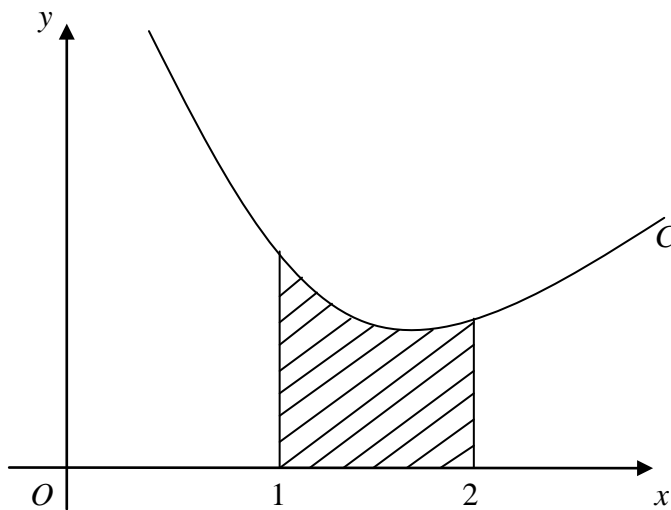


1. The curve C has equation $5x^2 + 2xy - 3y^2 + 3 = 0$. The point P on the curve C has coordinates $(1, 2)$.
- (a) Find the gradient of the curve at P . (5)
- (b) Find the equation of the normal to the curve C at P , in the form $y = ax + b$, where a and b are constants. (3)
-

2. **Figure 1**



In Fig. 1, the curve C has equation $y = f(x)$, where

$$f(x) = x + \frac{2}{x^2}, \quad x > 0.$$

The shaded region is bounded by C , the x -axis and the lines with equations $x = 1$ and $x = 2$. The shaded region is rotated through 2π radians about the x -axis.

Using calculus, calculate the volume of the solid generated. Give your answer in the form $\pi(a + \ln b)$, where a and b are constants. (8)

3.

Figure 2

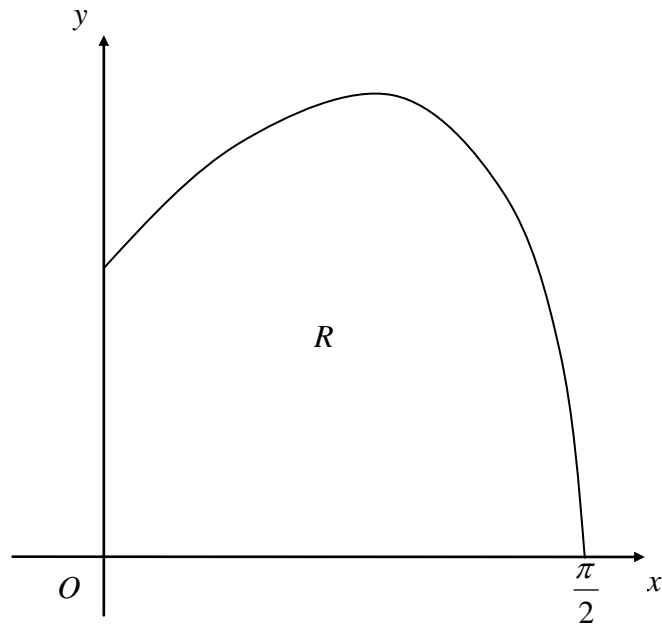


Figure 2 shows part of the curve with equation

$$y = e^x \cos x, \quad 0 \leq x \leq \frac{\pi}{2}.$$

The finite region R is bounded by the curve and the coordinate axes.

