Mark Scheme (Results)

January 2014

Pearson Edexcel International Advanced Level

Core Mathematics 3 (6665A)

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January 2014
Publications Code IA037653

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- $\sqsubset$ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \ldots$

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to x = …

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula
Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.
Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers
Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working
The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Question Number	Scheme		Marks
1 (a)	Radians: f(0.2) = - 0.4 , f(0.4) = 0.3 or considers smaller subset of [0.2, 0.4]	Degrees: f(0.2) = - 0.4 , f(0.4) = 0.2 or considers smaller subset of [0.2, 0.4]	M1
	Change of sign implies root		A1 (2)
(b)	$\sec x+3x-2=0 \Rightarrow 3x=2-\sec x$ and so $x=\frac{2}{3}-\frac{1}{3\cos x}$ *		B1 (1)
(c)	Radians: $x_1=0.3177$, $x_2=0.3158$, $x_3=0.3160$	Degrees: $x_1=0.3333$, $x_2=0.3333$, $x_3=0.3333$	M1, A1, A1 (3)
(d)	0.316 (radians)	0.333 (degrees)	B1 (1)
			[7]

Notes

(a) **M1:** Gives two answers with at least one correct to 1sf. Candidates may work in degrees or in radians in this question, but there is a maximum of 6/7 for those working in degrees. (May choose smaller interval between 0.2 and 0.4 e.g. f(0.3) and f(0.35) but this must span the root which is near to 0.316 in radians and 0.333 in degrees) **If they choose a larger interval then this is M0**

A1: Both their values correct to at least one decimal place, **and** reason given (e.g. change of sign or f(0.2)<0, f(0.4)>0 or product f(0.2)f(0.4)<0 or equivalent) **and** conclusion e.g. root

(b) **B1:** Starts with equation equal to zero, rearranges correctly with **no errors** and at least one intermediate step

(c) **M1**:Substitutes $x_0=0.3$ into $x=\frac{2}{3}-\frac{1}{3\cos x} \Rightarrow x_1=$

This can be implied by $x_1=\frac{2}{3}-\frac{1}{3\cos 0.3}$, or answers which round to 0.32 (rads) or 0.33 (degrees)

A1: x_1 awrt 0.3177 4dp (rads) or to awrt 0.3333 4dp (degrees)

Mark as the first value given. Don't be concerned by the subscript

A1: $x_2=$ awrt 0.3158, $x_3=$ awrt 0.3160 (rads) – NOT just 0.316

NB $x_2=$ awrt 0.3333, $x_3=$ awrt 0.3333 (degrees). **This mark is A0.** They cannot score **A1** if working in degrees

Mark the second and third values given. Don't be concerned by the subscripts Ignore extra values.

(d) **B1**: 0.316 stated to 3dp (independent of part (c)) for radians or 0.333 for degrees

The whole answer must maintain consistent units – either degrees, or radians. Use answer to (c) to determine units being used. NB Degree answers have maximum of M1A1B1M1A1A0B1 ie 6/7

