C4 INTEGRATION *Worksheet F*

1 Using integration by parts, show that

$$\int x\cos x\ \mathrm{d}x = x\sin x + \cos x + c.$$

2 Use integration by parts to find

a $\int xe^{x}\ \mathrm{d}x$ **b** $\int 4x\sin x\ \mathrm{d}x$ **c** $\int x\cos 2x\ \mathrm{d}x$

d $\int x\sqrt{x+1}\ \mathrm{d}x$ **e** $\int \frac{x}{e^{3x}}\ \mathrm{d}x$ **f** $\int x\sec^2 x\ \mathrm{d}x$

3 Using

i integration by parts,
ii the substitution $u = 2x + 1$,

find $\int x(2x+1)^3\ \mathrm{d}x$, and show that your answers are equivalent.

4 Show that

$$\int_0^2 xe^{-x}\ \mathrm{d}x = 1 - 3e^{-2}.$$

5 Evaluate

a $\int_0^{\frac{\pi}{6}} x\cos x\ \mathrm{d}x$ **b** $\int_0^1 xe^{2x}\ \mathrm{d}x$ **c** $\int_0^{\frac{\pi}{4}} x\sin 3x\ \mathrm{d}x$

6 Using integration by parts twice in each case, show that

a $\int x^2e^{x}\ \mathrm{d}x = e^{x}(x^2 - 2x + 2) + c,$

b $\int e^{x}\sin x\ \mathrm{d}x = \frac{1}{2}e^{x}(\sin x - \cos x) + c.$

7 Find

a $\int x^2\sin x\ \mathrm{d}x$ **b** $\int x^2e^{3x}\ \mathrm{d}x$ **c** $\int e^{-x}\cos 2x\ \mathrm{d}x$

8 **a** Write down the derivative of $\ln x$ with respect to x.

b Use integration by parts to find

$$\int \ln x\ \mathrm{d}x.$$

9 Find

a $\int \ln 2x\ \mathrm{d}x$ **b** $\int 3x\ln x\ \mathrm{d}x$ **c** $\int (\ln x)^2\ \mathrm{d}x$

10 Evaluate

a $\int_{-1}^{0} (x+2)e^{x}\ \mathrm{d}x$ **b** $\int_1^2 x^2\ln x\ \mathrm{d}x$ **c** $\int_{\frac{1}{3}}^{1} 2xe^{3x-1}\ \mathrm{d}x$

d $\int_0^3 \ln(2x+3)\ \mathrm{d}x$ **e** $\int_0^{\frac{\pi}{2}} x^2\cos x\ \mathrm{d}x$ **f** $\int_0^{\frac{\pi}{4}} e^{3x}\sin 2x\ \mathrm{d}x$