

### C3 Numerical Answers

#### June 2005

- (b)  $\theta_1 = 131.8^\circ; \theta_2 = 228.2^\circ$
- (a) (i)  $6 \sin x \cos x + 2 \sec 2x \tan 2x$  (ii)  $3(x + \ln 2x)^2(1 + \frac{1}{x})$
- (b)  $\frac{2+x}{x}$  (c)  $x = \pm 2$
- (a)  $3e^x - \frac{1}{2x}$   
(c)  $x_1 = 0.0613\dots, x_2 = 0.1568\dots, x_3 = 0.1425\dots, x_4 = 0.1445\dots$
- (c)  $R = 7.21; \alpha = 0.588$  (d)  $\theta = 0, 2.12$
- (c)  $a = -2, b = -1$  (d)  $x = -\frac{1}{6}$
- (b) 14 years

#### January 2006

- $\frac{x+3}{x+1}$
- $y = -3x + 9$
- (a) (i)  $3x^2e^{3x+2} + 2xe^{3x+2}$  (ii)  $\frac{-18x^3 \sin(2x^3) - 3 \cos(2x^3)}{9x^2}$   
(b)  $\frac{1}{8 \cos\left(\arcsin\left(\frac{x}{4}\right)\right)}$
- (b)  $x_1 = 1.41, x_2 = 1.39, x_3 = 1.39$
- (a)  $R = \sqrt{160}; \alpha \approx 18.43^\circ$  (b)  $x = 38.0^\circ, 285.2^\circ$   
(c) (i)  $-\sqrt{160}$ ; (ii)  $x \approx 161.57^\circ$
- (c)  $\theta = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$
- (c)  $\mathbb{Z}^+$  (d)  $x \approx -0.418$

#### June 2006

- (a)  $\frac{3x+2}{x+1}$  (b)  $\frac{3x-1}{x}$
- (a)  $3e^{3x} + \frac{1}{x}$  (b)  $3x(5+x^2)^{\frac{1}{2}}$
- (a)  $425^\circ\text{C}$  (b)  $t = 7.49$  (c)  $(\pm) 1.64^\circ\text{C}/\text{min}$
- (b)  $x_1 = 0.2670, x_2 = 0.2809, x_3 = 0.2746, x_4 = 0.2774$
- (c)  $135^\circ$
- (b)  $-\infty < f(x) < \infty$  (c)  $\ln\left(\frac{3k}{2}\right)$  (d)  $1\frac{1}{2}$
- (a)  $-\frac{3\sqrt{7}}{8}$

#### January 2007

- (b)  $\frac{9\sqrt{3}}{16}$
- (c)  $y = -\sqrt{2x+2} + \frac{\pi}{4}$
- (i)  $(3, \frac{1}{6}), (-3, -\frac{1}{6})$  (ii) 18
- (a)  $y = 2 \sin\left(x + \frac{\pi}{3}\right)$  (b)  $x = \frac{\pi}{2}, \frac{11\pi}{6}$
- (a)  $\mathbb{R}$  (b)  $f^{-1}(x) < 2$  and  $f^{-1}(x) \in \mathbb{R}$  (c) 1.5,  $\ln 4$   
(d)  $x_1 = -0.3704, x_2 = -0.3452$  (e)  $k = -0.352$
- (b) (1, -11) (c)  $a = 2, b = 4, c = 4$
- (b) (ii)  $\arcsin x = \frac{\pi}{2} - y$  (ii)  $\frac{\pi}{2}$

**June 2007**

- (a)  $x = 2$  (b)  $x = \ln 3, x = 0$
- (b)  $\frac{8}{(2x-1)^2}$
- (a)  $\frac{dy}{dx} = x^2 e^x + 2xe^x$  (b)  $x = 0, y = 0$  and  $x = -2, y = 4e^{-2}$   
(c)  $\frac{d^2y}{dx^2} = x^2 e^x + 2xe^x + 2xe^x + 2e^x$   
(d)  $x = 0$  is a min,  $x = -2$  is a max
- (b)  $x_2 = 0.6455, x_3 = 0.6517, x_4 = 0.6526$
- (a)  $\ln 3$  (b)  $f^{-1}(x) = \frac{1}{2}(e^x + 1), x \in \mathbb{R}$  (d)  $x = 3\frac{2}{3}, x = 2\frac{1}{3}$
- (a)  $\sqrt{13} \sin(x + 0.588)$  (b) 169 (c)  $x = 2.273$  or  $x = 5.976$
- (c)  $\theta = 20.9^\circ, 69.1^\circ, 200.9^\circ, 249.1^\circ$
- (a) 5.353 (c)  $T = 13.06\dots$

