C3 TRIGONOMETRY — Answers - Worksheet G

1

$\sec^2 x - 1 - \sec x = 1$
$\sec^2 x - \sec x - 2 = 0$
$(\sec x + 1)(\sec x - 2) = 0$
$\sec x = -1$ or 2
$\cos x = -1$ or $\frac{1}{2}$
$x = 180$ or $60, 360 - 60$
$x = 60°, 180°, 300°$

2 **a** $2 \cos x + 5 \sin x$
$= R \cos x \cos \alpha + R \sin x \sin \alpha$
$\Rightarrow R \cos \alpha = 2, \; R \sin \alpha = 5$
$\therefore R = \sqrt{4+25} = \sqrt{29} = 5.39$
$\tan \alpha = \frac{5}{2}, \; \alpha = 68.2$
$\therefore 2 \cos x + 5 \sin x = 5.39 \cos (x - 68.2)°$

b $\sqrt{29} \cos (x - 68.199) = 3$
$\cos (x - 68.199) = \frac{3}{\sqrt{29}} = 0.5571$
$x - 68.199 = 56.145, -56.145$
$x = 12.1, 124.3$

3 **a** $\arctan 2x = \frac{\pi}{6}$
$2x = \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$
$x = \frac{1}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{1}{6}\sqrt{3}$

b $4 \sin x \cos x = 3 \cos x$
$\cos x(4 \sin x - 3) = 0$
$\cos x = 0$ or $\sin x = \frac{3}{4}$
$x = 90, 360 - 90$ or $48.6, 180 - 48.6$
$x = 48.6°$ (1dp), $90°$, $131.4°$ (1dp), $270°$

4 **a** $\sin (A + B) \equiv \sin A \cos B + \cos A \sin B$
$\sin (A - B) \equiv \sin A \cos B - \cos A \sin B$

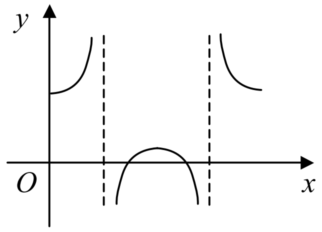
subtracting

$\sin (A + B) - \sin (A + B) \equiv 2 \cos A \sin B$
let $P = A + B$ (1) and $Q = A - B$ (2)
(1) + (2) $\Rightarrow 2A = P + Q \Rightarrow A = \frac{P+Q}{2}$
(1) – (2) $\Rightarrow 2B = P - Q \Rightarrow B = \frac{P-Q}{2}$
$\therefore \sin P - \sin Q \equiv 2 \cos \frac{P+Q}{2} \sin \frac{P-Q}{2}$

b $\sin 4x - \sin 2x = 0$
$2 \cos \frac{4x+2x}{2} \sin \frac{4x-2x}{2} = 0$
$\cos 3x \sin x = 0$
$\cos 3x = 0$ or $\sin x = 0$
$3x = 90, 360 - 90, 360 + 90$ or $x = 0, 180$
$3x = 90, 270, 450$ or $x = 0, 180$
$x = 0, 30°, 90°, 150°, 180°$

5 **a** LHS = $4\sin^2\theta - 4 + \text{cosec}^2\theta$
$= 4(1 - \cos^2\theta) - 4 + \text{cosec}^2\theta$
$= \text{cosec}^2\theta - 4\cos^2\theta$ = RHS

b **i**



ii (0, 5)

iii $3 + 2\sec x = 0$

$\sec x = -\frac{3}{2}$, $\cos x = -\frac{2}{3}$

$x = \pi - 0.841, \pi + 0.841 = 2.30, 3.98$

$\therefore$ (2.30, 0) and (3.98, 0) [x to 2dp]

6 **a** $\cos x - \sin x = R\cos x\cos\alpha - R\sin x\sin\alpha$
$\Rightarrow R\cos\alpha = 1,\ R\sin\alpha = 1$
$\therefore R = \sqrt{1+1} = \sqrt{2}$
$\tan\alpha = 1,\ \alpha = \frac{\pi}{4}$

b $\cos x - \sin x + \sqrt{2}\cos(3x - \frac{\pi}{4}) = 0$

$\sqrt{2}\cos(x + \frac{\pi}{4}) + \sqrt{2}\cos(3x - \frac{\pi}{4}) = 0$

$2\sqrt{2}\cos\frac{4x}{2}\cos\frac{-2x+\frac{\pi}{2}}{2} = 0$

$\cos 2x\cos(\frac{\pi}{4} - x) = 0$

$\cos 2x\cos(x - \frac{\pi}{4}) = 0$

$\cos 2x = 0$ or $\cos(x - \frac{\pi}{4}) = 0$

$2x = \frac{\pi}{2}, 2\pi - \frac{\pi}{2}, 2\pi + \frac{\pi}{2}, 4\pi - \frac{\pi}{2}$

or $x - \frac{\pi}{4} = \frac{\pi}{2}, 2\pi - \frac{\pi}{2}$

$2x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$ or $x - \frac{\pi}{4} = \frac{\pi}{2}, \frac{3\pi}{2}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

7 **a** LHS $= \frac{\cos 2x}{\sin 2x} + \frac{1}{\sin 2x}$
$= \frac{\cos 2x + 1}{\sin 2x}$
$= \frac{2\cos^2 x - 1 + 1}{2\sin x\cos x}$
$= \frac{2\cos^2 x}{2\sin x\cos x}$
$= \frac{\cos x}{\sin x}$
$= \cot x$ = RHS

b $\cot x = 6 - \cot^2 x$
$\cot^2 x + \cot x - 6 = 0$

$(\cot x + 3)(\cot x - 2) = 0$

$\cot x = -3$ or 2
$\tan x = -\frac{1}{3}$ or $\frac{1}{2}$
$x = \pi - 0.3218, 2\pi - 0.3218$
or $0.4636, \pi + 0.4636$
$x = 0.46, 2.82, 3.61, 5.96$

