C4 PARTIAL FRACTIONS — *Answers - Worksheet A*

1 **a** $x = -4 \Rightarrow -12 = -6A \Rightarrow A = 2$
$x = 2 \Rightarrow -6 = 6B \Rightarrow B = -1$

b $x = -2 \Rightarrow -5 = -5A \Rightarrow A = 1$
$x = \frac{1}{2} \Rightarrow 10 = \frac{5}{2}B \Rightarrow B = 4$

2 **a** $2 \equiv A(x+3) + B(x+1)$
$x = -1 \Rightarrow 2 = 2A \Rightarrow A = 1$
$x = -3 \Rightarrow 2 = -2B \Rightarrow B = -1$

b $x - 3 \equiv A(x-1) + Bx$
$x = 0 \Rightarrow -3 = -A \Rightarrow A = 3$
$x = 1 \Rightarrow B = -2$

c $x + 1 \equiv A(x-5) + B(x-3)$
$x = 3 \Rightarrow 4 = -2A \Rightarrow A = -2$
$x = 5 \Rightarrow 6 = 2B \Rightarrow B = 3$

d $x + 10 \equiv A(2-x) + B(1+x)$
$x = -1 \Rightarrow 9 = 3A \Rightarrow A = 3$
$x = 2 \Rightarrow 12 = 3B \Rightarrow B = 4$

e $\frac{4x-1}{(x+2)(x-1)} \equiv \frac{A}{x+2} + \frac{B}{x-1}$
$4x - 1 \equiv A(x-1) + B(x+2)$
$x = -2 \Rightarrow -9 = -3A \Rightarrow A = 3$
$x = 1 \Rightarrow 3 = 3B \Rightarrow B = 1$

f $\frac{x-9}{(x-1)(x-3)} \equiv \frac{A}{x-1} + \frac{B}{x-3}$
$x - 9 \equiv A(x-3) + B(x-1)$
$x = 1 \Rightarrow -8 = -2A \Rightarrow A = 4$
$x = 3 \Rightarrow -6 = 2B \Rightarrow B = -3$

3 **a** $\frac{8}{(x-1)(x+3)} \equiv \frac{A}{x-1} + \frac{B}{x+3}$
$8 \equiv A(x+3) + B(x-1)$
$x = 1 \Rightarrow 8 = 4A \Rightarrow A = 2$
$x = -3 \Rightarrow 8 = -4B \Rightarrow B = -2$
$\therefore \frac{8}{(x-1)(x+3)} \equiv \frac{2}{x-1} - \frac{2}{x+3}$

b $\frac{x-1}{(x+2)(x+3)} \equiv \frac{A}{x+2} + \frac{B}{x+3}$
$x - 1 \equiv A(x+3) + B(x+2)$
$x = -2 \Rightarrow A = -3$
$x = -3 \Rightarrow -4 = -B \Rightarrow B = 4$
$\therefore \frac{x-1}{(x+2)(x+3)} \equiv \frac{4}{x+3} - \frac{3}{x+2}$

c $\frac{10x}{(x+4)(x-1)} \equiv \frac{A}{x+4} + \frac{B}{x-1}$
$10x \equiv A(x-1) + B(x+4)$
$x = -4 \Rightarrow -40 = -5A \Rightarrow A = 8$
$x = 1 \Rightarrow 10 = 5B \Rightarrow B = 2$
$\therefore \frac{10x}{(x+4)(x-1)} \equiv \frac{8}{x+4} + \frac{2}{x-1}$

d $\frac{5x+7}{x(x+1)} \equiv \frac{A}{x} + \frac{B}{x+1}$
$5x + 7 \equiv A(x+1) + Bx$
$x = 0 \Rightarrow A = 7$
$x = -1 \Rightarrow 2 = -B \Rightarrow B = -2$
$\therefore \frac{5x+7}{x^2+x} \equiv \frac{7}{x} - \frac{2}{x+1}$

e $\frac{x+2}{(x-1)(x-4)} \equiv \frac{A}{x-1} + \frac{B}{x-4}$
$x + 2 \equiv A(x-4) + B(x-1)$
$x = 1 \Rightarrow 3 = -3A \Rightarrow A = -1$
$x = 4 \Rightarrow 6 = 3B \Rightarrow B = 2$
$\therefore \frac{x+2}{x^2-5x+4} \equiv \frac{2}{x-4} - \frac{1}{x-1}$