**January 2008**

- $a = 2, b = 0, c = -1, d = 1, e = 0$
- (b)  $y = x$
- (b)  $x_1 = 2.50408, x_2 = 2.50498, x_3 = 2.50518$
- (a) (5, 4), (-5, 4) (b) (5, 4), (-5, 4) (c) (4, 8), (-6, -8)
- (a) 1000 (b) 0.000121 (c) 62.5
- (a)  $4 \cos^3 x - 3 \cos x$  (b) (ii)  $x = \frac{\pi}{3}, \frac{5\pi}{3}$
- (a)  $y - 4 = -\frac{1}{6}x$  (b)  $5 \sin(2x + 0.927)$   
(c)  $x = -2.03, -0.46, 1.11, 2.68$
- (a)  $f^{-1}: x \mapsto \left(\frac{1-x}{2}\right)^{\frac{1}{3}}$  (c)  $x = \frac{1}{2}$  (d)  $x = 0, y = -1$

**June 2008**

- (a)  $x = \frac{1}{2}(\ln 2 - 1)$  (b)  $y = 16x + 16 - 8 \ln 2$
- (a)  $R = 13, \alpha = 1.176$  (b)  $x = 0.0849$   
(c) (i)  $R_{\max} = 13$  (ii)  $x = 1.176$
- (c)  $P(-1, 2), Q(0, 1), R(1, 0)$  (d)  $x = -6$
- (b)  $\left(0, \frac{1}{4}\right)$  (c)  $\left(0, \frac{1}{4}\right)$  (d)  $x = \pm\sqrt{5}$
- (b)  $\theta = 11.5^\circ, 168.5^\circ$
- (a) (i)  $e^{3x}(\sin x + 7 \cos x)$  (ii)  $3x^2 \ln(5x + 2) + \frac{5x^3}{5x + 2}$   
(c)  $x = 1, -3$
- (c)  $x_1 = 1.4371, x_2 = 1.4347, x_3 = 1.4355$

**January 2009**

- (a)  $\frac{46}{3}$  (b)  $\frac{2x^2 \cos 2x - 2x \sin 2x}{x^4}$
- (a)  $\frac{1-x}{x-3}$
- $y = \frac{1}{2}x + \frac{\pi}{4}$
- (a)  $g(x) \geq 1$  (c)  $fg(x) \geq 3$  (d)  $x = 0, 6$
- (a) (ii)  $\theta = \frac{\pi}{18}, \frac{5\pi}{18}$
- (a)  $(-1, -3e^{-1} - 1)$   
(b)  $x_1 = 0.2596, x_2 = 0.2571, x_3 = 0.2578$
- (a)  $5 \cos\left(\theta - \frac{4}{3}\right)$  (b) max value = 5 where  $\theta = \alpha = \frac{4}{3}$   
(c)  $5^\circ$  (d)  $t = 15.5$

**June 2009**

- (a)  $x_1 = 2.32$ ,  $x_2 = 2.371581451\dots$ ,  $x_3 = 2.355593575\dots$ ,  
 $x_4 = 2.360436923\dots$
- (b)  $120^\circ$
- (a) 80 (b) 12.6286... (c)  $\frac{dP}{dt} = 16e^{\frac{t}{2}}$  (d) 250
- (i) (a)  $\frac{dy}{dx} = 2x \cos 3x - 3x^2 \sin 3x$   
(b)  $\frac{dy}{dx} = \frac{\left(\frac{2x}{x^2+1}\right)(x^2+1) - 2x \ln(x^2+1)}{(x^2+1)^2}$  (ii)  $2x - 3y + 5 = 0$
- (c)  $f(x) > -k$  (d)  $f^{-1}(x) = \frac{1}{2} \ln(x+k)$  (e)  $x > -k$
- (c)  $5 \cos(2x - 36.87)$  (d)  $x = 51.6^\circ, 165.2^\circ$
- (c)  $x = \ln 4$  or  $x = 0$
- (a)  $\sin 2x = 2 \sin x \cos x$  (b) 0.13, 1.44

**January 2010**

- $\frac{4}{3(x-1)(3x+1)}$
- (b)  $x_2 = 2.345$ ,  $x_3 = 2.037$ ,  $x_4 = 2.059$
- (a)  $\sqrt{34} \cos(x + 0.5404)$  (b)  $x = (0.27, 4.93)$
- (i)  $\frac{2}{(x^2+1)} - \frac{1}{x^2 \ln(x^2+1)}$
- (b)  $2e^{2x} \sec 3x + 3e^{2x} \sec 3x \tan x$   
(c)  $a = -0.196$ ,  $b = 0.812$
- (a)  $x = 22.5, 112.5$
- (i) (a)  $x = \frac{e^5 + 7}{3}$  (b)  $x = \frac{-2 + \ln 15}{7 + \ln 3}$   
(ii) (a)  $\frac{1}{2} \ln(x-3)$  (b)  $(x-1)^2 + 3$ ,  $y > 3$

