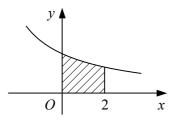
C4

INTEGRATION

Worksheet G

1



The diagram shows part of the curve with parametric equations

$$x = 2t - 4$$
, $y = \frac{1}{t}$.

The shaded region is bounded by the curve, the coordinate axes and the line x = 2.

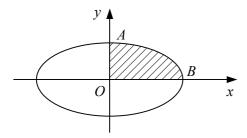
a Find the value of the parameter t when x = 0 and when x = 2.

b Show that the area of the shaded region is given by $\int_{2}^{3} \frac{2}{t} dt$.

c Hence, find the area of the shaded region.

d Verify your answer to part **c** by first finding a cartesian equation for the curve.

2



The diagram shows the ellipse with parametric equations

$$x = 4 \cos \theta$$
, $y = 2 \sin \theta$, $0 \le \theta < 2\pi$,

which meets the positive coordinate axes at the points A and B.

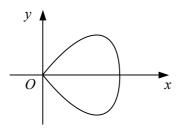
a Find the value of the parameter θ at the points A and B.

b Show that the area of the shaded region bounded by the curve and the positive coordinate axes is given by

$$\int_0^{\frac{\pi}{2}} 8\sin^2\theta \ d\theta.$$

c Hence, show that the area of the region enclosed by the ellipse is 8π .

3



The diagram shows the curve with parametric equations

$$x = 2 \sin t$$
, $y = 5 \sin 2t$, $0 \le t < \pi$.

a Show that the area of the region enclosed by the curve is given by $\int_0^{\frac{\pi}{2}} 20 \sin 2t \cos t \, dt$.

b Evaluate this integral.