

4.

Figure 1

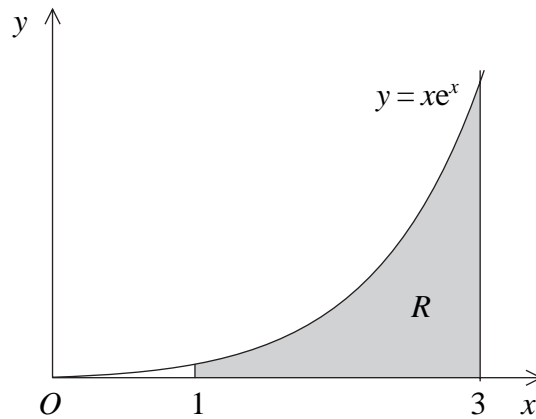


Figure 1 shows the finite shaded region, R , which is bounded by the curve $y = xe^x$, the line $x = 1$, the line $x = 3$ and the x -axis.

The region R is rotated through 360 degrees about the x -axis.

Use integration by parts to find an exact value for the **volume** of the solid generated.

(8)



6. The line l_1 has vector equation

$$\mathbf{r} = 8\mathbf{i} + 12\mathbf{j} + 14\mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} - \mathbf{k}),$$

where λ is a parameter.

The point A has coordinates $(4, 8, a)$, where a is a constant. The point B has coordinates $(b, 13, 13)$, where b is a constant. Points A and B lie on the line l_1 .

(a) Find the values of a and b . (3)

Given that the point O is the origin, and that the point P lies on l_1 such that OP is perpendicular to l_1 ,

(b) find the coordinates of P . (5)

(c) Hence find the distance OP , giving your answer as a simplified surd. (2)



7. The volume of a spherical balloon of radius r cm is V cm³, where $V = \frac{4}{3}\pi r^3$.

(a) Find $\frac{dV}{dr}$. (1)

The volume of the balloon increases with time t seconds according to the formula

$$\frac{dV}{dt} = \frac{1000}{(2t+1)^2}, \quad t \geq 0.$$

(b) Using the chain rule, or otherwise, find an expression in terms of r and t for $\frac{dr}{dt}$. (2)

(c) Given that $V = 0$ when $t = 0$, solve the differential equation $\frac{dV}{dt} = \frac{1000}{(2t+1)^2}$, to obtain V in terms of t . (4)

(d) Hence, at time $t = 5$,

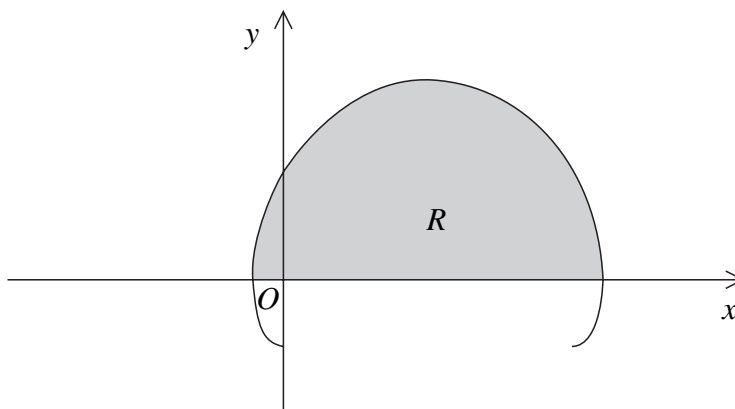
(i) find the radius of the balloon, giving your answer to 3 significant figures, (3)

(ii) show that the rate of increase of the radius of the balloon is approximately 2.90×10^{-2} cm s⁻¹. (2)



8.

Figure 2



The curve shown in Figure 2 has parametric equations

$$x = t - 2 \sin t, \quad y = 1 - 2 \cos t, \quad 0 \leq t \leq 2\pi.$$

- (a) Show that the curve crosses the x -axis where $t = \frac{\pi}{3}$ and $t = \frac{5\pi}{3}$. (2)

The finite region R is enclosed by the curve and the x -axis, as shown shaded in Figure 2.

- (b) Show that the area of R is given by the integral

$$\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}} (1 - 2 \cos t)^2 dt. \tag{3}$$

- (c) Use this integral to find the exact value of the shaded area. (7)



