P1-P3 Mark Schemes

There aren't separate files for the mark schemes of relevant questions for C1-C4. This file contains all the mark schemes for the P1-P3 papers. They are in chronological order. You also use the bookmarks to navigate through or use the search function.

EDEXCEL FOUNDATION - LONDON EXAMINATIONS

Stewart House 32 Russell Square London WC1B 5DN

January 2001

Advanced Supplementary/Advanced Level

General Certificate of Education

Subject **PURE MATHEMATICS 6671** Paper No. **P1**

Question number	Scheme	Marks
1. (a)	$8+4\sqrt{7}-2\sqrt{7}-7=$ $\quad 1+2\sqrt{7}$	M1 A1 (2)
(b)	$\frac{2+\sqrt{7}}{4+\sqrt{7}}\times\frac{4-\sqrt{7}}{4-\sqrt{7}}=\frac{1+2\sqrt{7}}{16-7}$	M1 A1√
	$c=\frac{1}{9}\quad d=\frac{2}{9}$	A1 (3) ⑤
2. (a)	$(x+k)^2$, $-k^2+c\ (=0)$	M1, A1
	$(x+k)^2=k^2-c\quad x=-k\pm\sqrt{(k^2-c)}$. *	M1 A1 c.s.o. (4)
(b)	(Discriminant = 0, $k^2=81$) $\quad$ <u>$k=9$, or -9</u>	B1, B1 (2) ⑥
3. (a)	~~($\theta+75=60$), 300, 420~~ $\theta=-15°$, $\theta=345°$ One of these…	B1
	$\theta+75=360-$"60"	M1
	<u>$\theta=225,\ 345$</u>	[redacted] A1 (3)
(b)	$(2\theta)=44.4$	B1
	$(2\theta)=\quad 135.6$ One more soln.	B1√
	$(2\theta)=404.4,\quad 495.6$ Other 2 in range	B1√
	<u>$\theta=\ 22.2, 67.8, 202.2, 247.8$</u> $\quad(\div 2)$	M1 A1 (5) ⑧

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Question number	Scheme	Marks
4. (a)	$5+2x-x^2=2$ OR $x^2-2x-3=0$	M1
	$(x-3)(x+1)=0$ $\quad x=-1,\ x=3$ S.C. One correct answer: B1	M1 A1 (3)
(b)	$\int(5+2x-x^2)\,dx = [\,5x+x^2-\frac{1}{3}x^3\,]$	M1 A1
	Using limits: $(15+9-9)-(-5+1+\frac{1}{3})$ $\quad (=18\frac{2}{3})$	M1 A1
	Shaded area $=18\frac{2}{3}-8=10\frac{2}{3}$.	M1 A1 (6) ⑨
	Or: Consider $(5+2x-x^2)-2$	M1
	$\int(5+2x-x^2)-2\ dx = 3x+x^2-\frac{1}{3}x^3$	M1A2,1,0
	$=(9+9-9)-(-3+1+\frac{1}{3})=10\frac{2}{3}$	M1 A1
5. (a)	$r=5.12\div 6.4 \quad = \underline{0.8}$	M1 A1 (2)
(b)	$a=6.4\div 0.64 \quad = \underline{10}$ $\quad$ (√: 3 s.f. or better reqd.)	M1 A1√ (2)
(c)	Sum to $\infty = a\div(1-r)=10\div(1-0.8) \quad = \underline{50}$	M1 A1 (2)
(d)	$S_{25}=10(1-0.8^{25})\div(1-0.8) \quad (=49.8111)$	M1 A1√
	$50-49.8111=\underline{0.189}$ $\quad$ a.w.r.t 0.19	M1 A1 (4) ⑩