f $\frac{4x+6}{(x+3)(x-3)} \equiv \frac{A}{x+3} + \frac{B}{x-3}$
$4x + 6 \equiv A(x-3) + B(x+3)$
$x = -3 \Rightarrow -6 = -6A \Rightarrow A = 1$
$x = 3 \Rightarrow 18 = 6B \Rightarrow B = 3$
$\therefore \frac{4x+6}{x^2-9} \equiv \frac{1}{x+3} + \frac{3}{x-3}$

g $\frac{3x+2}{(x-6)(x+4)} \equiv \frac{A}{x-6} + \frac{B}{x+4}$
$3x + 2 \equiv A(x+4) + B(x-6)$
$x = 6 \Rightarrow 20 = 10A \Rightarrow A = 2$
$x = -4 \Rightarrow -10 = -10B \Rightarrow B = 1$
$\therefore \frac{3x+2}{x^2-2x-24} \equiv \frac{2}{x-6} + \frac{1}{x+4}$

h $\frac{38-x}{(4+x)(3-x)} \equiv \frac{A}{4+x} + \frac{B}{3-x}$
$38 - x \equiv A(3-x) + B(4+x)$
$x = -4 \Rightarrow 42 = 7A \Rightarrow A = 6$
$x = 3 \Rightarrow 35 = 7B \Rightarrow B = 5$
$\therefore \frac{38-x}{12-x-x^2} \equiv \frac{6}{4+x} + \frac{5}{3-x}$