8 **a** LHS = $\cos x\cos 30 - \sin x\sin 30 + \sin x$

$= \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x + \sin x$

$= \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x$

$= \cos x\cos 30 + \sin x\sin 30$

$= \cos(x - 30)°$ = RHS

b let $x = 45$
$\cos 75° + \sin 45° = \cos 15°$
$\therefore \cos 75° - \cos 15° = -\sin 45°$
$= -\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = -\frac{1}{2}\sqrt{2}$

c $3\cos(x + 30) + 3\sin x - 2\sin x$
$= 3\cos(x - 30) + 1$

$-2\sin x = 1$
$\sin x = -\frac{1}{2}$
$x = -30, 30 - 180$
$x = -150, -30$

9 **a** $a = 3$

$b \sin x° + c \cos x°$ can be expressed in the form $k \sin (x + \alpha)°$ which will vary between $-k$ and $+k$

$\therefore\ a + k = 5$ and $a - k = 1$, hence $a = 3$

b $3 + k = 5 \quad \therefore\ k = 2$

$60 + \alpha = 90 \quad \therefore\ \alpha = 30$

c $f(x) = 3 + 2 \sin (x + 30)$

$= 3 + 2 \sin x \cos 30 + 2 \cos x \sin 30$

$= 3 + \sqrt{3} \sin x + \cos x$

$\therefore\ b = \sqrt{3},\ c = 1$

10 **a** LHS $= \dfrac{1 - (1 - 2\sin^2 \frac{x}{2})}{1 + (2\cos^2 \frac{x}{2} - 1)}$

$= \dfrac{2\sin^2 \frac{x}{2}}{2\cos^2 \frac{x}{2}}$

$= \tan^2 \dfrac{x}{2} =$ RHS

b **i** let $x = \frac{\pi}{6}$, $\dfrac{1 - \cos\frac{\pi}{6}}{1 + \cos\frac{\pi}{6}} = \tan^2 \frac{\pi}{12}$

$\tan^2 \frac{\pi}{12} = \dfrac{1 - \frac{\sqrt{3}}{2}}{1 + \frac{\sqrt{3}}{2}} = \dfrac{2 - \sqrt{3}}{2 + \sqrt{3}} \times \dfrac{2 - \sqrt{3}}{2 - \sqrt{3}}$

$= \dfrac{4 - 4\sqrt{3} + 3}{4 - 3} = 7 - 4\sqrt{3}$

ii $\tan^2 \dfrac{x}{2} = 1 - \sec \dfrac{x}{2}$

$\sec^2 \dfrac{x}{2} - 1 = 1 - \sec \dfrac{x}{2}$

$\sec^2 \dfrac{x}{2} + \sec \dfrac{x}{2} - 2 = 0$

$(\sec \dfrac{x}{2} + 2)(\sec \dfrac{x}{2} - 1) = 0$

$\sec \dfrac{x}{2} = -2$ or 1

$\cos \dfrac{x}{2} = -\frac{1}{2}$ or 1

$\dfrac{x}{2} = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ or 0

$x = 0,\ \frac{4\pi}{3}$

11 **a** $6 \cot^2 x - \operatorname{cosec} x + 5 = 0$

$\Rightarrow\ 6(\operatorname{cosec}^2 x - 1) - \operatorname{cosec} x + 5 = 0$

$6 \operatorname{cosec}^2 x - \operatorname{cosec} x - 1 = 0$

$(3 \operatorname{cosec} x + 1)(2 \operatorname{cosec} x - 1) = 0$

$\operatorname{cosec} x = -\frac{1}{3},\ \frac{1}{2}$

for real x, $|\operatorname{cosec} x| \geq 1$

$\therefore$ no real solutions

b $\cos 5y - \cos y = 0$

$-2 \sin \frac{5y+y}{2} \sin \frac{5y-y}{2} = 0$

$\sin 3y \sin 2y = 0$

$\sin 3y = 0$ or $\sin 2y = 0$

$3y = 0, 180, 360, 540$ or $2y = 0, 180, 360$

$y = 0, 60°, 90°, 120°, 180°$

12 **a** $\cos (A - B) \equiv \cos A \cos B + \sin A \sin B$

$\cos (A + B) \equiv \cos A \cos B - \sin A \sin B$

subtracting

$\cos (A - B) - \cos (A + B) \equiv 2 \sin A \sin B$

$\therefore\ \sin A \sin B \equiv \frac{1}{2}[\cos (A - B) - \cos (A + B)]$

b $4 \sin (x + \frac{\pi}{3}) = \dfrac{1}{\sin(x - \frac{\pi}{6})}$

$4 \sin (x + \frac{\pi}{3}) \sin (x - \frac{\pi}{6}) = 1$

$2[\cos \frac{\pi}{2} - \cos (2x + \frac{\pi}{6})] = 1$

$2[0 - \cos (2x + \frac{\pi}{6})] = 1$

$\cos (2x + \frac{\pi}{6}) = -\frac{1}{2}$

$2x + \frac{\pi}{6} = \pi - \frac{\pi}{3},\ \pi + \frac{\pi}{3}$

$= \frac{2\pi}{3},\ \frac{4\pi}{3}$

$2x = \frac{\pi}{2},\ \frac{7\pi}{6}$

$x = \frac{\pi}{4},\ \frac{7\pi}{12}$