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Question number	Scheme	Marks
6. (a)	$AB: \ m = -\frac{4}{3}$, $BC: \ m = \frac{3}{4}$ (s.c. $AB: \frac{4}{3}$, $BC: \frac{3}{4}$ B1)	B1, M1 A1√ (3)
(b)	$BC = \sqrt{(8^2 + (k-4)^2)}$ $(= \sqrt{(k^2 - 8k + 80)})$ $BC^2 = \ldots$ M1	M1 A1 (2)
(c)	$(k^2 - 8k + 80) = 100$ (Their $BC^2 = 100$)	M1
	$k^2 - 8k - 20 = 0$ $(k-10)(k+2) = 0$	M1 A1
	$\underline{k = 10}$, $k = -2$ (rejected)	A1, [redacted] (4)
(d)	$\underline{(11, 6)}$	B1 B1 (2)
		(11)
7. (a)	$100 = 81 + 25 - (2 \times 9 \times 5 \cos BAC)$	M1 A1
	$\cos BAC = \frac{81 + 25 - 100}{90}$ $\left(= \frac{1}{15}\right)$, $BAC = \underline{1.504 \text{ radians}}$. *	A1 (3)
(b)	$\frac{1}{2} r^2 \theta = \frac{1}{2} \times 9 \times 1.504 = \underline{6.768 \text{ cm}^2}$ (6.77)	M1 A1 (2)
(c)	Area of triangle $= \frac{1}{2} \times 45 \times \sin 1.504$ $(= 22.450 \text{ cm}^2)$	M1 A1
	Shaded area $= 22.450 - 6.768 = \underline{15.682 \text{ cm}^2}$ (15.68, 15.7)	A1 (3)
(d)	Arc length $= r\theta = 3 \times 1.504$ $(= 4.512 \text{ cm})$	M1 A1
	Perimeter $= 10 + 6 + 2 + 4.512 = \underline{22.512 \text{ cm}}$ (22.51, 22.5)	M1 A1√ (4)
		(12)

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Question number	Scheme	Marks
8. (a)	$2x^2 h = 1030$, $h = \frac{515}{x^2}$	M1, A1 (2)
(b)	$A = 4x^2 + 6xh$ (Or unsimplified version)	B1
	$A = 4x^2 + \frac{3090}{x}$. *	M1 A1 (3)
(c)	$\frac{dA}{dx} = 8x - 3090x^{-2}$	M1 A1
	$8x - 3090x^{-2} = 0$	M1
	$x^3 = (386.25)$	M1
	$\underline{x = 7.283}$ (7.28, 7.3)	A1 (5)
(d)	$A = 4 \times 7.28^2 + \frac{3090}{7.28}$	M1
	$= 636.4 \text{ cm}^2$ (636, 640)	A1 (2)
(e)	Second derivative $= 8 + 6180x^{-3}$ Attempt to diff.	M1
	Correct deriv, > 0, $\therefore$ Min.	A1 (2)
	(Or equivalent method).	(14)

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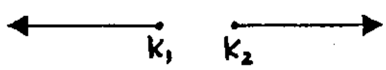
Question number	Scheme	Marks
1. (a)	$k = 3$	B1 (1)
(b)	$(2^2)^x = (2^3)^{2-x}$ (A1 for $2x$ and $3(2-x)$)	M1 A1
	$2x = 3(2-x)$ $5x = 6$ $\underline{x = 1.2}$	M1 A1 (4) ⑤
2. (a)	$\sin 2\theta \div \cos 2\theta = \tan 2\theta$, $\tan 2\theta = 0.5$ *	M1 (1)
(b)	$\tan 2\theta = 0.5$, $2\theta = 26.6°$	B1
	$2\theta = 206.6$, One more soln.	B1ft
	386.6, 566.6 Other 2 solns in range	B1ft
	$\underline{\theta = 13.3,\ 103.3,\ 193.3,\ 283.3}$ (M: dividing by 2)	M1 A1 (5) ⑥
3. (a)	$b^2 - 4ac \geq 0$ $(5k)^2 - 8k \geq 0$, $k(25k - 8) \geq 0$ *	M1, A1 (2)
(b)	Critical values: $k = 0$, $k = \frac{8}{25}$	B1 B1
	$k \leq 0$, $k \geq \frac{8}{25}$  [redacted]	M1 A1ft (4)
	(A1 requires $\leq$ and $\geq$)	
(c)	$k = 0$ $k = \frac{8}{25}$ (Clearly seen as a soln. for (c))	B1 (1) ⑦

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ubject **PURE MATHEMATICS 6671** Paper No. **P1**

