Practice paper

1 Use the binomial theorem to expand $\frac{1}{(2+x)^2}$, $|x| < 2$, in ascending powers of x, as far as the term in x^3, giving each coefficient as a simplified fraction. (6)

2 The curve C has equation

$$x^2 + 2y^2 - 4x - 6yx + 3 = 0$$

Find the gradient of C at the point (1, 3). (7)

3 Use the substitution $u = 5x + 3$, to find an exact value for

$$\int_0^3 \frac{10x}{(5x+3)^3}\,dx$$ (9)

4 **a** Find the values of A and B for which

$$\frac{1}{(2x+1)(x-2)} \equiv \frac{A}{2x+1} + \frac{B}{x-2}$$ (3)

b Hence find $\int \frac{1}{(2x+1)(x-2)}\,dx$, giving your answer in the form $\ln f(x)$. (4)

c Hence, or otherwise, obtain the solution of

$$(2x+1)(x-2)\frac{dy}{dx} = 10y,\ y > 0,\ x > 2$$

for which $y = 1$ at $x = 3$, giving your answer in the form $y = f(x)$. (5)

5 A population grows in such a way that the rate of change of the population P at time t in days is proportional to P.

a Write down a differential equation relating P and t. (2)

b Show, by solving this equation or by differentiation, that the general solution of this equation may be written as $P = Ak^t$, where A and k are positive constants. (5)

Initially the population is 8 million and 7 days later it has grown to 8.5 million.

c Find the size of the population after a further 28 days. (5)

6 Referred to an origin O the points A and B have position vectors $\mathbf{i} - 5\mathbf{j} - 7\mathbf{k}$ and $10\mathbf{i} + 10\mathbf{j} + 5\mathbf{k}$ respectively. P is a point on the line AB.

a Find a vector equation for the line passing through A and B. (3)

b Find the position vector of point P such that OP is perpendicular to AB. (5)

c Find the area of triangle OAB. (4)

d Find the ratio in which P divides the line AB. (2)

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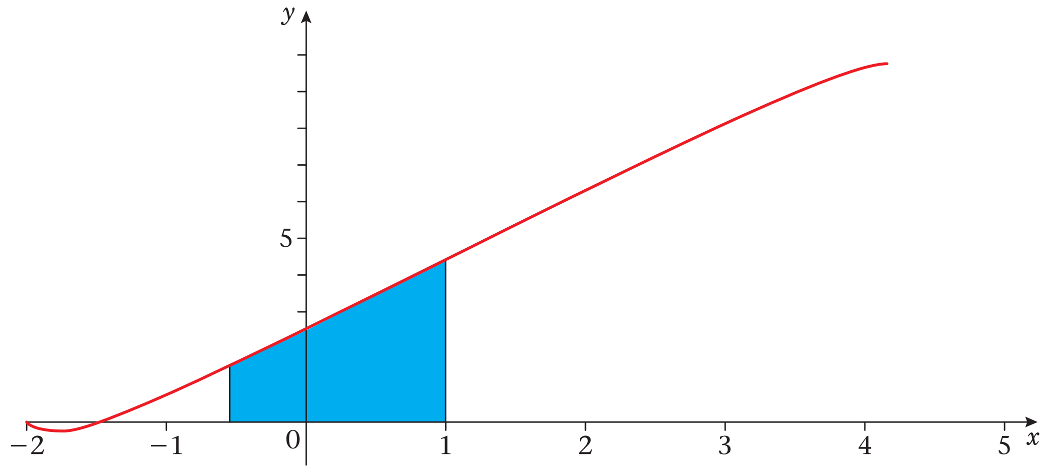

The curve C, shown has parametric equations

$$x = 1 - 3\cos t,\ y = 3t - 2\sin 2t,\ 0 < t < \pi.$$

a Find the gradient of the curve at the point P where $t = \dfrac{\pi}{6}$. (4)

b Show that the area of the finite region beneath the curve, between the lines $x = -\frac{1}{2}$, $x = 1$ and the x-axis, shown shaded in the diagram, is given by the integral

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 9t\sin t\,dt - \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 12\sin^2 t\cos t\,dt.$$ (4)

c Hence, by integration, find an exact value for this area. (7)