Question Number	Scheme	Marks
2. **(a)**	Use of common denominator e.g. $\frac{15}{3x+4}-\frac{2x}{x-1}+\frac{14}{(3x+4)(x-1)}=\frac{15(x-1)-2x(3x+4)+14}{(3x+4)(x-1)}$	M1
	$=\frac{-6x^2+7x-1}{(3x+4)(x-1)}$	A1
	$=\frac{-(6x-1)\cancel{(x-1)}}{(3x+4)\cancel{(x-1)}}$	M1
	$=\frac{(1-6x)}{(3x+4)}$ or $=\frac{(-6x+1)}{(3x+4)}$ or $=-\frac{(6x-1)}{(3x+4)}$ o.e.	A1 (4)
First Alternative for (a)	$\frac{15}{3x+4}-\frac{2x}{x-1}+\frac{14}{(3x+4)(x-1)}=\frac{15}{3x+4}+\frac{-2x(3x+4)+14}{(3x+4)(x-1)}$	M1
	$=\frac{15}{3x+4}+\frac{-6x^2-8x+14}{(3x+4)(x-1)}$	A1
	$=\frac{15}{3x+4}+\frac{-2\cancel{(x-1)}(3x+7)}{(3x+4)\cancel{(x-1)}}$	M1
	$=\frac{(1-6x)}{(3x+4)}$ or $=\frac{(-6x+1)}{(3x+4)}$ or $=-\frac{(6x-1)}{(3x+4)}$ o.e.	A1 (4)
Second Alternative for (a)	$\frac{15}{3x+4}-\frac{2x}{x-1}+\frac{14}{(3x+4)(x-1)}=\frac{15(3x+4)(x-1)-2x(3x+4)^2+14(3x+4)}{(3x+4)^2(x-1)}$	M1
	$=\frac{(3x+4)(-6x^2+7x-1)}{(3x+4)^2(x-1)}$ or $=\frac{(x-1)(-18x^2-21x+4)}{(3x+4)^2(x-1)}$	A1
	$=\frac{\cancel{(3x+4)}^2(1-6x)}{\cancel{(3x+4)}^2(x-1)}$, $=\frac{(1-6x)}{(3x+4)}$ or $=\frac{(-6x+1)}{(3x+4)}$ or $=-\frac{(6x-1)}{(3x+4)}$ o.e.	M1, A1 (4)
(b)	$f'(x)=\frac{(3x+4)\times(-6)-(1-6x)\times 3}{(3x+4)^2}$	M1 A1ft
	$=\frac{-27}{(3x+4)^2}$	A1cao (3)
Alternative for (b)	Or $f'(x)=(3x+4)^{-1}\times(-6)+(1-6x)\times(-3)\times(3x+4)^{-2}$	M1 A1ft
	$=\frac{-27}{(3x+4)^2}$	A1cao (3)
Second Alternative for (b)	Or $f(x)=-2+\frac{9}{3x+4}$ so $f'(x)=9\times(-3)\times(3x+4)^{-2}$ $=\frac{-27}{(3x+4)^2}$	M1 A1ft A1cao (3)

Question Number	Scheme	Marks
Third Alternative for (b)	Differentiates original expression: $\frac{-45}{(3x+4)^2}-\left[\frac{2(x-1)-2x}{(x-1)^2}\right]+\frac{-14(6x+1)}{(3x+4)^2(x-1)^2}$	M1 A1
	$=\frac{-27}{(3x+4)^2}$	A1cao **[7]**

Notes

(a) **M**1: Combines two or three fractions into single fraction with **correct** use of common denominator
A1: correct answer with collected terms giving three term quadratic numerator
M1: Factorises their quadratic following usual rules in numerator:
A1 cao (but may be written in different ways – see m-s above)

(b) **M1**: Applies product or quotient rule correctly to **their** fraction (must have x terms in numerator and denominator of their answer to (a) which may be linear, quadratic, or even cubic; **not just constant numerator**) but it should be clear that they are using the correct rule with correct signs and correct term squared (in the case of quotient rule) i.e. using $\frac{vu'-uv'}{v^2}$ and states $u=\ ,\ v=\ ,\frac{du}{dx}=\ ,\ \frac{dv}{dx}=$ or an answer of the form $\frac{(3x+4)\times A-(1-6x)\times B}{(3x+4)^2}$ implies the method.
Similarly for the product rule : If the formula is quoted it must be correct. There must have been some attempt to differentiate both terms.
If the rule is not quoted nor implied by their working, meaning that term are written out $u=\text{"}1-6x\text{"},\ v=(\text{"}3x+4\text{"})^{-1},\ u'=..,\ v'=...$ followed by their $vu'+uv'$, then only accept answers of the form $(\text{"}3x+4\text{"})^{-1}\times A\pm\text{"}3\text{"}(\text{"}3x+4\text{"})^{-2}\times B$.
Condone invisible brackets for the M mark.
For the **third alternative method**, need an attempt at all three differentiations in line with the guidance above. (N.B. the first A1 is not ft for this method).