Question number	Scheme	Marks
4. (a)	$a+(n-1)d = 500 + 39\times 50 = \underline{£2450}$	M1 A1 (2)
(b)	$\frac{1}{2}n\,(a+l) = 20\,(500+2450) = \underline{£59000}$	M1 A1√ (2)
(c)	Brian: $20\,(1780 + 39d) = $ (b)	M1 A1√
	Solve: $\underline{d = 30}$	M1 A1 (4) ⑧
5. (a)	$f''(x) = 2x - 2x^{-3}$	M1 A1
	$= 8 - \frac{2}{64} = 7\frac{31}{32}$ (7.97)	A1 (3)
(b)	$f(x) = \frac{1}{3}x^3 - 2x - \frac{1}{x}$ (+C)	M1 A1
	$0 = 9 - 6 - \frac{1}{3} + C$ $\quad C = -\frac{8}{3}$ (or -2.67)	M1 A1 (4)
(c)	$f'(x) > 0$ needed, or $f'(x) \geq 0$, or "as x increases, f(x) increases"	B1
	$f'(x) = (x - \frac{1}{x})^2$, >0 always, or ≥ 0 always.	M1, A1 (3) ⑩
	s.c. for last 2 marks in (c): B1. Noting that $f'(1) = 0$. B1. Convincing argument, e.g. f(x) is not increasing because $f'(x)$ is not always positive in its domain.	

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Question number	Scheme	Marks
6. (a)	Area of $X = 2d^2 + \frac{1}{2}\pi d^2$, Area of $Y = \frac{1}{2}(4d^2)\theta$	B1, M1 A1
	Equate and divide by d^2 : $2 + \frac{1}{2}\pi = 2\theta$, $\theta = 1 + \frac{1}{4}\pi$. ✱	M1 A1 (5)
(b)	$12 + 3\pi$	B1 B1 (2)
(c)	$4d + r\theta$, $= 12 + 6\,(1 + \frac{1}{4}\pi)$, $= 18 + \frac{3}{2}\pi$	M1, A1, A1 (3)
(d)	$X: 12 + 3\pi = 21.425$ cm, $Y: 18 + \frac{3}{2}\pi = 22.712$ cm	
	Difference = 13 mm (or 12.9mm) or 12·88 mm	M1 A1 (2) (12)
7. (a)	$y = x(x^2 - 6x + 9) = x(x-3)^2$, ✱ $A\,(3,0)$	B1, B1 (2)
(b)	$\frac{dy}{dx} = 3x^2 - 12x + 9$	M1 A1
	$3(x^2 - 4x + 3) = 0$ $3(x-1)(x-3) = 0$	M1 A1
	At B, $x = 1$ $y = 4$ $(1,4)$	A1 (5)
(c)	$\int (x^3 - 6x^2 + 9x)\,dx = \frac{1}{4}x^4 - 2x^3 + \frac{9}{2}x^2$	M1 A2,1,0
	$\left[\frac{1}{4}x^4 - 2x^3 + \frac{9}{2}x^2\right]_0^3 = \frac{81}{4} - 54 + \frac{81}{2} = 6\frac{3}{4}$	M1 A1 (5) (12)

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Question number	Scheme	Marks
8. (a)	Gradient of $AB = \frac{4}{8} = \frac{1}{2}$	M1 A1 (2)
(b)	Gradient of $C = -2$, $\frac{4-2}{k-7} = -2$ (or full Pythag.method)	M1
	$\underline{k = 6}$	A1 (2)
(c)	$AB = \sqrt{(4^2 + 8^2)}$	M1 A1
	$= \sqrt{80} = \sqrt{16}\sqrt{5} = \underline{4\sqrt{5}}$	A1 (3)
(d)	$BC = \sqrt{(1^2 + 2^2)} = \sqrt{5}$ (or $AC = \sqrt{(7^2 + 6^2)} = \sqrt{85}$)	B1ft
	Area of $ABC = \frac{1}{2}(4\sqrt{5} \times \sqrt{5}) = \underline{10}$	M1 A1 (3)
	Other <u>exact</u> methods can score M1 A2.	
	Non-exact methods score M1 A0 (but may gain the B1).	
(e)	$y - 2 = -2(x - 7)$	[redacted] B1
	$2x + y - 16 = 0$	[redacted] B1 (2)
(f)	When $y = 0$, $x = 8$ $D(8, 0)$	
	When $x = 0$, $y = 16$ $E(0, 16)$ (both)	B1√
	Mid-point $\underline{(4, 8)}$	M1 A1ft (3)
		(15)

