

**Mark Scheme 4722
June 2007**

<p>1 (i) $u_2 = 12$ $u_3 = 9.6, u_4 = 7.68$ (or any exact equivalents)</p> <p>(ii) $S_{20} = \frac{15(1-0.8^{20})}{1-0.8}$ $= 74.1$</p> <p style="text-align: center;">OR</p>	<p>B1 B1✓ 2</p> <p>M1 A1 A1 3</p> <p>M1 A2</p> <p style="text-align: center;">5</p>	<p>State $u_2 = 12$ Correct u_3 and u_4 from their u_2</p> <p>Attempt use of $S_n = \frac{a(1-r^n)}{1-r}$, with $n = 20$ or 19 Obtain correct unsimplified expression Obtain 74.1 or better</p> <p>List all 20 terms of GP Obtain 74.1</p>
<p>2 $(x + \frac{2}{x})^4 = x^4 + 4x^3(\frac{2}{x}) + 6x^2(\frac{2}{x})^2 + 4x(\frac{2}{x})^3 + (\frac{2}{x})^4$</p> <p style="text-align: center;">OR</p> <p>$= x^4 + 8x^2 + 24 + \frac{32}{x^2} + \frac{16}{x^4}$ (or equiv)</p> <p style="text-align: center;">OR</p>	<p>M1*</p> <p>M1* A1dep* A1 A1 5</p> <p>M1* M1*</p> <p>A1dep* A1 A1</p> <p style="text-align: center;">5</p>	<p>Attempt expansion, using powers of x and $\frac{2}{x}$ (or the two terms in their bracket), to get at least 4 terms Use binomial coefficients of 1, 4, 6, 4, 1 Obtain two correct, simplified, terms Obtain a further one correct, simplified, term Obtain a fully correct, simplified, expansion</p> <p>Attempt expansion using all four brackets Obtain expansion containing the correct 5 powers only (could be unsimplified powers eg $x^3 \cdot x^{-1}$)</p> <p>Obtain two correct, simplified, terms Obtain a further one correct, simplified, term Obtain a fully correct, simplified, expansion</p>
<p>3 $\log 3^{(2x+1)} = \log 5^{200}$ $(2x+1)\log 3 = 200\log 5$</p> <p style="text-align: center;">OR</p> <p>$2x + 1 = \frac{200\log 5}{\log 3}$ $x = 146$</p> <p>$(2x + 1) = \log_3 5^{200}$ $2x + 1 = 200\log_3 5$</p>	<p>M1 M1 A1 M1 A1 5</p> <p>M1 M1 A1 M1 A1</p> <p style="text-align: center;">5</p>	<p>Introduce logarithms throughout Drop power on at least one side Obtain correct linear equation (now containing no powers) Attempt solution of linear equation Obtain $x = 146$, or better</p> <p>Introduce \log_3 on right-hand side Drop power of 200 Obtain correct equation Attempt solution of linear equation Obtain $x = 146$, or better</p>
<p>4 (i) $\text{area} \approx \frac{1}{2} \times \frac{1}{2} \times \{ \sqrt{5} + 2(\sqrt{7} + \sqrt{9} + \sqrt{11}) + \sqrt{13} \}$</p> <p style="text-align: center;">OR</p> <p>$\approx 0.25 \times 23.766\dots$ ≈ 5.94</p> <p>(ii) This is an underestimate..... ...as the tops of the trapezia are below the curve</p>	<p>M1 M1 A1 A1 4</p> <p>*B1 B1dep*B1 2</p> <p style="text-align: center;">6</p>	<p>Attempt y-values for at least 4 of the $x = 1, 1.5, 2, 2.5, 3$ only Attempt to use correct trapezium rule Obtain $\frac{1}{2} \times \frac{1}{2} \times \{ \sqrt{5} + 2(\sqrt{7} + \sqrt{9} + \sqrt{11}) + \sqrt{13} \}$, or decimal equiv Obtain 5.94 or better (answer only is 0/4)</p> <p>State underestimate Correct statement or sketch</p>