A1ft: may be **unsimplified** e.g. $\frac{(3x+4)\times(-6)-(1-6x)\times 3}{(3x+4)^2}$ but should be correct for their answer to (a)

A1: correct **simplified** cao but accept $=\frac{-27}{9x^2+24x+16}$, as alternative

So a wrong answer in (a) can only achieve a maximum mark of M1A1A0 in part (b)

Question Number	Scheme	Marks
3 (a)	Let $y = (\sin x)^{-1}$, then $\frac{dy}{dx} = -1(\sin x)^{-2} \times \cos x$	M1 A1
	i.e. $\frac{dy}{dx} = \frac{-1}{\sin x} \times \frac{\cos x}{\sin x} = -\operatorname{cosec} x \cot x$ *	B1* (3)
Alternative Method (a)	Use of quotient rule $\frac{dy}{dx} = \frac{\sin x \times 0 - 1\cos x}{\sin^2 x}$	M1A1
	i.e. $\frac{dy}{dx} = \frac{-1}{\sin x} \times \frac{\cos x}{\sin x} = -\operatorname{cosec} x \cot x$ *	B1* (3)
(b)	$\frac{dy}{dx} = 3e^{3x} \operatorname{cosec} 2x + e^{3x} \times -2\operatorname{cosec} 2x \cot 2x$	M1 A1 A1 (3)
(c)	$\frac{dy}{dx} = e^{3x} \operatorname{cosec} 2x(3 - 2\cot 2x) = 0$	M1
	(So cot $2x = 1.5$) tan $2x = 2/3$ so $x = \frac{1}{2}\arctan\frac{2}{3}$ (or $k = 2/3$)	A1 (2) **[8]**

Notes

(a) **M1**: Use of chain rule so $\frac{dy}{dx} = -1(\sin x)^{-2} \times (\pm\cos x)$

A1: cao

B1: Use of definitions of cosec$x$ and cot$x$ and conclusion, with no errors (need at least intermediate step shown in scheme which may be written $\frac{dy}{dx} = \frac{-\cos x}{\sin x \sin x}$). This mark is dependent on the M1.

Alternative: **M1**: If quotient rule is used need to see $\frac{dy}{dx} = \frac{\sin x \times 0 - 1(\pm\cos x)}{\sin^2 x}$, then **A1** is cao

(b) **M1**: If the rule is not quoted nor implied by their working, meaning that terms are written out $u = e^{3x}, v = \operatorname{cosec} 2x, u' = .., v' = ...$ followed by their $vu' + uv'$, then only accept answers of the form $\mu e^{3x} \operatorname{cosec} 2x + e^{3x} \times \lambda \operatorname{cosec} 2x \cot 2x$.

A1: one term correct, A1 both terms correct (need not simplify isw)

(c) **M1**: Puts $\frac{dy}{dx} = 0$ and factorises or cancels by $e^{3x} \operatorname{cosec} 2x$ concluding that $a \pm b\cot 2x = 0$ or $\cot 2x = \pm\frac{a}{b}$

A1: Draws correct conclusion $\frac{1}{2}\arctan\frac{2}{3}$ or $k = 2/3$

Question Number	Scheme	Marks
4.	(a) When $t = 0$, $\theta = 85$ so $85 = A + 60$, $A = 25$	B1 (1)
	(b) $58 = "25" + 60e^{-k15}$	
	$"33" = 60e^{-k15} \rightarrow e^{-k15} = \frac{"33"}{60}$ or $e^{k15} = \frac{60}{"33"}$	M1
	So $-15k = \ln\left(\frac{"11"}{20}\right)$ or $15k = \ln\left(\frac{20}{"11"}\right)$	M1
	$k = -\frac{1}{15}\ln\left(\frac{11}{20}\right) = \frac{1}{15}\ln\left(\frac{20}{11}\right)$ *	A1cso* (3)
	(c) $50 = "25" + 60e^{-kt} \rightarrow e^{-kt} =$ or $e^{kt} =$ or $(.96)^t =$	M1
	$(e^{-kt}) = \frac{25}{60}$ (or awrt 0.42) or $(0.96^t) = \frac{25}{60}$ or $(e^{kt}) = \frac{60}{25}$	A1
	$t = \frac{\ln\left(\frac{"25"}{60}\right)}{-k}$ or $t = \frac{\ln\left(\frac{60}{"25"}\right)}{k}$	M1
	$\frac{\ln\left(\frac{25}{60}\right)}{-\frac{1}{15}\ln\left(\frac{20}{11}\right)} = (21.96) =$ 22 mins (approx) or 11.22 or $t = 22$	A1 (4) **[8]**

