Paper 1: AS Pure Mathematics Mock paper Spring 2018 Answer ALL questions.

1. A curve has equation $y = 2x^3 - 2x^2 - 2x + 8$.

(a) Find $\frac{dy}{dx}$. **(2)**

(b) Hence find the range of values of x for which y is increasing.
Write your answer in set notation. **(4)**

(Total for Question 1 is 6 marks)

---

2. The quadrilateral $OABC$ has $\overrightarrow{OA} = 4\mathbf{i} + 2\mathbf{j}$, $\overrightarrow{OB} = 6\mathbf{i} - 3\mathbf{j}$ and $\overrightarrow{OC} = 8\mathbf{i} - 20\mathbf{j}$.

(a) Find $\overrightarrow{AB}$. **(2)**

(b) Show that quadrilateral $OABC$ is a trapezium. **(2)**

(Total for Question 2 is 4 marks)

---

3. A tank, which contained water, started to leak from a hole in its base.
The volume of water in the tank 24 minutes after the leak started was 4 m^3.
The volume of water in the tank 60 minutes after the leak started was 2.8 m^3.
The volume of water, V m^3, in the tank t minutes after the leak started, can be described by a linear model between V and t.

(a) Find an equation linking V with t. **(4)**

Use this model to find

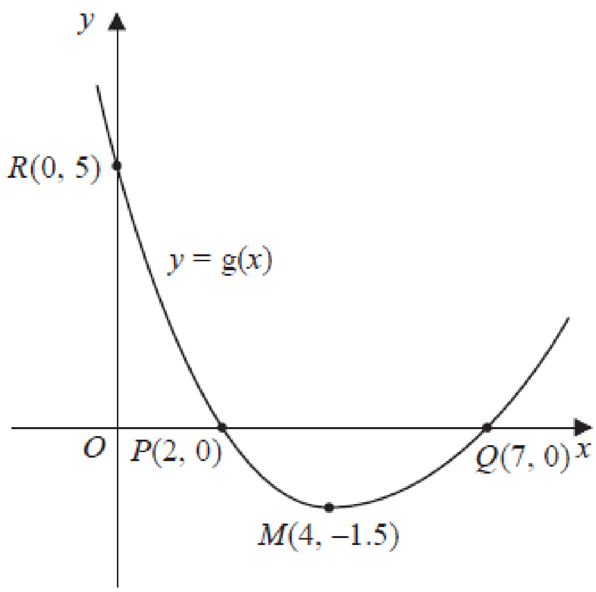
(b) (i) the initial volume of water in the tank,
(ii) the time taken for the tank to empty. **(3)**

(c) Suggest a reason why this linear model may not be suitable. **(1)**

(Total for Question 3 is 8 marks)

4.

The diagram shows a sketch of the curve with equation $y = g(x)$.
The curve has a single turning point, a minimum, at the point $M(4, -1.5)$.
The curve crosses the x-axis at two points, $P(2, 0)$ and $Q(7, 0)$.
The curve crosses the y-axis at a single point $R(0, 5)$.



(a) State the coordinates of the turning point on the curve with equation $y = 2g(x)$. **(1)**

(b) State the largest root of the equation $g(x + 1) = 0$. **(1)**

(c) State the range of values of x for which $g'(x) \le 0$. **(1)**

Given that the equation $g(x) + k = 0$, where k is a constant, has no real roots,

(d) state the range of possible values for k. **(1)**

(Total for Question 4 is 4 marks)

5. $\mathrm{f}(x) = x^3 + 3x^2 - 4x - 12.$

(*a*) Using the factor theorem, explain why f(*x*) is divisible by $(x + 3)$. **(2)**

(*b*) Hence fully factorise f(*x*). **(3)**

(*c*) Show that $\dfrac{x^3+3x^2-4x-12}{x^3+5x^2+6x}$ can be written in the form $A + \dfrac{B}{x}$, where *A* and *B* are integers to be found. **(3)**

(Total for Question 5 is 8 marks)

---

6. (i) Use a counterexample to show that the following statement is false.