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Subject **PURE MATHEMATICS 6672** Paper No. **P2**

Question number	Scheme	Marks
1.	$y = 2e^x + 3x^2 + 2$ $\frac{dy}{dx} = 2e^x + 6x$ Evidence of differentiation M1 correct $\frac{dy}{dx}$ A1 At (0, 4) $\frac{dy}{dx} = 2$ Tangent at (0, 4) $y - 4 = 2x$ (any correct linear form A1)	M1 A1 A1 f.t. M1 A1cso [5]
2.	$f(x) = x + \ln 2x - 4$; $x_{n+1} = 4 - \ln 2x_n$, $x_0 = 2.4$ (a) $x_1 = 2.431$ ... A single sound application of iteration $x_2 = 2.418$ ... $x_3 = 2.423$ ... At least x_3 reached Root = 2.422 (A2) 2.42 or "correct" unrounded to 3 d.p. answer A1	M1 M1 A2,1,0 (4)
	(b) Choosing an appropriate interval e.g. [2.4215, 2.4225] Establishing change of sign + conclusion	M1 A1 (2)
3.	Estimate for $M^2 = \frac{0.25}{2}\left[(48^2 + 29^2) + 2(207^2 + 37^2 + 161^2)\right]$ For squaring V values M1, outside multiplier $\frac{0.25}{2}$ B1, inside bracket M1. Evaluating this estimate to 17900 (AWRT) $M \approx 134$ (133.9), (130) If no attempt made to present final answer for M to nearest integer or 1 d.p. withhold this final mark.	M1 B1 M1 M1 A1 A1 (6)

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Subject **PURE MATHEMATICS 6672** Paper No. **P2**

Question number	Scheme	Marks
4(i)	Choosing values of A and B and attempting to evaluate LHS and RHS of statement	M1
	Showing that LHS $\neq$ RHS + conclusion	A1 (2)
	(ii) Using $\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$	M1
	to obtain $\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$	A1
	Using $\cos^2\theta + \sin^2\theta \equiv 1$	M1
	Using $2\sin\theta\cos\theta \equiv \sin 2\theta$	M1
	leading without any error or fudge to $2\operatorname{cosec}2\theta$	A1cso [5]
5	Realising that $(x^2+3)^2$ required (i.e. y^2 in terms of x)	M1
	$(x^2+3)^2 = x^4 + 6x^2 + 9$	B1
	$\int y^2\,dx \rightarrow \frac{x^5}{5} + \frac{6x^3}{3} + 9x$ (two terms ✓ B1)	B2,1,0
	$\left[\;\right]_1^3$ Using limits top-bottom or bottom-top	M1
	Volume $\pi\int_1^3 y^2\,dx \rightarrow 118.4\pi$ [M1 method complete]	M1 A1
		(7)
	Notes on special cases:	
	π omitted throughout – loses 1st M1 A1 $\rightarrow \frac{5}{7}$	
	trivialised to $\int \pi y\,dx$ or just $\int y\,dx$ – only M1 for limits $\rightarrow \frac{1}{7}$	
	$(x^2+3)^2$ taken as x^4+9 scores M1 B0 B1 M1 M1 A0 $\rightarrow \frac{4}{7}$ or $\frac{3}{7}$ if π omitted	

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Question number	Scheme	Marks
6 (a)	Attempting to get to $a^6 =$ from $800 = \frac{2000a^6}{4+a^6}$	M1
	$a^6 = \frac{3200}{1200}$	A1
	$a = \left(\frac{3200}{1200}\right)^{\frac{1}{6}} \rightarrow 1.1776$ (4 d.p.)	M1 A1cao (4)
(b)	Substituting $P = 1800$ into formula with a^t as unknown	M1
	$a^t = 36 \rightarrow, t = 22$	A1, M1
	Number of years needed for P from 800 to 1800 $= 16$ years	A1 f.t. (4)
(c)	$P = \frac{2000}{1+4a^{-t}}$, $4a^{-t} \rightarrow 0$ as $t \rightarrow \infty$ so $P \rightarrow 2000$ but does not exceed it	B1 (1)
7 (a)	Using $x^2 - 1 \equiv (x-1)(x+1)$ somewhere in solution	M1
	Using a common denominator e.g. $\frac{x-(x-1)}{(x-1)(x+1)}$	M1
	clear, sound, complete proof of $f(x) = \frac{1}{(x-1)(x+1)}$	A1 (3)
(b)	Range of f is y, where $y > 0$	B2
	If $y \geqslant 0$ given allow B1.	(2)
(c)	$gf(x) = g\left(\frac{1}{(x-1)(x+1)}\right) = 2(x-1)(x+1)$	M1 A1
	M1 requires correct order and $g(x) = \frac{2}{x}$ used	
	$2(x-1)(x+1) = 70$	M1
	M1 is independent of previous work	
	$x = 6$ (treat -6 extra as ISW)	A1 (4)