i $\dfrac{4x-5}{(2x+1)(x-3)} \equiv \dfrac{A}{2x+1} + \dfrac{B}{x-3}$

$4x - 5 \equiv A(x-3) + B(2x+1)$

$x = -\frac{1}{2} \Rightarrow -7 = -\frac{7}{2}A \Rightarrow A = 2$

$x = 3 \Rightarrow 7 = 7B \Rightarrow B = 1$

$\therefore \dfrac{4x-5}{(2x+1)(x-3)} \equiv \dfrac{2}{2x+1} + \dfrac{1}{x-3}$

j $\dfrac{1-3x}{(3x+4)(2x+1)} \equiv \dfrac{A}{3x+4} + \dfrac{B}{2x+1}$

$1 - 3x \equiv A(2x+1) + B(3x+4)$

$x = -\frac{4}{3} \Rightarrow 5 = -\frac{5}{3}A \Rightarrow A = -3$

$x = -\frac{1}{2} \Rightarrow \frac{5}{2} = \frac{5}{2}B \Rightarrow B = 1$

$\therefore \dfrac{1-3x}{(3x+4)(2x+1)} \equiv \dfrac{1}{2x+1} - \dfrac{3}{3x+4}$

k $\dfrac{x+1}{x(1-3x)} \equiv \dfrac{A}{x} + \dfrac{B}{1-3x}$

$x + 1 \equiv A(1-3x) + Bx$

$x = 0 \Rightarrow A = 1$

$x = \frac{1}{3} \Rightarrow \frac{4}{3} = \frac{1}{3}B \Rightarrow B = 4$

$\therefore \dfrac{x+1}{x-3x^2} \equiv \dfrac{1}{x} + \dfrac{4}{1-3x}$

l $\dfrac{5}{(2x-1)(x+2)} \equiv \dfrac{A}{2x-1} + \dfrac{B}{x+2}$

$5 \equiv A(x+2) + B(2x-1)$

$x = \frac{1}{2} \Rightarrow 5 = \frac{5}{2}A \Rightarrow A = 2$

$x = -2 \Rightarrow 5 = -5B \Rightarrow B = -1$

$\therefore \dfrac{5}{2x^2+3x-2} \equiv \dfrac{2}{2x-1} - \dfrac{1}{x+2}$

m $\dfrac{2x+10}{(4x-1)(2x+3)} \equiv \dfrac{A}{4x-1} + \dfrac{B}{2x+3}$

$2x + 10 \equiv A(2x+3) + B(4x-1)$

$x = \frac{1}{4} \Rightarrow \frac{21}{2} = \frac{7}{2}A \Rightarrow A = 3$

$x = -\frac{3}{2} \Rightarrow 7 = -7B \Rightarrow B = -1$

$\therefore \dfrac{2(x+5)}{8x^2+10x-3} \equiv \dfrac{3}{4x-1} - \dfrac{1}{2x+3}$

n $\dfrac{3x-7}{(x+1)(x-3)} \equiv \dfrac{A}{x+1} + \dfrac{B}{x-3}$

$3x - 7 \equiv A(x-3) + B(x+1)$

$x = -1 \Rightarrow -10 = -4A \Rightarrow A = \frac{5}{2}$

$x = 3 \Rightarrow 2 = 4B \Rightarrow B = \frac{1}{2}$

$\therefore \dfrac{3x-7}{x^2-2x-3} \equiv \dfrac{5}{2(x+1)} + \dfrac{1}{2(x-3)}$

o $\dfrac{1-3x}{(1+x)(1-2x)} \equiv \dfrac{A}{1+x} + \dfrac{B}{1-2x}$

$1 - 3x \equiv A(1-2x) + B(1+x)$

$x = -1 \Rightarrow 4 = 3A \Rightarrow A = \frac{4}{3}$

$x = \frac{1}{2} \Rightarrow -\frac{1}{2} = \frac{3}{2}B \Rightarrow B = -\frac{1}{3}$

$\therefore \dfrac{1-3x}{1-x-2x^2} \equiv \dfrac{4}{3(1+x)} - \dfrac{1}{3(1-2x)}$

4 **a** $x = 4 \Rightarrow 84 = 21A \Rightarrow A = 4$

$x = -3 \Rightarrow -56 = 28B \Rightarrow B = -2$

$x = 1 \Rightarrow -12 = -12C \Rightarrow C = 1$

b $x = \frac{1}{3} \Rightarrow \frac{20}{3} = -\frac{20}{9}A \Rightarrow A = -3$

$x = 2 \Rightarrow 30 = 15B \Rightarrow B = 2$

$x = -1 \Rightarrow -12 = 12C \Rightarrow C = -1$

c $x = -5 \Rightarrow 32 = 16A \Rightarrow A = 2$

$x = -1 \Rightarrow 12 = 4C \Rightarrow C = 3$

coeffs of $x^2 \Rightarrow 1 = A + B \Rightarrow B = -1$

d $x = 3 \Rightarrow 196 = 49A \Rightarrow A = 4$

$x = -\frac{1}{2} \Rightarrow 21 = -\frac{7}{2}C \Rightarrow C = -6$

coeffs of $x^2 \Rightarrow 20 = 4A + 2B \Rightarrow B = 2$

5 **a** $8x + 14 \equiv A(x + 1)(x + 3) + B(x - 2)(x + 3) + C(x - 2)(x + 1)$

$x = 2 \Rightarrow 30 = 15A \Rightarrow A = 2$

$x = -1 \Rightarrow 6 = -6B \Rightarrow B = -1$

$x = -3 \Rightarrow -10 = 10C \Rightarrow C = -1$

b $2x^2 - 6x + 20 \equiv A(x + 2)(x - 6) + B(x + 1)(x - 6) + C(x + 1)(x + 2)$

$x = -1 \Rightarrow 28 = -7A \Rightarrow A = -4$

$x = -2 \Rightarrow 40 = 8B \Rightarrow B = 5$

$x = 6 \Rightarrow 56 = 56C \Rightarrow C = 1$

c $9x - 14 \equiv A(x - 1)^2 + B(x + 4)(x - 1) + C(x + 4)$

$x = -4 \Rightarrow -50 = 25A \Rightarrow A = -2$

$x = 1 \Rightarrow -5 = 5C \Rightarrow C = -1$

coeffs of $x^2 \Rightarrow 0 = A + B \Rightarrow B = 2$

d $3x^2 - 7x - 4 \equiv A(x - 2)^2 + B(x - 3)(x - 2) + C(x - 3)$

$x = 3 \Rightarrow A = 2$

$x = 2 \Rightarrow -6 = -C \Rightarrow C = 6$

coeffs of $x^2 \Rightarrow 3 = A + B \Rightarrow B = 1$

6 **a** $\frac{2x^2 + 4}{x(x-1)(x-4)} \equiv \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x-4}$

$2x^2 + 4 \equiv A(x - 1)(x - 4) + Bx(x - 4) + Cx(x - 1)$

$x = 0 \Rightarrow 4 = 4A \Rightarrow A = 1$

$x = 1 \Rightarrow 6 = -3B \Rightarrow B = -2$

$x = 4 \Rightarrow 36 = 12C \Rightarrow C = 3 \quad \therefore \frac{2x^2 + 4}{x(x-1)(x-4)} \equiv \frac{1}{x} - \frac{2}{x-1} + \frac{3}{x-4}$

b $\frac{9}{(x-2)(x+1)^2} \equiv \frac{A}{x-2} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$