Notes

(a) **B1**: Gives answer $A = 25$ – any work seen should be correct

(b) **M1**: Uses values 58 and 15 with their A to form equation in k and isolate $e^{-k15} =$ or $e^{k15} =$

M1: Uses logs correctly (*following correct log rules and only applying log to positive quantities)* with their value of A to find k . Need to see line shown in mark scheme.

A1cso: There needs to be a step between $-15k = \ln\left(\frac{"11"}{20}\right)$ and the printed answer. The printed answer needs to be stated. No errors should be seen reaching it. Use of decimals giving 0.03985 **as part of the proof** will result in A0

N.B. This proof must be seen in part (b) to be credited with marks in part (b).

(c) **M1**: Uses 50 with their A and makes their e^{-kt} subject

A1: correct numerical fraction (any correct form- if given as decimal accept awrt 0.42)[ignore LHS]

M1: Uses logs correctly then rearranges correctly to obtain $t = \frac{\ln\left(\frac{"25"}{60}\right)}{-k}$ (Allow 50 – their A instead of 25 in numerator)

A1: awrt 22 minutes. Accept 11.22 i.e. 24 hour clock or $t = 22$ or $t = 22$ minutes but not $t = 22$ degrees C.

Special case: A common error is to reach $0.96t = \frac{25}{60}$; this is a result of log errors- so allow M1A1M0A0

Another common error is to miscopy 15 as 5 (usually part way through the answer). Answer is usually 7.3 and this achieves M1A1M1A0

Question Number	Scheme	Marks
5.	(a) $R\cos\alpha = 3, \quad R\sin\alpha = 3$	
	$R = 3\sqrt{2}$ or $\sqrt{18}$ or awrt 4.24	B1
	$\tan\alpha = 1, \Rightarrow \alpha = \frac{\pi}{4}$ or 0.785	M1, A1 (3)
	(b) $\left(\frac{dx}{dy}\right) = 3\cos y - 3\sin y$ or $3\sqrt{2}\cos(y+\frac{\pi}{4})$	M1 A1
	Puts $3\sqrt{2}\cos(y+\alpha) = 2$ | or puts $-3\sqrt{2}\sin(y-\alpha) = 2$	B1
	So $\cos(y+\alpha) = \frac{\sqrt{2}}{3}$ and $y = "1.0799" - \alpha$ | or $y = "-0.491" + \alpha$	M1
	y= 0.295 and x = 3.742 (or 3.743)	A1 A1 (6) **[9]**

Notes

(a) **B1**: $(R = \sqrt{(3^2 + 3^2)}) = 3\sqrt{2}$ (accept $\pm 3\sqrt{2}$ but not just $-3\sqrt{2}$) No working need be seen. Accept decimal answers which round to 4.24.

M1: For $\tan\alpha = \pm\frac{3}{3}$ If R is used then accept $\sin\alpha = \pm\frac{3}{R}$ or $\cos\alpha = \pm\frac{3}{R}$

A1: Accept awrt 0.785 BUT 45 degrees is A0

(b) **M1**: Attempts differentiation (may be sign errors)

A1: correct in either form shown on scheme – answer is A0 if clearly in degrees.