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Question number	Scheme	Marks
8 (a)	EITHER expanding	
	Using coefficients 1, 5, 10, 10, 5, 1 as necessary	M1
	Using powers x^5 $2x^4$ 2^2x^3 etc as necessary	M1
	Completing the method	M1
	$A = 64$	B1
	$B = 160$, $C = 20$	A2,1,0 (6)
	[OR Substituting values for x	
	$x = \quad \rightarrow A = 64$	B1
	Forming a first equation in B and C	M1
	Forming a second equation in B and C	M1
	Solving to complete the method down to either $B =$ or $C =$	M1
	$B = 160$, $C = 20$	A2,1,0]
(b)	Candidate's values of A, B, C used to form	
	$20x^4 + 160x^2 + 64 = 349$	M1
	$4y^2 + 32y - 57 = 0$ [3 term quadratic needed]	A1 f.t.
	Solving for y	M1
	replacing by x^2 and completion to obtain <u>all</u> relevant values of x	M1
	$\pm\sqrt{\frac{3}{2}}$ or AWRT ± 1.22	A1cao (5)

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Question number	Scheme	Marks
9.	(a) $R = \sqrt{29} = 5.39$	B1
	$\tan\alpha = \frac{5}{2} \rightarrow \alpha = 1.19, 0.379\pi, 68.2^\circ$	M1 A1 (3)
	(b) Max $= \sqrt{29}$ (or as in (a))	B1 f.t.
	at $\theta = 1.19$ (or as in (a) above)	B1 f.t. (2)
	(c) $T = 15 + \sqrt{29}\cos\left(\frac{\pi t}{12} - 1.19\right)$	
	Max. $T = 15 + \sqrt{29}$	M1
	$20.4^\circ C$ (accept 20° AWRT)	A1
	occurs when $t = \frac{12 \times 1.19}{\pi}$	M1
	$= 4\frac{5}{6}$ hours (accept AWRT 4.5 or 4.6)	A1 (4)
	(d) $12 = 15 + \sqrt{29}\cos\left(\frac{\pi t}{12} - 1.19\right)$	M1
	$\cos\left(\frac{\pi t}{12} - 1.19\right) = -\frac{3}{\sqrt{29}}$ (using cand.'s values of R & α)	A1 f.t.
	$\frac{\pi t}{12} - 1.19 = 2.16(2)$ or $4.12(2)$	M1 + M1
	$t = 12.8(0)$ or $20.2(9)$ (either)	A1
	i.e. 0100 or 0830 (both)	A1 (6)

H.M.K.
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Subject **PURE MATHEMATICS 6673** Paper No. **P3**

Question number	Scheme	Marks
1. (a)	centre is $(5, -3)$ [M1 if sign errors]	M1 A1 (2)
(b)	radius is 7 [M1 attempts $\sqrt{g^2+f^2-c}$]	M1 A1 (2)
2. (a)	uses $f(1)=9, \Rightarrow a+b=2$ (o.e.)	M1, A1 (2)
(b)	uses $f(-2)=0, \Rightarrow -8a+4b=-28$ (o.e.)	M1, A1 (2)
	$\therefore \underline{a=3}, \underline{b=-1}$ (solves to find both values - M1)	M1 A1 cao. (2)
3. (a)	$x \ln a \equiv kx \ln e$	M1
	$\therefore k \ln e = \ln a \Rightarrow k = \ln a$ ⊛	A1 (2)
(b)	$y = e^{kx} \Rightarrow \frac{dy}{dx} = ke^{kx}$ and $k = \ln 2$	M1
	$\therefore y = 2^x \Rightarrow \frac{dy}{dx} = \ln 2 . 2^x$ ⊛. as $e^{\ln 2} = 2$	A1 (2).
(c)	when $x=2$, $\frac{dy}{dx} = 4\ln 2, = \ln 2^4 = \ln 16$ ⊛	M1, A1 (2).