$9 \equiv A(x + 1)^2 + B(x - 2)(x + 1) + C(x - 2)$

$x = 2 \Rightarrow 9 = 9A \Rightarrow A = 1$

$x = -1 \Rightarrow 9 = -3C \Rightarrow C = -3$

coeffs of $x^2 \Rightarrow 0 = A + B \Rightarrow B = -1 \quad \therefore \frac{9}{(x-2)(x+1)^2} \equiv \frac{1}{x-2} - \frac{1}{x+1} - \frac{3}{(x+1)^2}$

c $\frac{x^2 + 11x - 21}{(2x+1)(x-2)(x-3)} \equiv \frac{A}{2x+1} + \frac{B}{x-2} + \frac{C}{x-3}$

$x^2 + 11x - 21 \equiv A(x - 2)(x - 3) + B(2x + 1)(x - 3) + C(2x + 1)(x - 2)$

$x = -\frac{1}{2} \Rightarrow -\frac{105}{4} = \frac{35}{4}A \Rightarrow A = -3$

$x = 2 \Rightarrow 5 = -5B \Rightarrow B = -1$

$x = 3 \Rightarrow 21 = 7C \Rightarrow C = 3 \quad \therefore \frac{x^2 + 11x - 21}{(2x+1)(x-2)(x-3)} \equiv \frac{3}{x-3} - \frac{3}{2x+1} - \frac{1}{x-2}$

d $\frac{10x + 9}{(x-4)(x+3)^2} \equiv \frac{A}{x-4} + \frac{B}{x+3} + \frac{C}{(x+3)^2}$

$10x + 9 \equiv A(x + 3)^2 + B(x - 4)(x + 3) + C(x - 4)$

$x = 4 \Rightarrow 49 = 49A \Rightarrow A = 1$

$x = -3 \Rightarrow -21 = -7C \Rightarrow C = 3$

coeffs of $x^2 \Rightarrow 0 = A + B \Rightarrow B = -1 \quad \therefore \frac{10x + 9}{(x-4)(x+3)^2} \equiv \frac{1}{x-4} - \frac{1}{x+3} + \frac{3}{(x+3)^2}$

e $\dfrac{x^2+4x+5}{(x+1)(x+2)^2} \equiv \dfrac{A}{x+1} + \dfrac{B}{x+2} + \dfrac{C}{(x+2)^2}$

$x^2 + 4x + 5 \equiv A(x+2)^2 + B(x+1)(x+2) + C(x+1)$

$x = -1 \quad \Rightarrow \quad A = 2$

$x = -2 \quad \Rightarrow \quad 1 = -C \quad \Rightarrow \quad C = -1$

coeffs of x^2 $\Rightarrow \quad 1 = A + B \quad \Rightarrow \quad B = -1 \quad \therefore \quad \dfrac{x^2+4x+5}{(x+1)(x+2)^2} \equiv \dfrac{2}{x+1} - \dfrac{1}{x+2} - \dfrac{1}{(x+2)^2}$

f $\dfrac{16-2x}{(x-3)(x+2)(x-2)} \equiv \dfrac{A}{x-3} + \dfrac{B}{x+2} + \dfrac{C}{x-2}$

$16 - 2x \equiv A(x+2)(x-2) + B(x-3)(x-2) + C(x-3)(x+2)$

$x = 3 \quad \Rightarrow \quad 10 = 5A \quad \Rightarrow \quad A = 2$

$x = -2 \quad \Rightarrow \quad 20 = 20B \quad \Rightarrow \quad B = 1$

$x = 2 \quad \Rightarrow \quad 12 = -4C \quad \Rightarrow \quad C = -3 \quad \therefore \quad \dfrac{16-2x}{(x-3)(x^2-4)} \equiv \dfrac{2}{x-3} + \dfrac{1}{x+2} - \dfrac{3}{x-2}$

g $\dfrac{2-9x}{(x-3)(2x-1)^2} \equiv \dfrac{A}{x-3} + \dfrac{B}{2x-1} + \dfrac{C}{(2x-1)^2}$

$2 - 9x \equiv A(2x-1)^2 + B(x-3)(2x-1) + C(x-3)$

$x = 3 \quad \Rightarrow \quad -25 = 25A \quad \Rightarrow \quad A = -1$

$x = \frac{1}{2} \quad \Rightarrow \quad -\frac{5}{2} = -\frac{5}{2}C \quad \Rightarrow \quad C = 1$

coeffs of x^2 $\Rightarrow \quad 0 = 4A + 2B \quad \Rightarrow \quad B = 2 \quad \therefore \quad \dfrac{2-9x}{(x-3)(2x-1)^2} \equiv \dfrac{2}{2x-1} + \dfrac{1}{(2x-1)^2} - \dfrac{1}{x-3}$

h $\dfrac{3+24x-4x^2}{(x+1)(x-4)^2} \equiv \dfrac{A}{x+1} + \dfrac{B}{x-4} + \dfrac{C}{(x-4)^2}$

