

1. (a) Given that $\sin^2\theta + \cos^2\theta \equiv 1$, show that $1 + \tan^2\theta \equiv \sec^2\theta$. (2)

(b) Solve, for $0 \leq \theta < 360^\circ$, the equation

$$2 \tan^2\theta + \sec\theta = 1,$$

giving your answers to 1 decimal place. (6)



2. (a) Differentiate with respect to x

(i) $3 \sin^2x + \sec 2x,$ **(3)**

(ii) $\{x + \ln(2x)\}^3.$ **(3)**

Given that $y = \frac{5x^2 - 10x + 9}{(x-1)^2}, \quad x \neq 1,$

(b) show that $\frac{dy}{dx} = -\frac{8}{(x-1)^3}.$ **(6)**



3. The function f is defined by

$$f : x \rightarrow \frac{5x+1}{x^2+x-2} - \frac{3}{x+2}, \quad x > 1.$$

(a) Show that $f(x) = \frac{2}{x-1}$, $x > 1$. (4)

(b) Find $f^{-1}(x)$. (3)

The function g is defined by

$$g : x \rightarrow x^2 + 5, \quad x \in \mathbb{R}.$$

(c) Solve $fg(x) = \frac{1}{4}$. (3)



4.

$$f(x) = 3e^x - \frac{1}{2}\ln x - 2, \quad x > 0.$$

(a) Differentiate to find $f'(x)$.

(3)

The curve with equation $y = f(x)$ has a turning point at P . The x -coordinate of P is α .

(b) Show that $\alpha = \frac{1}{6}e^{-\alpha}$.

(2)

The iterative formula

$$x_{n+1} = \frac{1}{6}e^{-x_n}, \quad x_0 = 1,$$

is used to find an approximate value for α .

(c) Calculate the values of x_1, x_2, x_3 and x_4 , giving your answers to 4 decimal places.

(2)

(d) By considering the change of sign of $f'(x)$ in a suitable interval, prove that $\alpha = 0.1443$ correct to 4 decimal places.

(2)



5. (a) Using the identity $\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$, prove that

$$\cos 2A \equiv 1 - 2 \sin^2 A. \tag{2}$$

(b) Show that

$$2 \sin 2\theta - 3 \cos 2\theta - 3 \sin \theta + 3 \equiv \sin \theta (4 \cos \theta + 6 \sin \theta - 3). \tag{4}$$

(c) Express $4 \cos \theta + 6 \sin \theta$ in the form $R \sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2} \pi$.

(d) Hence, for $0 \leq \theta < \pi$, solve

$$2 \sin 2\theta = 3(\cos 2\theta + \sin \theta - 1),$$

giving your answers in radians to 3 significant figures, where appropriate.

6.

Figure 1

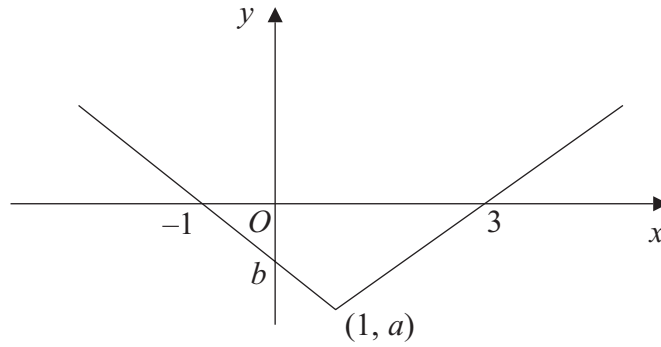


Figure 1 shows part of the graph of $y = f(x)$, $x \in \mathbb{R}$. The graph consists of two line segments that meet at the point $(1, a)$, $a < 0$. One line meets the x -axis at $(3, 0)$. The other line meets the x -axis at $(-1, 0)$ and the y -axis at $(0, b)$, $b < 0$.

In separate diagrams, sketch the graph with equation

(a) $y = f(x + 1)$, (2)

(b) $y = f(|x|)$. (3)

Indicate clearly on each sketch the coordinates of any points of intersection with the axes.

Given that $f(x) = |x - 1| - 2$, find

(c) the value of a and the value of b , (2)

(d) the value of x for which $f(x) = 5x$. (4)



Question 6 continued



7. A particular species of orchid is being studied. The population p at time t years after the study started is assumed to be

$$p = \frac{2800ae^{0.2t}}{1 + ae^{0.2t}}, \text{ where } a \text{ is a constant.}$$

Given that there were 300 orchids when the study started,

(a) show that $a = 0.12$,

(3)

(b) use the equation with $a = 0.12$ to predict the number of years before the population of orchids reaches 1850.

(4)

(c) Show that $p = \frac{336}{0.12 + e^{-0.2t}}$.

(1)

(d) Hence show that the population cannot exceed 2800.

(2)