**June 2010**

- (b)  $26.6^\circ$ ,  $-153.4^\circ$
- $x - 18y + 52 = 0$
- (c)  $x_1 = 1.3038$ ,  $x_2 = 1.2867$ ,  $x_3 = 1.2917$
- (b)  $x = -\frac{10}{3}$  (c)  $fg(2) = 11$  (d)  $-3 \leq g(x) \leq 6$
- (a) (0, 2) (b)  $x = \frac{1}{2}$  (c)  $(4x-5)e^{-x} - (2x^2 - 5x + 2)e^{-x}$   
(d)  $(1, -e^{-1})$ ,  $\left(\frac{7}{2}, 9e^{\frac{7}{2}}\right)$
- (a) (i) (3, 4) (ii) (6, -8) (c)  $f(x) = (x-3)^2 - 4$
- (a) 0.6435 (b) (i) 2.5 (ii) 2.21 (c) 4.41 (d) 14:06, 18:43
- (a)  $\frac{(2x-1)}{(x-3)}$  (b)  $x = \frac{3e-1}{e-2}$

**January 2011**

- (a)  $25 \cos(x + 1.287)$  (b) 25 (c)  $x = 3.84, 6.16$
- (a)  $\frac{4x+1}{2x-1}$  (c)  $f'(x) = \frac{-6}{(2x-1)^2}$ ,  $f'(2) = -\frac{2}{3}$
- $\theta = 54^\circ, 126^\circ, 198^\circ, 342^\circ$
- (a)  $A = 70$  (c) 2.426 °C per minute
- (a)  $A(1, 0)$ ,  $B(8, 0)$  (b)  $f'(x) = -\ln x + \frac{8-x}{x}$   
(e)  $x_1 = 3.529$ ,  $x_2 = 3.538$ ,  $x_3 = 3.534$
- (a)  $f^{-1}(x) = \frac{3+5x}{x+2}$  (b)  $-9 \leq g(x) \leq 4$  (c) -6  
(d) 5 (f)  $-9 \leq x \leq 4$
- (b)  $y = -2x + (\pi + 3)$
- (b)  $\frac{dx}{dy} = 2 \sec 2y \tan 2y$  (c)  $\frac{dy}{dx} = \frac{1}{2x\sqrt{(x^2-1)}}$

**June 2011**

- (a)  $\frac{2x+3}{(x^2+3x+5)}$  (b)  $\frac{-x \sin x - 2 \cos x}{x^3}$
- (b)  $x_1 = 0.80219$ ,  $x_2 = 0.80133$ ,  $x_3 = 0.80167$
- (a)  $R = (0, -6)$  (b)  $R = (-4, 3)$
- (a)  $f^{-1}(x) = e^{4-x} - 2$  (b)  $x \leq 4$  (c)  $fg(x) = 4 - x^2$   
(d)  $fg(x) \leq 4$
- (a)  $p = 7.5$  (c)  $t = 4$
- (b) (ii)  $x = 22.5^\circ$ ,  $112.5^\circ$ ,  $202.5^\circ$ ,  $292.5^\circ$
- (b)  $y + \frac{5}{2} = \frac{4}{15}(x+1)$
- (a)  $\sqrt{13} \cos(3x + 0.983)$  (c)  $x = 0.196$

**January 2012**

- (a)  $2x \ln 3x + x$  (b)  $\frac{4x \cos 4x - 3 \sin 4x}{x^4}$
- (a)  $(-5, 0)$ ,  $(0, -12)$  (b)  $(-3, 0)$ ,  $(2, 4)$
- (a)  $20 \text{ mm}^2$  (b)  $12.28 \text{ p.m.}$
- $\left(y - \frac{\pi}{4}\right) = -8(x - 2\sqrt{3})$
- $6.5^\circ$ ,  $53.5^\circ$ ,  $126.5^\circ$ ,  $173.5^\circ$
- (c)  $x_1 = 1.921$ ,  $x_2 = 1.910$ ,  $x_3 = 1.908$
- (b)  $f^{-1}(x) = \frac{1+x}{2x}$  (c)  $x > 0$  (d)  $x = e^4 = 1$
- (c)  $\frac{5\pi}{12}$ ,  $\frac{11\pi}{12}$