$3 + 24x - 4x^2 \equiv A(x-4)^2 + B(x+1)(x-4) + C(x+1)$

$x = -1 \quad \Rightarrow \quad -25 = 25A \quad \Rightarrow \quad A = -1$

$x = 4 \quad \Rightarrow \quad 35 = 5C \quad \Rightarrow \quad C = 7$

coeffs of x^2 $\Rightarrow \quad -4 = A + B \quad \Rightarrow \quad B = -3 \quad \therefore \quad \dfrac{3+24x-4x^2}{(x+1)(x-4)^2} \equiv \dfrac{7}{(x-4)^2} - \dfrac{3}{x-4} - \dfrac{1}{x+1}$

i $\dfrac{9x^2-2x-12}{x(x+3)(x-2)} \equiv \dfrac{A}{x} + \dfrac{B}{x+3} + \dfrac{C}{x-2}$

$9x^2 - 2x - 12 \equiv A(x+3)(x-2) + Bx(x-2) + Cx(x+3)$

$x = 0 \quad \Rightarrow \quad -12 = -6A \quad \Rightarrow \quad A = 2$

$x = -3 \quad \Rightarrow \quad 75 = 15B \quad \Rightarrow \quad B = 5$

$x = 2 \quad \Rightarrow \quad 20 = 10C \quad \Rightarrow \quad C = 2 \quad \therefore \quad \dfrac{9x^2-2x-12}{x^3+x^2-6x} \equiv \dfrac{2}{x} + \dfrac{5}{x+3} + \dfrac{2}{x-2}$

j $\dfrac{5x^2+3x-20}{x^2(x+4)} \equiv \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x+4}$

$5x^2 + 3x - 20 \equiv Ax(x+4) + B(x+4) + Cx^2$

$x = 0 \quad \Rightarrow \quad -20 = 4B \quad \Rightarrow \quad B = -5$

$x = -4 \quad \Rightarrow \quad 48 = 16C \quad \Rightarrow \quad C = 3$

coeffs of x^2 $\Rightarrow \quad 5 = A + C \quad \Rightarrow \quad A = 2 \quad \therefore \quad \dfrac{5x^2+3x-20}{x^3+4x^2} \equiv \dfrac{2}{x} - \dfrac{5}{x^2} + \dfrac{3}{x+4}$

k $\dfrac{13-3x^2}{(2x+3)(x-1)^2} \equiv \dfrac{A}{2x+3} + \dfrac{B}{x-1} + \dfrac{C}{(x-1)^2}$

$13 - 3x^2 \equiv A(x-1)^2 + B(2x+3)(x-1) + C(2x+3)$

$x = -\frac{3}{2} \quad \Rightarrow \quad \frac{25}{4} = \frac{25}{4}A \quad \Rightarrow \quad A = 1$

$x = 1 \quad \Rightarrow \quad 10 = 5C \quad \Rightarrow \quad C = 2$

coeffs of x^2 $\Rightarrow \quad -3 = A + 2B \quad \Rightarrow \quad B = -2 \quad \therefore \quad \dfrac{13-3x^2}{(2x+3)(x-1)^2} \equiv \dfrac{1}{2x+3} - \dfrac{2}{x-1} + \dfrac{2}{(x-1)^2}$

l $\dfrac{26-x-x^2}{(x-1)(x+3)(x+5)} \equiv \dfrac{A}{x-1} + \dfrac{B}{x+3} + \dfrac{C}{x+5}$

$26 - x - x^2 \equiv A(x + 3)(x + 5) + B(x - 1)(x + 5) + C(x - 1)(x + 3)$

$x = 1 \quad \Rightarrow \quad 24 = 24A \quad \Rightarrow \quad A = 1$

$x = -3 \quad \Rightarrow \quad 20 = -8B \quad \Rightarrow \quad B = -\frac{5}{2}$

$x = -5 \quad \Rightarrow \quad 6 = 12C \quad \Rightarrow \quad C = \frac{1}{2} \quad \therefore \quad \dfrac{26-x-x^2}{(x-1)(x+3)(x+5)} \equiv \dfrac{1}{x-1} - \dfrac{5}{2(x+3)} + \dfrac{1}{2(x+5)}$