<p>5 (i) $3(1 - \sin^2 \theta) = \sin \theta + 1$ $3 - 3\sin^2 \theta = \sin \theta + 1$ $3\sin^2 \theta + \sin \theta - 2 = 0$</p> <p>(ii) $(3\sin \theta - 2)(\sin \theta + 1) = 0$ $\sin \theta = \frac{2}{3}$ or -1 $\theta = 42^\circ, 138^\circ, 270^\circ$</p>	<p>M1 A1 2 M1 A1 A1 A1 A1 A1√ 5</p>	<p>Use $\cos^2 \theta = 1 - \sin^2 \theta$</p> <p>Show given equation correctly</p> <p>Attempt to solve quadratic equation in $\sin \theta$</p> <p>Both values of $\sin \theta$ correct</p> <p>Correct answer of 270°</p> <p>Correct answer of 42°</p> <p>For correct non-principal value answer, following their first value of θ in the required range (any extra values for θ in required range is max 4/5) (radians is max 4/5)</p> <p>SR: answer only (or no supporting method) is B1 for 42°, B1√ for 138°, B1 for 270°</p> <p style="text-align: center;">7</p>
<p>6 (a) (i) $\int x^3 - 4x = \frac{1}{4}x^4 - 2x^2 + c$</p> <p>(ii) $\left[\frac{1}{4}x^4 - 2x^2\right]_1^6$ $= (324 - 72) - (\frac{1}{4} - 2)$ $= 253\frac{3}{4}$</p> <p>(b) $\int 6x^{-3} dx = -3x^{-2} + c$</p>	<p>M1 A1 B1 3 M1 A1 2 B1 M1 A1 3</p>	<p>Expand and attempt integration</p> <p>Obtain $\frac{1}{4}x^4 - 2x^2$ (A0 if \int or dx still present) + c (mark can be given in (b) if not gained here)</p> <p>Use limits correctly in integration attempt (ie F(6) - F(1))</p> <p>Obtain $253\frac{3}{4}$ (answer only is M0A0)</p> <p>Use of $\frac{1}{x^3} = x^{-3}$</p> <p>Obtain integral of the form kx^{-2}</p> <p>Obtain correct $-3x^{-2} (+c)$ (A0 if \int or dx still present, but only penalise once in question)</p> <p style="text-align: center;">8</p>
<p>7 (a) $S_{70} = \frac{70}{2} \{(2 \times 12) + (70 - 1)d\}$ $35(24 + 69d) = 12915$ $d = 5$</p> <p>OR $\frac{70}{2} \{12 + l\} = 12915$ $l = 357$ $12 + 69d = 357$ $d = 5$</p> <p>(b) $ar = -4$ $\frac{a}{1-r} = 9$ $\frac{-4}{r} = 9 - 9r$ or $a = 9 - (9 \times \frac{-4}{a})$ $9r^2 - 9r - 4 = 0$ $a^2 - 9a - 36 = 0$ $(3r - 4)(3r + 1) = 0$ $(a + 3)(a - 12) = 0$ $r = \frac{4}{3}, r = -\frac{1}{3}$ $a = -3, a = 12$ Hence $r = -\frac{1}{3}$</p>	<p>M1 A1 M1 A1 4 M1 A1 M1 A1 B1 B1 M1 A1 M1 A1 A1 7</p>	<p>Attempt S_{70}</p> <p>Obtain correct unsimplified expression</p> <p>Equate attempt at S_{70} to 12915, and attempt to find d</p> <p>Obtain $d = 5$</p> <p>Attempt to find d by first equating $^n/2 (a + l)$ to 12915</p> <p>Obtain $l = 357$</p> <p>Equate u_{70} to l</p> <p>Obtain $d = 5$</p> <p>Correct statement for second term</p> <p>Correct statement for sum to infinity</p> <p>Attempt to eliminate either a or r</p> <p>Obtain correct equation (no algebraic denominators/brackets)</p> <p>Attempt solution of three term quadratic equation</p> <p>Obtain at least $r = -\frac{1}{3}$ (from correct working only)</p> <p>Obtain $r = -\frac{1}{3}$ only (from correct working only)</p> <p>SR: answer only / T&I is B2 only</p> <p style="text-align: center;">11</p>

