac{\pi}{3} = \frac{\pi}{3}, \frac{5\pi}{3}$

$x = \frac{\pi}{6}, \frac{3\pi}{2}$

8 **a** $6\sin x + 8\cos x = R\sin(x + \alpha)$

$= R\sin x\cos\alpha + R\cos x\sin\alpha$

$\Rightarrow R\cos\alpha = 6,\ R\sin\alpha = 8$

$\therefore R = \sqrt{36+64} = 10$

$\tan\alpha = \frac{4}{3},\ \alpha = 0.9273$

$\therefore 10\sin(x + 0.9273) = 5$

$\sin(x + 0.9273) = \frac{1}{2}$

$x + 0.9273 = \pi - \frac{\pi}{6}, 2\pi + \frac{\pi}{6}$

$= \frac{5\pi}{6}, \frac{13\pi}{6}$

$x = 1.69, 5.88$

b $2\cos x - 2\sin x = R\cos(x + \alpha)$

$= R\cos x\cos\alpha + R\sin x\sin\alpha$

$\Rightarrow R\cos\alpha = 2,\ R\sin\alpha = 2$

$\therefore R = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$\tan\alpha = 1,\ \alpha = \frac{\pi}{4}$

$\therefore 2\sqrt{2}\cos(x + \frac{\pi}{4}) = 1$

$\cos(x + \frac{\pi}{4}) = \frac{1}{2\sqrt{2}}$

$x + \frac{\pi}{4} = 1.2094, 2\pi - 1.2094$

$= 1.2094, 5.0738$

$x = 0.42, 4.29$

c $7 \sin x - 24 \cos x = R \sin (x - \alpha)$
$= R \sin x \cos \alpha - R \cos x \sin \alpha$
$\Rightarrow R \cos \alpha = 7,\ R \sin \alpha = 24$
$\therefore R = \sqrt{49 + 576} = 25$
$\tan \alpha = \frac{24}{7},\ \alpha = 1.2870$
$\therefore 25 \sin (x - 1.2870) - 10 = 0$
$\sin (x - 1.2870) = \frac{2}{5}$
$x - 1.2870 = 0.4115, \pi - 0.4115$
$= 0.4115, 2.7301$
$x = 1.70, 4.02$

d $3 \cos x + \sin x = R \cos (x - \alpha)$
$= R \cos x \cos \alpha + R \sin x \sin \alpha$
$\Rightarrow R \cos \alpha = 3,\ R \sin \alpha = 1$
$\therefore R = \sqrt{9 + 1} = \sqrt{10}$
$\tan \alpha = \frac{1}{3},\ \alpha = 0.3218$
$\therefore \sqrt{10} \cos (x - 0.3218) + 1 = 0$
$\cos (x - 0.3218) = -\frac{1}{\sqrt{10}}$
$x - 0.3218 = \pi - 1.2490, \pi + 1.2490$
$= 1.8925, 4.3906$
$x = 2.21, 4.71$

e $\cos 2x + 4 \sin 2x = R \cos (2x - \alpha)$
$= R \cos 2x \cos \alpha + R \sin 2x \sin \alpha$
$\Rightarrow R \cos \alpha = 1,\ R \sin \alpha = 4$
$\therefore R = \sqrt{1 + 16} = \sqrt{17}$
$\tan \alpha = 4,\ \alpha = 1.3258$
$\therefore \sqrt{17} \cos (2x - 1.3258) = 3$
$\cos (2x - 1.3258) = \frac{3}{\sqrt{17}}$
$2x - 1.3258 = 0.7560, 2\pi - 0.7560,$
$2\pi + 0.7560, -0.7560$
$= -0.7560, 0.7560, 5.5272, 7.0392$
$2x = 0.5698, 2.0818, 6.8530, 8.3650$
$x = 0.28, 1.04, 3.43, 4.18$

f $5 \sin x - 8 \cos x = R \sin (x - \alpha)$
$= R \sin x \cos \alpha + R \cos x \sin \alpha$
$\Rightarrow R \cos \alpha = 5,\ R \sin \alpha = 8$
$\therefore R = \sqrt{25 + 64} = \sqrt{89}$
$\tan \alpha = \frac{8}{5},\ \alpha = 1.0122$
$\therefore \sqrt{89} \sin (x - 1.0122) + 7 = 0$
$\sin (x - 1.0122) = -\frac{7}{\sqrt{89}}$
$x - 1.0122 = -0.8360, \pi + 0.8360$
$= -0.8360, 3.9776$
$x = 0.18, 4.99$

9 **a** $\sin x + \cos x = R \sin (x + \alpha)$
$= R \sin x \cos \alpha + R \cos x \sin \alpha$
$\Rightarrow R \cos \alpha = 1,\ R \sin \alpha = 1$
$\therefore R = \sqrt{1 + 1} = \sqrt{2}$
$\tan \alpha = 1,\ \alpha = 45°$
$\therefore \sqrt{2} \sin (x + 45°) = 1$
$\sin (x + 45°) = \frac{1}{\sqrt{2}}$
$x + 45 = 45, 180 - 45$
$= 45, 135$
$x = 0, 90°$

b $4 \cos x - \sin x = R \cos (x + \alpha)$
$= R \cos x \cos \alpha - R \sin x \sin \alpha$
$\Rightarrow R \cos \alpha = 4,\ R \sin \alpha = 1$
$\therefore R = \sqrt{16 + 1} = \sqrt{17}$
$\tan \alpha = \frac{1}{4},\ \alpha = 14.04°$
$\therefore \sqrt{17} \cos (x + 14.04°) + 2 = 0$
$\cos (x + 14.04°) = -\frac{2}{\sqrt{17}}$
$x + 14.04 = 180 - 60.98, 60.98 - 180$
$= -119.02, 119.02$
$x = -133.1°, 105.0°$

c $\cos \frac{x}{2} + 5 \sin \frac{x}{2} = R \cos (\frac{x}{2} - \alpha)$
$= R \cos \frac{x}{2} \cos \alpha + R \sin \frac{x}{2} \sin \alpha$
$\Rightarrow R \cos \alpha = 1,\ R \sin \alpha = 5$
$\therefore R = \sqrt{1 + 25} = \sqrt{26}$
$\tan \alpha = 5,\ \alpha = 78.69°$
$\therefore \sqrt{26} \cos (\frac{x}{2} - 78.69°) - 4 = 0$
$\cos (\frac{x}{2} - 78.69°) = \frac{4}{\sqrt{26}}$
$\frac{x}{2} - 78.69 = -38.33$
$\frac{x}{2} = 40.36$
$x = 80.7°$

d $6 \sin x + 3 \cos x = R \sin (x + \alpha)$
$= R \sin x \cos \alpha + R \cos x \sin \alpha$
$\Rightarrow R \cos \alpha = 6,\ R \sin \alpha = 3$
$\therefore R = \sqrt{36 + 9} = \sqrt{45} = 3\sqrt{5}$
$\tan \alpha = \frac{1}{2},\ \alpha = 26.57°$
$\therefore 3\sqrt{5} \sin (x + 26.57°) = 5$
$\sin (x + 26.57°) = \frac{\sqrt{5}}{3}$
$x + 26.57 = 48.19, 180 - 48.19$
$= 48.19, 131.81$
$x = 21.6°, 105.2°$