7 **a** $x^2 \equiv A(x - 2)(x - 6) + B(x - 6) + C(x - 2)$

$x = 2 \quad \Rightarrow \quad 4 = -4B \quad \Rightarrow \quad B = -1$

$x = 6 \quad \Rightarrow \quad 36 = 4C \quad \Rightarrow \quad C = 9$

coeffs of $x^2 \quad \Rightarrow \quad A = 1$

b $\dfrac{x^2+2x+9}{(x-1)(x+5)} \equiv A + \dfrac{B}{x-1} + \dfrac{C}{x+5}$

$x^2 + 2x + 9 \equiv A(x - 1)(x + 5) + B(x + 5) + C(x - 1)$

$x = 1 \quad \Rightarrow \quad 12 = 6B \quad \Rightarrow \quad B = 2$

$x = -5 \quad \Rightarrow \quad 24 = -6C \quad \Rightarrow \quad C = -4$

coeffs of $x^2 \quad \Rightarrow \quad A = 1$

8 **a**

$$\begin{array}{r} x + 3 \\ x^2 + x - 2 \overline{)\ x^3 + 4x^2 + 0x - 2} \\ \underline{x^3 + x^2 - 2x} \\ 3x^2 + 2x - 2 \\ \underline{3x^2 + 3x - 6} \\ -x + 4 \end{array}$$

quotient: $x + 3$ remainder: $-x + 4$

b $\dfrac{x^3+4x^2-2}{x^2+x-2} \equiv x + 3 + \dfrac{4-x}{x^2+x-2}$

$\dfrac{4-x}{(x+2)(x-1)} \equiv \dfrac{A}{x+2} + \dfrac{B}{x-1}$

$4 - x \equiv A(x - 1) + B(x + 2)$

$x = -2 \quad \Rightarrow \quad 6 = -3A \quad \Rightarrow \quad A = -2$

$x = 1 \quad \Rightarrow \quad 3 = 3B \quad \Rightarrow \quad B = 1 \quad \therefore \quad \dfrac{x^3+4x^2-2}{x^2+x-2} \equiv x + 3 - \dfrac{2}{x+2} + \dfrac{1}{x-1}$

9 **a** $(x - 3)(x + 1) = x^2 - 2x - 3$

$$\begin{array}{r} 1 \\ x^2 - 2x - 3 \overline{)\ x^2 + 0x + 3} \\ \underline{x^2 - 2x - 3} \\ 2x + 6 \end{array}$$

$\therefore \quad \dfrac{x^2+3}{(x-3)(x+1)} \equiv 1 + \dfrac{2x+6}{(x-3)(x+1)}$

$\dfrac{2x+6}{(x-3)(x+1)} \equiv \dfrac{A}{x-3} + \dfrac{B}{x+1}$

$2x + 6 \equiv A(x + 1) + B(x - 3)$

$x = 3 \quad \Rightarrow \quad 12 = 4A \quad \Rightarrow \quad A = 3$

$x = -1 \quad \Rightarrow \quad 4 = -4B \quad \Rightarrow \quad B = -1 \quad \therefore \quad \dfrac{x^2+3}{(x-3)(x+1)} \equiv 1 + \dfrac{3}{x-3} - \dfrac{1}{x+1}$

b

$$\begin{array}{r} x - 3 \\ x^2 - 4 \overline{)\, x^3 - 3x^2 - x + 2} \\ \underline{x^3 + 0x^2 - 4x} \\ -3x^2 + 3x + 2 \\ \underline{-3x^2 + 0x + 12} \\ 3x - 10 \end{array}$$

$\therefore \dfrac{x^3-3x^2-x+2}{x^2-4} \equiv x - 3 + \dfrac{3x-10}{x^2-4}$

$\dfrac{3x-10}{(x+2)(x-2)} \equiv \dfrac{A}{x+2} + \dfrac{B}{x-2}$

$3x - 10 \equiv A(x-2) + B(x+2)$

$x = -2 \quad \Rightarrow \quad -16 = -4A \quad \Rightarrow \quad A = 4$

$x = 2 \quad \Rightarrow \quad -4 = 4B \quad \Rightarrow \quad B = -1 \qquad \therefore \dfrac{x^3-3x^2-x+2}{x^2-4} \equiv x - 3 + \dfrac{4}{x+2} - \dfrac{1}{x-2}$

c

$$\begin{array}{r} 2 \\ x^2 + 6x + 8 \overline{)\, 2x^2 + 7x + 0} \\ \underline{2x^2 + 12x + 16} \\ -5x - 16 \end{array}$$