B1: Obtains equation given in scheme, or $\frac{1}{3\sqrt{2}\cos(y+\alpha)} = \frac{1}{2}$, or equivalent. 3 cos$y$ – 3 sin y = 2 (without further work) is B0 but may be written as $-3\sqrt{2}\sin(y-\alpha) = 2$ which would be B1. **It may also be solved by "*t*" formulae (see below)**

M1: Allow **in degrees or radians** for $\arccos\left(\frac{\pm 2}{R}\right) \pm \alpha$ or for $\arcsin\left(\frac{\pm 2}{R}\right) \pm \alpha$ or for $\arccos\left(\frac{\pm\frac{1}{2}}{R}\right) \pm \alpha$ or for $\arcsin\left(\frac{\pm\frac{1}{2}}{R}\right) \pm \alpha$

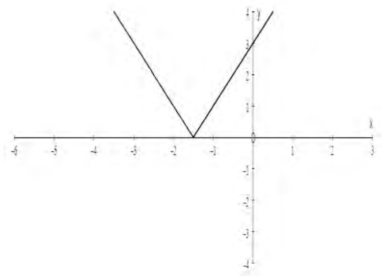
A1: one correct answer – allow 3.742 or 3.743 following incorrect y value **A1**: two correct answers (Accept awrt in both cases)

Do not accept mixed units- unless recovery yields a correct final answer.

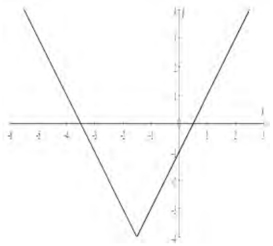
Special case: Candidate works solely in **degrees:** In part (a) max mark is B1M1A0 In part (b) they can have M1A1 for $\frac{dx}{dy} = 3\cos y - 3\sin y$ or M1A0 for $3\sqrt{2}\cos(y+45)$) then B1 is possible and M1 if solution is completed in degrees. The value for y in degrees is not appropriate but correct work in degrees may lead to correct value for x, so A1, A0 could be earned.

Ignore extra answer outside range. (PTO for little *t* formula method in part (b))

Question Number	Scheme	Marks
5. Alternative for last four marks in (b)	Contd "little t formula method"	
	(b) $\frac{\mathrm{d}x}{\mathrm{d}y} = 3\cos y - 3\sin y$ (as before)	M1A1
	$3\cos y - 3\sin y = 2$ so $3\frac{1-t^2}{1+t^2} - 3\frac{2t}{1+t^2} = 2$	B1
	Attempt to solve $5t^2 + 6t - 1 = 0$ and use $y = 2 \times \arctan "0.148"$	M1
	y= 0.295 and x = 3.742 (or 3.743)	A1 A1 (6) **[9]**

<table>
<tr><th>Question Number</th><th colspan="2">Scheme</th><th>Marks</th></tr>
<tr><td rowspan="3">6.</td><td>(a) (i)
</td><td>V shape in correct position i.e. touches – ve x – axis as shown

(-a/2,0) and (0, a)</td><td>B1

B1</td></tr>
<tr><td>(ii)
</td><td>Translation down of previous V shape ft or correct position if starts again

((b –a) /2, 0) and (-(a + b)/2, 0)
Completely correct graph with y intercept at (0, a - b)</td><td>B1 ft

B1, B1

B1
(6)</td></tr>
<tr><td colspan="2">(b) $(2x+a)-b=\frac{1}{3}x\rightarrow "\frac{5}{3}x=b-a"$
So $x=\frac{3}{5}(b-a)$
And $-(2x+a)-b=\frac{1}{3}x\rightarrow -2x-\frac{1}{3}x=a+b$
So $x=-\frac{3}{7}(a+b)$</td><td>M1
A1
M1
A1

(4)
[10]</td></tr>
</table>

Notes

(a) (i) **B1**: V shape correct orientation and position. Could be a tick shape (i.e. not whole of V)

B1: (-a/2, 0) **and** (0, a) accept –a/2 and a marked on the correct axes or even (0, -a/2) on x axis and (a, 0) on y axis

There must be a graph for these marks to be awarded in part (a).

(ii) **B1ft:** Translation down of previous V shaped graph by any amount (may be in wrong position) **or** correct V in correct position if candidate starts again and does not relate this to their graph in part (a)

B1: one x coordinate correct **B1**: both correct (may be shown on x axis as ((b –a) /2) and (-(a + b)/2) or even (0, (b –a) /2) and (0, -(a + b)/2)). (May be shown on wrong parts of x – axis, or interchanged) Marks may be given for **correct** coordinates **i.e.** ((b –a) /2, 0) and (-(a + b)/2, 0) **without graph**.