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Question number	Scheme	Marks
4 (a)	$1 - 3x, + 9x^2; - 27x^3 + \ldots$	B1, B1; B1 (3)
(b)	$(1+x)(1-3x+9x^2-27x^3\ldots)$ uses product.	M1
	$= 1 + (x-3x) + (9x^2-3x^2) + (9x^3-27x^3) \ldots$	
	$= 1 - 2x + 6x^2 - 18x^3$. ⊛	A1 (2)
(c)	$x = \cdot 01$	B1
	$1 - \cdot 02 + \cdot 0006 - \cdot 000018, = \cdot 98058$ substitutes.	M1, A1 cao (3)
5 (a)	$x\tan x, - \int \tan x \, dx$	M1, A1
	$= x\tan x + [\ln\cos x]$ (or equivalent)	[M1 A1]
	$= \frac{\pi}{4} + \ln\cos\frac{\pi}{4}$ use of limits	M1
	$= \frac{\pi}{4} + \ln\frac{1}{\sqrt{2}} = \frac{\pi}{4} - \frac{1}{2}\ln 2$ ⊛	A1. (6)
(b)	$V = \pi\int_0^{\pi/4} x\sec^2 x \, dx$ uses $\pi \times \int$	M1
	$= \frac{\pi^2}{4} - \frac{\pi}{2}\ln 2 = 1.38$	A1 (2)
(c)	$\frac{dy}{dx} = \frac{1}{2}x^{-1/2}\sec x + x^{1/2}\sec x\tan x$ uses product rule.	M1 A1
	$= 2.05$ (or $\frac{\sqrt{2\pi}}{2\pi}(2+\pi)$)	A1 (3)

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Question number	Scheme	Marks
6(a)	$1\times4-2\times1-2\times1=0$ ie: $\begin{pmatrix}1\\-2\\2\end{pmatrix}\cdot\begin{pmatrix}4\\1\\-1\end{pmatrix}=0 \therefore \perp^{\text{r}}$.	M1 A1 (2)
(b)	$3+\lambda = 9+4\mu$ and either $4-2\lambda = 1+\mu$ or $-5+2\lambda = -2-\mu$ Eliminate to obtain, $\mu=-1$ or $\lambda=2$ point is $(5,0,-1)$ (no check needed) vector is $\begin{pmatrix}5\\0\\-1\end{pmatrix}$.	M1 A1 M1, A1 B1 (5)
(c)	$\lambda=-3 \Rightarrow$ point lies on 1st line l_1 show contradiction for $\mu \Rightarrow$ point not on l_2	M1A1 B1. (3)
(d)	$\sqrt{(5^2+10^2+10^2)} = 15, \Rightarrow 1.5\text{km}$	M1, A1 (2)
7. (a)	$\int \frac{dx}{(1-2x)(1-4x)} = \int k\,dt$ $\int \frac{-1}{1-2x} + \frac{2}{1-4x}\,dx, = \int k\,dt$ $\frac{1}{2}\ln(1-2x) - \frac{1}{2}\ln(1-4x), = kt \left(+\frac{c}{2}\right)$ $\ln\frac{1-2x}{1-4x} = 2kt + c$ ⊛	B1 M1A1, M1A1, A1 A1 cso (7)
(b)	use $x=0$ when $t=0 \Rightarrow c=0$ $\therefore \frac{1-2x}{1-4x} = e^{2kt}$ $\therefore x(4e^{2kt}-2) = e^{2kt}-1$, $\therefore x = \frac{e^{2kt}-1}{4e^{2kt}-2}$	B1 M1 M1, A1 oae. (4)
(c)	As $t\to\infty$, $x \to \frac{1}{4}$.	M1A1 (2)

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Advanced Supplementary/Advanced Level