"$n^2 - n - 1$ is a prime number, for $3 \leq n \leq 10$." **(2)**

(ii) Prove that the following statement is always true.

"The difference between the cube and the square of an odd number is even."

For example, $5^3 - 5^2 = 100$ is even. **(4)**

(Total for Question 6 is 6 marks)

---

7. (*a*) Expand $\left(1+\dfrac{3}{x}\right)^2$, simplifying each term. **(2)**

(*b*) Use the binomial expansion to find, in ascending powers of *x*, the first four terms in the expansion of $\left(1+\dfrac{3}{4}x\right)^6$, simplifying each term. **(4)**

(*c*) Hence find the coefficient of *x* in the expansion of $\left(1+\dfrac{3}{x}\right)^2\left(1+\dfrac{3}{4}x\right)^6$. **(2)**

(Total for Question 7 is 8 marks)

8.

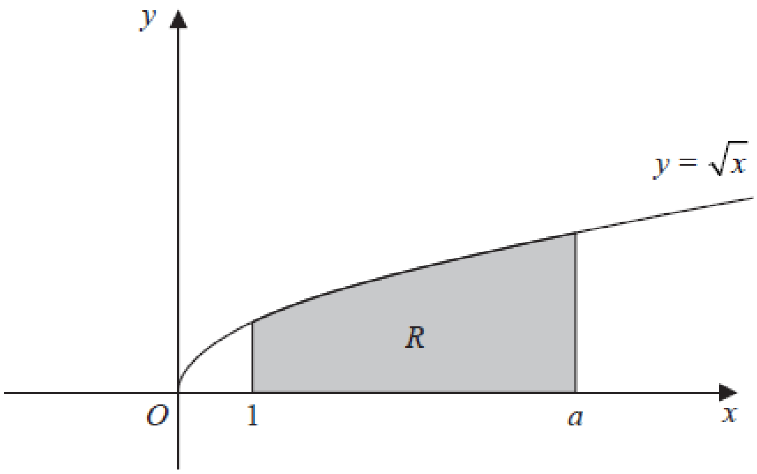


Figure 2 shows a sketch of the curve with equation $y = \sqrt{x}$, $x \geq 0$.

The region *R*, shown shaded in Figure 2, is bounded by the curve, the line with equation $x = 1$, the *x*-axis and the line with equation $x = a$, where *a* is a constant.

Given that the area of *R* is 10,

(*a*) find, in simplest form, the value of

(i) $\displaystyle\int_1^a \sqrt{8x}\ \mathrm{d}x$, (ii) $\displaystyle\int_0^a \sqrt{x}\ \mathrm{d}x$, **(4)**

(*b*) show that $a = 2^k$, where *k* is a rational constant to be found. **(4)**

(Total for Question 8 is 8 marks)

---

9. Find any real values of *x* such that $2\log_4(2-x) - \log_4(x+5) = 1$.

(Total for Question 9 is 6 marks)

---

10. A circle C has centre (2, 5). Given that the point $P(–2, 3)$ lies on C.

(*a*) find an equation for C. **(3)**

The line l is the tangent to C at the point P. The point $Q(2, k)$ lies on l.

(*b*) Find the value of k. **(5)**

(Total for Question 10 is 8 marks)

---

11. (i) Solve, for $-90° \le \theta < 270°$, the equation, $\sin(2\theta + 10°) = -0.6$, giving your answers to one decimal place. **(5)**

(ii) (*a*) A student's attempt at the question

"Solve, for $-90° < x < 90°$, the equation $7 \tan x = 8 \sin x$"

is set out below.

$$7 \tan x = 8 \sin x$$

$$7 \times \frac{\sin x}{\cos x} = 8 \sin x$$

$$7 \sin x = 8 \sin x \cos x$$

$$7 = 8 \cos x$$

$$\cos x = \frac{7}{8}$$

$$x = 29.0° \text{ (to 3 sf)}$$

Identify two mistakes made by this student, giving a brief explanation of each mistake. **(2)**