B1: The graph must be completely correct. Intercept m**ust be on negative *y* axis and there should be two *x*-intercepts, one positive and one negative. The** y coordinate must be correct (may be shown on y axis as a – b or even (a – b, 0))

(b) **M1**:Attempts first +ve solution correctly using (2x + a) and obtains equation with multiple of x only on LHS

A1: any equivalent to $x=\frac{3}{5}(b-a)$ e.g. $x=\frac{3}{5}b-\frac{3}{5}a$

M1: Attempts second –ve solution **correctly** using - (2x + a) and obtains equation with multiple of x on LHS

A1: any equivalent to $x=-\frac{3}{7}(a+b)$ e.g. $x=-\frac{3}{7}a-\frac{3}{7}b$

Question Number	Scheme	Marks
7 (i)(a)	$\cos 3\theta = \cos 2\theta \cos\theta - \sin 2\theta \sin\theta$	M1
	$= (2\cos^2\theta - 1)\cos\theta - 2\sin\theta\cos\theta\sin\theta$	M1
	$= 2\cos^3\theta - \cos\theta - 2(1-\cos^2\theta)\cos\theta$	dM1
	$= 4\cos^3\theta - 3\cos\theta$ *	A1 * (4)
(b)	$8\cos^3\theta - 6\cos\theta + (2\cos^2\theta - 1) + 1 = 0$	M1
	$8\cos^3\theta + 2\cos^2\theta - 6\cos\theta = 0$	A1
	$2\cos\theta(4\cos\theta - 3)(\cos\theta + 1) = 0$ so $\cos\theta =$	dM1
	$\cos\theta = \frac{3}{4}$ (or 0 or -1)	A1
	$\theta = 0.723$ and no extra answers in range , or $\theta = \frac{\pi}{2}$ and π (or 90° and 180°)	A1, B1 (6)
(ii)	($\sin\theta = x$ and so) $\cos\theta = \sqrt{(1-x^2}$ — Or uses right angled triangle with sides 1, x and $\sqrt{(1-x^2}$	M1
	$(\cot\theta) = \dfrac{\cos\theta}{\sin\theta}$ — Indicates θ on diagram and implies $(\cot\theta) = \dfrac{adjacent}{opposite}$	M1
	$= \dfrac{\sqrt{(1-x^2)}}{x}$ *	A1* (3)
		[13]

Notes

(i) (a) **M1**: **Correct** statement for $\cos 3\theta$ as shown using compound angle formula

M1: Uses **correct** double angle formulae for $\sin 2\theta$ and $\cos 2\theta$ (any of the three) – allow invisible brackets

dM1: (dependent on both previous Ms). Uses $\sin^2\theta = (1-\cos^2\theta)$ o.e. to replace all sin terms by cos terms

A1: deduces result with no errors- allow recovery from invisible brackets or from occasional missing θ – need all 3 M marks

(b) **M1**: Replaces $\cos 3\theta$ and $\cos 2\theta$ by expression from (a) and by attempt at double angle formula resulting in expression in cosine only – may do this in one or several steps – allow slips

$8\cos^3\theta - 6\cos\theta + (\cos^2\theta - \sin^2\theta) + 1 = 0$ is not yet enough for M mark- but

$8\cos^3\theta - 6\cos\theta + (\cos^2\theta - (1-\cos^2\theta) + 1 = 0$ would get M1 but not yet the A mark

A1: correct cubic shown with 3 terms

dM1: Solves by any valid method (factorising, formula, completion of square or calculator or implied by 3/4) to give at least one non zero value for $\cos\theta =$ **A1**: for ¾

A1: 0.723 or answers which round to this and no extra answers in range. Do not accept degrees.