$\therefore \dfrac{2x^2+7x}{x^2+6x+8} \equiv 2 + \dfrac{-5x-16}{x^2+6x+8}$

$\dfrac{-5x-16}{(x+2)(x+4)} \equiv \dfrac{A}{x+2} + \dfrac{B}{x+4}$

$-5x - 16 \equiv A(x+4) + B(x+2)$

$x = -2 \quad \Rightarrow \quad -6 = 2A \quad \Rightarrow \quad A = -3$

$x = -4 \quad \Rightarrow \quad 4 = -2B \quad \Rightarrow \quad B = -2 \qquad \therefore \dfrac{2x^2+7x}{x^2+6x+8} \equiv 2 - \dfrac{3}{x+2} - \dfrac{2}{x+4}$

d $3(x+1)(x-1) = 3x^2 - 3$

$(x-4)(x+5) = x^2 + x - 20$

$$\begin{array}{r} 3 \\ x^2 + x - 20 \overline{)\, 3x^2 + 0x - 3} \\ \underline{3x^2 + 3x - 60} \\ -3x + 57 \end{array}$$

$\therefore \dfrac{3(x+1)(x-1)}{(x-4)(x+5)} \equiv 3 + \dfrac{57-3x}{(x-4)(x+5)}$

$\dfrac{57-3x}{(x-4)(x+5)} \equiv \dfrac{A}{x-4} + \dfrac{B}{x+5}$

$57 - 3x \equiv A(x+5) + B(x-4)$

$x = 4 \quad \Rightarrow \quad 45 = 9A \quad \Rightarrow \quad A = 5$

$x = -5 \quad \Rightarrow \quad 72 = -9B \quad \Rightarrow \quad B = -8 \qquad \therefore \dfrac{3(x+1)(x-1)}{(x-4)(x+5)} \equiv 3 + \dfrac{5}{x-4} - \dfrac{8}{x+5}$

e

$$x^2+4x+3 \overline{)\,3x^3+7x^2+0x+4}$$

$$\begin{array}{r} 3x-5 \\ \hline 3x^3+7x^2+0x+4 \\ 3x^3+12x^2+9x \\ \hline -5x^2-9x+4 \\ -5x^2-20x-15 \\ \hline 11x+19 \end{array}$$

$\therefore \quad \dfrac{3x^3+7x^2+4}{x^2+4x+3} \equiv 3x-5+\dfrac{11x+19}{x^2+4x+3}$

$\dfrac{11x+19}{(x+1)(x+3)} \equiv \dfrac{A}{x+1}+\dfrac{B}{x+3}$

$11x+19 \equiv A(x+3)+B(x+1)$

$x=-1 \quad \Rightarrow \quad 8=2A \quad \Rightarrow \quad A=4$

$x=-3 \quad \Rightarrow \quad -14=-2B \quad \Rightarrow \quad B=7$

$\therefore \quad \dfrac{3x^3+7x^2+4}{x^2+4x+3} \equiv 3x-5+\dfrac{4}{x+1}+\dfrac{7}{x+3}$

f

$$2x^2-7x+3 \overline{)\,4x^2-7x+5}$$

$$\begin{array}{r} 2 \\ \hline 4x^2-7x+5 \\ 2x^2-14x+6 \\ \hline 7x-1 \end{array}$$

$\therefore \quad \dfrac{4x^2-7x+5}{2x^2-7x+3} \equiv 2+\dfrac{7x-1}{2x^2-7x+3}$

$\dfrac{7x-1}{(2x-1)(x-3)} \equiv \dfrac{A}{2x-1}+\dfrac{B}{x-3}$

$7x-1 \equiv A(x-3)+B(2x-1)$

$x=\frac{1}{2} \quad \Rightarrow \quad \frac{5}{2}=-\frac{5}{2}A \quad \Rightarrow \quad A=-1$

$x=3 \quad \Rightarrow \quad 20=5B \quad \Rightarrow \quad B=4$

$\therefore \quad \dfrac{4x^2-7x+5}{2x^2-7x+3} \equiv 2-\dfrac{1}{2x-1}+\dfrac{4}{x-3}$

g

$$x^2-2x-3 \overline{)\,2x^2+0x+0}$$

$$\begin{array}{r} 2 \\ \hline 2x^2+0x+0 \\ 2x^2-4x-6 \\ \hline 4x+6 \end{array}$$