<p>8 (i) $\frac{1}{2} \times AB^2 \times 0.9 = 16.2$ $AB^2 = 36 \Rightarrow AB = 6$</p> <p>(ii) $\frac{1}{2} \times 6 \times AC \times \sin 0.9 = 32.4$ $AC = 13.8$ cm</p> <p>(iii) $BC^2 = 6^2 + 13.8^2 - 2 \times 6 \times 13.8 \times \cos 0.9$ Hence $BC = 11.1$ cm $BD = 6 \times 0.9 = 5.4$ cm Hence perimeter = $11.1 + 5.4 + (13.8 - 6)$ = 24.3 cm</p>	<p>M1 A1 2 16.2)</p> <p>M1* M1 dep* A1 3</p> <p>M1 A1√ A1</p> <p>B1 M1 A1 6</p> <p>11</p>	<p>Use $(\frac{1}{2})r^2\theta = 16.2$ Confirm $AB = 6$ cm (or verify $\frac{1}{2} \times 6^2 \times 0.9 = 16.2$)</p> <p>Use $\Delta = \frac{1}{2}bc \sin A$, or equiv Equate attempt at area to 32.4 Obtain $AC = 13.8$ cm, or better</p> <p>Attempt use of correct cosine formula in ΔABC Correct unsimplified equation, from their AC Obtain $BC = 11.1$ cm, or anything that rounds to this</p> <p>State $BD = 5.4$ cm (seen anywhere in question) Attempt perimeter of region BCD Obtain 24.3 cm, or anything that rounds to this</p>
<p>9 (i) (a) $f(-1) = -1 + 6 - 1 - 4 = 0$</p> <p>(b) $x = -1$ $f(x) = (x+1)(x^2 + 5x - 4)$</p> $x = \frac{-5 \pm \sqrt{25+16}}{2}$ $x = \frac{1}{2}(-5 \pm \sqrt{41})$ <p>(ii) (a) $\log_2(x+3)^2 + \log_2 x - \log_2(4x+2) = 1$</p> $\log_2\left(\frac{(x+3)^2 x}{4x+2}\right) = 1$ $\frac{(x+3)^2 x}{4x+2} = 2$ $(x^2 + 6x + 9)x = 8x + 4$ $x^3 + 6x^2 + x - 4 = 0$ <p>(b) $x > 0$, otherwise $\log_2 x$ is undefined $x = \frac{1}{2}(-5 + \sqrt{41})$</p>	<p>B1 1</p> <p>B1 M1 A1 A1 M1</p> <p>A1 6</p> <p>B1 M1 A1</p> <p>B1</p> <p>A1 5</p> <p>B1* B1√dep* 2</p> <p>14</p>	<p>Confirm $f(-1) = 0$, through any method</p> <p>State $x = -1$ at any point Attempt complete division by $(x + 1)$, or equiv Obtain $x^2 + 5x + k$ Obtain completely correct quotient</p> <p>Attempt use of quadratic formula, or equiv, find roots Obtain $\frac{1}{2}(-5 \pm \sqrt{41})$</p> <p>State or imply that $2\log(x+3) = \log(x+3)^2$ Add or subtract two, or more, of their algebraic logs correctly</p> <p>Obtain correct equation (or any equivalent, with single term on each side)</p> <p>Use $\log_2 a = 1 \Rightarrow a = 2$ at any point</p> <p>Confirm given equation correctly</p> <p>State or imply that $\log x$ only defined for $x > 0$ State $x = \frac{1}{2}(-5 + \sqrt{41})$ (or $x = 0.7$) only, following their single positive root in (i)(b)</p>