B1: for $\theta = \dfrac{\pi}{2}$ and π (allow decimals to 3sf 3.14 and 1.57 or degrees)

(ii) **M1:** States $\cos\theta = \sqrt{(1-x^2)}$, **or** see right angled triangle with sides 1, x and $\sqrt{(1-x^2)}$

M1: Implies $(\cot\theta) = \dfrac{\cos\theta}{\sin\theta}$ - not $(\cot\theta) = \dfrac{\cos}{\sin}\theta$ nor $(\cot\theta) = \dfrac{\cos}{\sin}$

or indicates angle on diagram and implies $(\cot\theta) = adjacent \div opposite$

A1: Clear explanation No errors, printed answer achieved. Needs both M marks

Question Number	Scheme		Marks	
8. (a)	Let $y = 3-2e^{-x}$, then $2e^{-x} = 3-y$	*Or* $2e^{-y} = 3-x$	M1	
	$-x = \ln\frac{3-y}{2}$ and $x =$	$-y = \ln\frac{3-x}{2}$ and $y =$	M1	
	$x = -\ln\frac{3-y}{2}$	This mark earned with next for correct answer by this method	A1	
	$\cancel{f^{-1}(x)} = \ln\frac{2}{3-x}$ or $-\ln\frac{3-x}{2}$ o.e.		A1	
	Domain is $x < 3$		B1	(5)
(b)	$\ln\frac{2}{3-x} = \ln x \rightarrow 2 = (3-x)x$		M1	
	$x^2 - 3x + 2 = 0$		A1	
	$x = 2$ or $x = 1$		M1 A1	(4)
(c)	$3-2e^{-t} = ke^{t} \quad \rightarrow ke^{2t} - 3e^{t} + 2 = 0$ or $\rightarrow ke^{2t} - 3e^{t} = -2$ o.e. (isw)		M1 A1	
	Use "$b^2 - 4ac = 0$" or "$b^2 = 4ac$" or attempts $e^{t} = \frac{3 \pm \sqrt{9-8k}}{2k}$		dM1	
	So $k = 1.125$ o.e. e.g. $\frac{9}{8}$ or $1\frac{1}{8}$		A1	(4) **[13]**

Notes

(a) **M1**: Puts $y = f(x)$ and makes e^{-x} term subject of formula so $2e^{-x} = 3-y$ or $e^{-x} = \frac{3-y}{2}$ or even $-2e^{-x} = y-3$ or $-e^{-x} = \frac{y-3}{2}$ - allow sign slips. Allow $f(x)$ instead of y in expression for both Ms

M1: Uses ln to get $x =$ (This mark is for knowing that $\ln x$ is inverse of e^x so allow sign errors and weak log work. These errors will be penalised in the A mark.)

A1: completely correct log work giving a correct unsimplified answer for $x =$ (then isw for this mark)

A1: any correct answer - do not need to see LHS of equation but variable **must** be x not y

NB Possible answers include $\frac{\log\frac{2}{3-x}}{\log e}$, $-\ln(3-x) + \ln 2, -\ln\left(-\frac{1}{2}x + \frac{3}{2}\right)$, or $\ln\frac{-2}{x-3}$ etc

If x and y interchanged at start – see alternative in scheme. Note this method gives A1A1 or A0A0

B1: For $x < 3$ (independent mark); allow $(-\infty, 3)$, but $x \le 3$ is B0

(b) **M1**: Removes ln correctly on both sides and multiplies across

A1: expands bracket to give three term quadratic equation, allow $x^2 - 3x = -2$

M1: Solves quadratic (may be implied by answers)

A1: Need both these correct answers

(c) **M1**: Sets $3-2e^{-t} = ke^{t}$ and attempts to multiply all terms by e^{t} or by e^{-t} (allow use of x instead of t)

A1: three term quadratic – allow x or t so $ke^{2x} - 3e^{x} + 2 = 0$ or $ke^{2x} - 3e^{x} = -2$ or $k = 3e^{-t} - 2e^{-2t}$ etc

dM1: Uses condition for equal roots to give expression in k – may not be simplified- or attempts to solve their quadratic equation in e^{t} using formula or completion of the square **A1**: See scheme

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