$\therefore \quad \dfrac{2x^2}{x^2-2x-3} \equiv 2+\dfrac{4x+6}{x^2-2x-3}$

$\dfrac{4x+6}{(x+1)(x-3)} \equiv \dfrac{A}{x+1}+\dfrac{B}{x-3}$

$4x+6 \equiv A(x-3)+B(x+1)$

$x=-1 \quad \Rightarrow \quad 2=-4A \quad \Rightarrow \quad A=-\frac{1}{2}$

$x=3 \quad \Rightarrow \quad 18=4B \quad \Rightarrow \quad B=\frac{9}{2}$

$\therefore \quad \dfrac{2x^2}{x^2-2x-3} \equiv 2-\dfrac{1}{2(x+1)}+\dfrac{9}{2(x-3)}$

h

$$x^2-6x+5\overline{)\;x^3-6x^2+6x+1}$$
$$\underline{x^3-6x^2+5x}$$
$$x+1$$

$\therefore \dfrac{x^3-6x^2+6x+1}{x^2-6x+5} \equiv x+\dfrac{x+1}{x^2-6x+5}$

$\dfrac{x+1}{(x-1)(x-5)} \equiv \dfrac{A}{x-1}+\dfrac{B}{x-5}$

$x+1 \equiv A(x-5)+B(x-1)$

$x=1 \quad \Rightarrow \quad 2=-4A \quad \Rightarrow \quad A=-\frac{1}{2}$

$x=5 \quad \Rightarrow \quad 6=4B \quad \Rightarrow \quad B=\frac{3}{2}$ $\quad \therefore \dfrac{x^3-6x^2+6x+1}{x^2-6x+5} \equiv x-\dfrac{1}{2(x-1)}+\dfrac{3}{2(x-5)}$

i

$$3x^2-4x-4\overline{)\;9x^3+0x^2-27x-2}$$
$$\underline{9x^3-12x^2-12x}$$
$$12x^2-15x-2$$
$$\underline{12x^2-16x-16}$$
$$x+14$$

$\therefore \dfrac{9x^3-27x-2}{3x^2-4x-4} \equiv 3x+4+\dfrac{x+14}{3x^2-4x-4}$

$\dfrac{x+14}{(3x+2)(x-2)} \equiv \dfrac{A}{3x+2}+\dfrac{B}{x-2}$

$x+14 \equiv A(x-2)+B(3x+2)$

$x=-\frac{2}{3} \quad \Rightarrow \quad \frac{40}{3}=-\frac{8}{3}A \quad \Rightarrow \quad A=-5$

$x=2 \quad \Rightarrow \quad 16=8B \quad \Rightarrow \quad B=2$ $\quad \therefore \dfrac{9x^3-27x-2}{3x^2-4x-4} \equiv 3x+4-\dfrac{5}{3x+2}+\dfrac{2}{x-2}$

10 **a** $\dfrac{x+5}{(x-1)(2x+1)} \equiv \dfrac{A}{x-1}+\dfrac{B}{2x+1}$

$x+5 \equiv A(2x+1)+B(x-1)$

$x=1 \quad \Rightarrow \quad 6=3A \quad \Rightarrow \quad A=2$

$x=-\frac{1}{2} \quad \Rightarrow \quad \frac{9}{2}=-\frac{3}{2}B \quad \Rightarrow \quad B=-3$

$\therefore \; f(x)=\dfrac{2}{x-1}-\dfrac{3}{2x+1}$

b $f'(x)=-2(x-1)^{-2}+3(2x+1)^{-2}\times 2$

$=\dfrac{6}{(2x+1)^2}-\dfrac{2}{(x-1)^2}$

SP: $\dfrac{6}{(2x+1)^2}-\dfrac{2}{(x-1)^2}=0$

$6(x-1)^2-2(2x+1)^2=0$

$x^2+10x-2=0$

$x=\dfrac{-10\pm\sqrt{100+8}}{2}$

$x=-5\pm 3\sqrt{3}$

11 **a** $x(4x+5) \equiv A(x+2)^2+B(x-1)(x+2)+C(x-1)$

$x=1 \quad \Rightarrow \quad 9=9A \quad \Rightarrow \quad A=1$

$x=-2 \quad \Rightarrow \quad 6=-3C \quad \Rightarrow \quad C=-2$

coeffs $x^2 \Rightarrow \quad 4=A+B \quad \Rightarrow \quad B=3$

b $x=-1 \;\; \therefore y=\frac{1}{2}$

$f(x)=(x-1)^{-1}+3(x+2)^{-1}-2(x+2)^{-2}$

$f'(x)=-(x-1)^{-2}-3(x+2)^{-2}+4(x+2)^{-3}$

grad $=-\frac{1}{4}-3+4=\frac{3}{4}$

$\therefore \; y-\frac{1}{2}=\frac{3}{4}(x+1)$

$4y-2=3x+3$

$3x-4y+5=0$