**June 2012**

- $\frac{6}{(3x-2)(3x+1)}$
- (b)  $x_1 = 0.41$ ,  $x_2 = 1.20$ ,  $x_3 = 1.31$
- (a)  $\frac{2\pi}{9}$  (b)  $y = -\frac{1}{3}x$
- (a)  $(-1.5, 0)$  and  $(0, 5)$  (b)  $(0, 5)$  (c)  $(0, 10)$  and  $(-0.5, 0)$
- (a)  $\frac{4}{(2 \sin \theta \cos \theta)^2} - \frac{1}{\sin^2 \theta}$  (c)  $\theta = \frac{\pi}{3}, \frac{2\pi}{3}$
- (a)  $f(x) > 2$  (b)  $fg(x) = x + 2$  (c)  $\ln 2 - \frac{3}{2}$   
(d)  $f^{-1}(x) = \ln(x-2)$ ,  $x > 2$  (e)  $(0, 3)$ ,  $(3, 0)$
- (a) (i)  $x^{-\frac{1}{2}}(\frac{1}{2} \ln 3x + 1)$  (ii)  $\frac{80x}{(2x-1)^6}$  (b)  $\frac{3}{18+2x^2}$
- (a)  $R = 25$ ,  $\alpha = 73.7^\circ$  (b)  $113.1^\circ$ ,  $173.1^\circ$   
(c)  $7 \cos 2x - 24 \sin 2x + 7$  (d)  $32$

**January 2013**

- (a)  $w = \frac{1}{2}$  (b)  $y = 160x - 112$
- (b)  $x_1 = 2.3863$ ,  $x_2 = 2.2847$ ,  $x_3 = 2.3125$
- (a)  $ff(-3) = 2$
- (a)  $a = 0.927$  (b) (i)  $2$  (ii)  $\theta = 4.07$
- (i) (a)  $3x^2 \ln 2x + x^2$  (b)  $3(x + \sin 2x)^2 \times (1 + 2 \cos 2x)$
- (i)  $1 + \frac{\sqrt{2}}{2}$  or  $1 + \frac{1}{\sqrt{2}}$  (ii) (b)  $0^\circ$ ,  $30^\circ$ ,  $150^\circ$ ,  $180^\circ$
- (b)  $h'(x) = \frac{10-2x^2}{(x^2+5)^2}$  (c)  $0 \leq h(x) \leq \frac{\sqrt{5}}{5}$
- (a)  $\text{£}19\,500$  (b)  $t = 4 \ln 2$   
(c) Decrease =  $\text{£}593$  per year

**June 2013**

1.  $a = 3, b = -2, c = 7, d = -8, e = 24$
3. (b)  $\theta = 7.8, 97.8, 187.8, 277.8$
4. (a)  $(0, -16)$  and  $(-1, 25e^{-2} - 16)$   
 (c)  $x_1 = 0.485, x_2 = 0.492, x_3 = 0.489$  (d)  $\alpha = 0.49$
5. (a)  $\frac{dx}{dy} = 6\sec^2 3y \tan 3y$  (c)  $\frac{d^2x}{dy^2} = \frac{2-3x}{12x^2(x-1)^{\frac{3}{2}}}$
6. (a)  $x = \frac{7}{5}$  (b)  $x = \frac{-1+\ln 10}{3+\ln 2}$
7. (a)  $0 \leq f(x) \leq 10$  (b)  $ff(0) = 3$  (c)  $g^{-1}(x) = \frac{5x-4}{3+x}$  (d)  $x = 0.4$
8. (a)  $R = 25, \alpha = 73.74^\circ$  (b)  $\frac{21}{25}$  (c) 7.29 m (d)  $\theta = 133.7, 13.7$

**June 2013 (R)**

1.  $\frac{1}{x+4}$
3. (a)  $R = 5\sqrt{2}, \alpha = 8.1^\circ$  (b)  $x = 53.1^\circ, 323.1^\circ$  (c)  $k = \pm\sqrt{50}$
4. (a)  $f(x) \geq 3$  (b)  $fg(1) = 5$   
 (c)  $g^{-1}(x) = \frac{3-x}{4}$  (d)  $x = 0, x = 0.5$
5. (a)  $\frac{-2\sqrt{x} \sin 2x - \frac{1}{2}x^{-\frac{1}{2}} \cos 2x}{x}$  (b)  $\mu = 6$  (c)  $\frac{dy}{dx} = \frac{3}{\sqrt{4-x^2}}$
6. (i)  $\lambda = \frac{1}{2}$  (ii)  $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \pi$
7. (a)  $x = -0.382, x = -2.618$   
 (b)  $f(x) = e^{x^2}(2x+3) + (x^2+3x+1)e^{x^2} \times 2x$   
 (d)  $x_1 = -2.420, x_2 = -2.427, x_3 = -2.430$
8. (a) 1000 (b) 8000 (c)  $k = 0.386$   
 (d) 6970 (e) 346