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Question number	Scheme	Marks
8(a)	$y^2 = 81\sin^2 2t$ squares + substitutes	M1
	$= 81 \times 4\sin^2 t\cos^2 t$ use of double angle	M1
	$= 4 \times 9(1-\cos^2 t) \times 9\cos^2 t$ use of $\sin^2 t = 1-\cos^2 t$	M1
	$= \underline{4(9-x^2)x^2}$	A1 (4)
(b)	$\int y\,dx = -\int_{\pi/2}^{0} 9\sin 2t \cdot 3\sin t\,dt$ uses $\int y\,dx$ for M1.	M1, A1
	$= \int_0^{\pi/2} 27\sin 2t\sin t\,dt$ (ie: $A = 27$)	B1 (3)
(c)	$27\int_0^{\pi/2} \sin 2t\sin t\,dt = 27\int 2\sin^2 t\cos t\,dt$	M1
	$= 27\left[\frac{2}{3}\sin^3 t\right], = 18$	M1A1, A1
	OR $-\frac{27}{2}\int(\cos 3t - \cos t)\,dt =$	M1
	$= -\frac{27}{2}\left[\frac{1}{3}\sin 3t - \sin t\right], = 18.$	M1A1, A1 (4)
(d)	Rectangular area $= 18 \times 6 = 108$	M1A1
	Red area = Rectangular $- 4 \times$ Blue $= 108 - 72 = 36$	M1 A1 (4)

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Advanced Supplementary/Advanced Level

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Subject **PURE MATHEMATICS 6671** Paper No. **P1**

Question number	Scheme	Marks
1.	(a) $x = -\frac{1}{2}$	B1
	$4 = 2^2$ and $\sqrt{2} = 2^{\frac{1}{2}}$, $y = 2\frac{1}{2}$	M1, A1 (3)
	(b) $y - x = 3$ $2^3 = 8$ (Or: $4\sqrt{2} \div \frac{1}{\sqrt{2}} = 8$)	M1 A1 (2) (5)
2.	(a) $64 - 16 - 28 + c = 0$ $c = -20$	M1 A1 (2)
	(b) $(x-4)(x^2 + 3x + 5)$ (B1 for $(x-4)$)	B1 M1 A1 (3)
	(c) For $x^2 + 3x + 5$, $b^2 - 4ac = -11 < 0$, $\therefore$ No real roots.	M1 A1ft (2) (7)
3.	$2\sin^2\theta - 2\sin\theta = 1 - \sin^2\theta$	M1
	$3\sin^2\theta - 2\sin\theta - 1 = 0$	A1
	$(3\sin\theta + 1)(\sin\theta - 1) = 0$ (or attempt by formula)	M1 A1ft
	$\sin\theta = -\frac{1}{3}$ $\sin\theta = 1$	A1 (5)
	$\theta = -19.5°, -160.5°, 90°$	A1 1ft A1 (3)
	Final 3 marks: subtract 1 for each extra soln in range. Ignore extra solutions outside range.	
	Special case, if the 2nd M mark has not been earned: Noting that $\sin\theta = 1$ (B1) so $\theta = 90°$ (B1)	(8)

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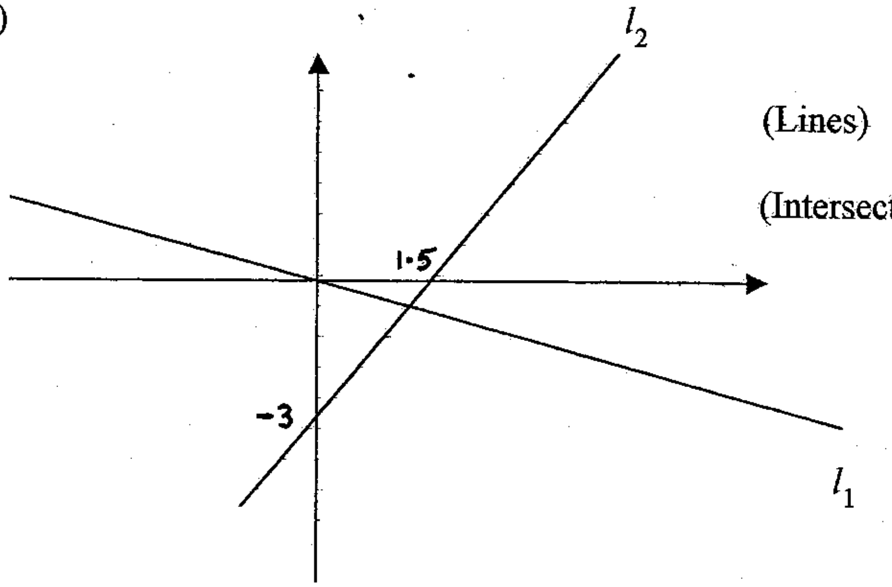
Question number	Scheme	Marks
4.	(a)  (Lines)	B1 B1
	(Intersections)	B1 (3)
	(b) $-\frac{1}{4}x = 2x - 3$ $\quad \frac{9}{4}x = 3$ $\quad x = \frac{4}{3}$ $\quad y = -\frac{1}{3}$	M1 A1 A1 (3)
	(c) Perp. to l_1 : $\quad m = 4$	B1
	$y + \frac{1}{3} = 4\left(x - \frac{4}{3}\right)$	M1
	$12x - 3y - 17 = 0$	A1 (3) (9)
	)	

