

3. The curve C has equation

$$x = 2 \sin y.$$

(a) Show that the point $P\left(\sqrt{2}, \frac{\pi}{4}\right)$ lies on C . (1)

(b) Show that $\frac{dy}{dx} = \frac{1}{\sqrt{2}}$ at P . (4)

(c) Find an equation of the normal to C at P . Give your answer in the form $y = mx + c$, where m and c are exact constants. (4)



5.

Figure 1

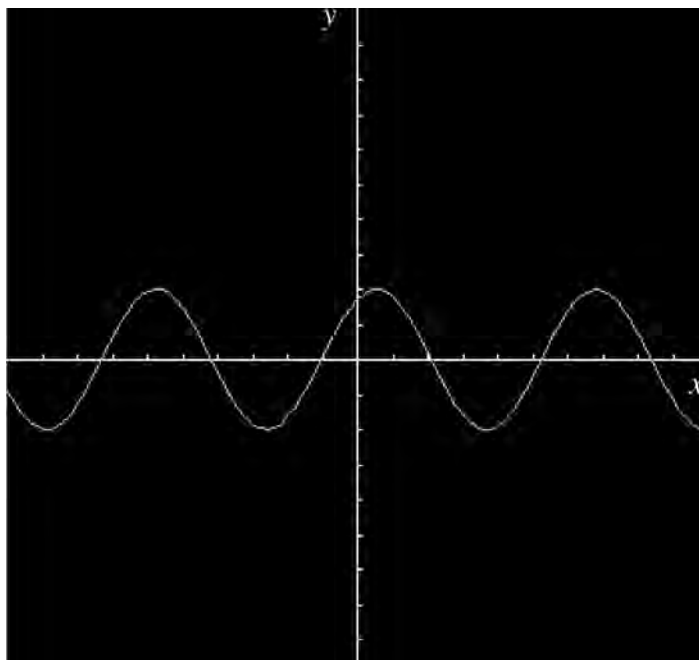


Figure 1 shows an oscilloscope screen.

The curve shown on the screen satisfies the equation

$$y = \sqrt{3} \cos x + \sin x.$$

(a) Express the equation of the curve in the form $y = R \sin(x + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. (4)

(b) Find the values of x , $0 \leq x < 2\pi$, for which $y = 1$. (4)



6. The function f is defined by

$$f : x \mapsto \ln(4 - 2x), \quad x < 2 \quad \text{and} \quad x \in \mathbb{R}.$$

(a) Show that the inverse function of f is defined by

$$f^{-1} : x \mapsto 2 - \frac{1}{2}e^x$$

and write down the domain of f^{-1} .

(4)

(b) Write down the range of f^{-1} .

(1)

(c) In the space provided on page 16, sketch the graph of $y = f^{-1}(x)$. State the coordinates of the points of intersection with the x and y axes.

(4)

The graph of $y = x + 2$ crosses the graph of $y = f^{-1}(x)$ at $x = k$.

The iterative formula

$$x_{n+1} = -\frac{1}{2}e^{x_n}, \quad x_0 = -0.3$$

is used to find an approximate value for k .

(d) Calculate the values of x_1 and x_2 , giving your answers to 4 decimal places.

(2)

(e) Find the value of k to 3 decimal places.

(2)



Question 6 continued



Question 7 continued

Q7

(Total 13 marks)