(a) Calculate, to 2 decimal places, the y -coordinates of the points on the curve where $x = 0, \frac{\pi}{6}, \frac{\pi}{3}$ and $\frac{\pi}{2}$. (3)

(b) Using the trapezium rule and all the values calculated in part (a), find an approximation for the area of R . (4)

(c) State, with a reason, whether your approximation underestimates or overestimates the area of R . (2)

4. A curve is given parametrically by the equations

$$x = 5 \cos t, \quad y = -2 + 4 \sin t, \quad 0 \leq t < 2\pi.$$

(a) Find the coordinates of all the points at which C intersects the coordinate axes, giving your answers in surd form where appropriate. (4)

(b) Sketch the graph of C . (2)

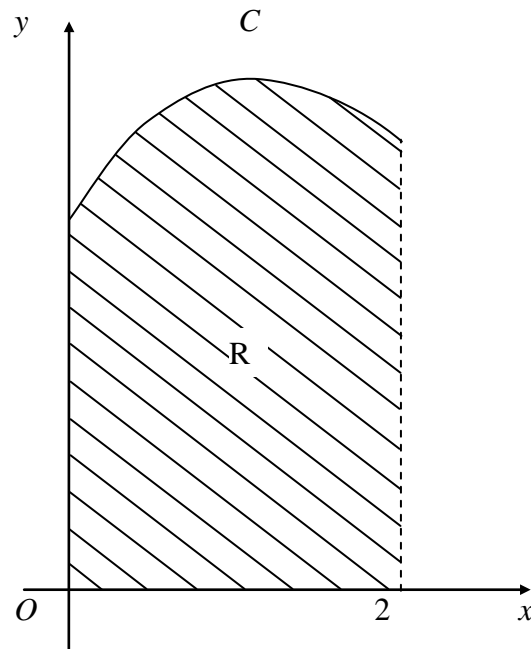
P is the point on C where $t = \frac{1}{6}\pi$.

(c) Show that the normal to C at P has equation

$$8\sqrt{3}y = 10x - 25\sqrt{3}. \quad (4)$$

5.

Figure 1



The curve C has equation $y = f(x)$, $x \in \mathbb{R}$. Figure 1 shows the part of C for which $0 \leq x \leq 2$.

Given that

$$\frac{dy}{dx} = e^x - 2x^2,$$

and that C has a single maximum, at $x = k$,

(a) show that $1.48 < k < 1.49$.

(3)

Given also that the point $(0, 5)$ lies on C ,

(b) find $f(x)$.

(4)

The finite region R is bounded by C , the coordinate axes and the line $x = 2$.

(c) Use integration to find the exact area of R .

(4)

6. When $(1 + ax)^n$ is expanded as a series in ascending powers of x , the coefficients of x and x^2 are -6 and 27 respectively.
- (a) Find the value of a and the value of n . (5)
- (b) Find the coefficient of x^3 . (2)
- (c) State the set of values of x for which the expansion is valid. (1)
-

7. Two submarines are travelling in straight lines through the ocean. Relative to a fixed origin, the vector equations of the two lines, l_1 and l_2 , along which they travel are

$$\mathbf{r} = 3\mathbf{i} + 4\mathbf{j} - 5\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$$

and $\mathbf{r} = 9\mathbf{i} + \mathbf{j} - 2\mathbf{k} + \mu(4\mathbf{i} + \mathbf{j} - \mathbf{k}),$

where λ and μ are scalars.

- (a) Show that the submarines are moving in perpendicular directions. (2)
- (b) Given that l_1 and l_2 intersect at the point A , find the position vector of A . (5)

The point b has position vector $10\mathbf{j} - 11\mathbf{k}$.

- (c) Show that only one of the submarines passes through the point B . (3)
- (d) Given that 1 unit on each coordinate axis represents 100 m, find, in km, the distance AB . (2)
-

8. In a chemical reaction two substances combine to form a third substance. At time t , $t \geq 0$, the concentration of this third substance is x and the reaction is modelled by the differential equation

$$\frac{dx}{dt} = k(1 - 2x)(1 - 4x), \text{ where } k \text{ is a positive constant.}$$

- (a) Solve this differential equation and hence show that

$$\ln \left| \frac{1 - 2x}{1 - 4x} \right| = 2kt + c, \text{ where } c \text{ is an arbitrary constant.} \tag{7}$$

- (b) Given that $x = 0$ when $t = 0$, find an expression for x in terms of k and t . (4)

- (c) Find the limiting value of the concentration x as t becomes very large. (2)

END