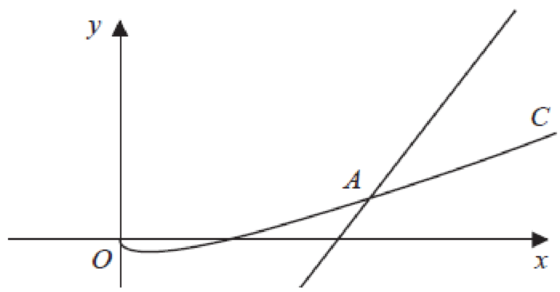
(*b*) Find the smallest positive solution to the equation

$$7 \tan(4\alpha + 199°) = 8 \sin(4\alpha + 199°).$$ **(2)**

(Total for Question 11 is 9 marks)

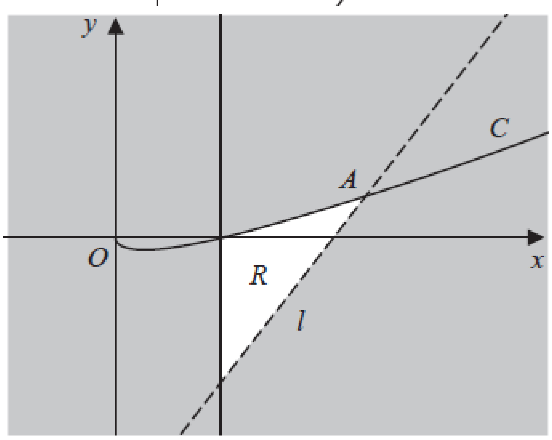
12.

The diagram shows a sketch of the curve C with equation $y = 3x - 2\sqrt{x}$, $x \ge 0$ and the line l with equation $y = 8x - 16$.



The line cuts the curve at point A as shown in Figure 3.

(*a*) Using algebra, find the x-coordinate of point A. **(5)**



The region R is shown unshaded in the diagram.

(b) Identify the inequalities that define R.

(3)

(Total for Question 12 is 8 marks)

---

13. The growth of pond weed on the surface of a pond is being investigated. The surface area of the pond covered by the weed, A m^2, can be modelled by the equation $A = 0.2e^{0.3t}$,where t is the number of days after the start of the investigation.

(a) State the surface area of the pond covered by the weed at the start of the investigation. **(1)**

(b) Find the rate of increase of the surface area of the pond covered by the weed, in m^2/day, exactly 5 days after the start of the investigation. **(2)**

Given that the pond has a surface area of 100 m^2,

(c) find, to the nearest hour, the time taken, according to the model, for the surface of the pond to be fully covered by the weed. **(4)**

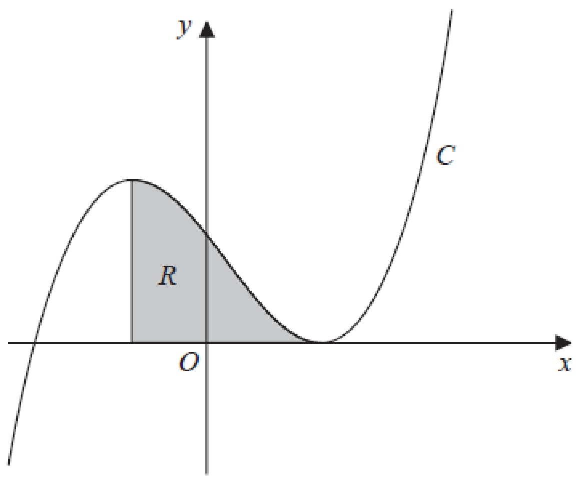
The pond was observed for one month. By the end of the month 90% of the surface area of the pond was covered by the weed.

(d) Evaluate the model in light of this information, giving a reason for your answer. **(1)**

(Total for Question 13 is 8 marks)

---

14. The diagram shows a sketch of the curve C with equation $y = (x - 2)^2 (x + 3)$.



The region R, shown shaded in Figure 5, is bounded by C, the vertical line passing through the maximum turning point of C and the x-axis.

Find the exact area of R.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(Total for Question 14 is 9 marks)

TOTAL FOR PAPER IS 100 MARKS