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Question number	Scheme	Marks
5.	(a) $\frac{dy}{dx} = 3x^2 - 10x + 5$	M1 A1 (2)
	(b) $3x^2 - 10x + 5 = 2$ $\quad 3x^2 - 10x + 3 = 0$	
	$(3x-1)(x-3) = 0 \quad x = \frac{1}{3}$	M1 A1 (2)
	(c) When $x = 3$, $\quad y = 27 - 45 + 15 + 2 = -1$	B1
	$y + 1 = 2(x-3) \quad y = 2x - 7$	M1 A1 (3)
	(d) $R: x = 0 \quad y = -7 \quad S: y = 0 \quad x = 3.5$ (Both for M1)	M1 A1ft
	$RS = \sqrt{(7^2 + (\frac{7}{2})^2)} = \frac{7}{2}\sqrt{5}$ (or equivalent)	M1 A1 (4) (11)
6.	(a) $\frac{1}{2}R^2\theta = \frac{49}{2}\theta$ or $\frac{1}{2}r^2\theta = \frac{25}{2}\theta$	B1
	$\frac{1}{2}R^2\theta - \frac{1}{2}r^2\theta = \frac{49}{2}\theta - \frac{25}{2}\theta = 12\theta$	M1 A1 (3)
	(b) $12\theta = 15 \quad \theta = 1.25$ (*)	M1 A1 (2)
	(c) $R\theta = 7 \times 1.25$ (or $r\theta = 5 \times 1.25$)	B1
	$R\theta + r\theta + 4 = 8.75 + 6.25 + 4 = 19$ m	M1 A1 (3)
	(d) $\sin 0.625 = \frac{x}{5} \quad AD = 2x$ (= 5.851 m)	M1
	$6.25 - 5.85 = 0.399$ 40 cm (M dep. on previous M)	M1 A1 (3) (11)

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Question number	Scheme	Marks
7.	(a) $S = a + ar + ar^2 + \ldots + ar^{n-1}$	B1
	$rS = ar + ar^2 + \ldots + ar^n$	M1
	Subtract: $S(1-r) = a(1-r^n)$ $\quad S = \dfrac{a(1-r^n)}{1-r}$	M1 A1 (4)
	(b) $ar = 3$ $\quad ar^3 = 1.08$	B1 B1
	Divide: $r^2 = 0.36$ $\quad r = 0.6$	M1 A1
	$a = 6 \div 1.2 = 5$	A1 (5)
	(c) $S = \dfrac{5}{1-0.6}$	M1 A1ft
	$= \underline{12.5}$	A1 (3) (12)

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Subject **PURE MATHEMATICS 6671** Paper No. **P1**

Question number	Scheme	Marks
8.	(a) $x+1=6x-x^2-3$	M1
	$x^2-5x+4=0$ $(x-1)(x-4)$ (or use of formula) $x=\ldots$	M1
	$x=1$ $x=4$	A1
	$y=2$ $y=5$	M1 A1 (5)
	(b) $\int(6x-x^2-3)\,dx = 3x^2-\frac{x^3}{3}-3x$	M1 A1
	Limits x_A and x_B: $(48-\frac{64}{3}-12)-(3-\frac{1}{3}-3)$ $(=15)$	M1 A1
	Trapezium : $\frac{1}{2}(2+5)\times 3=10.5$	B1ft
	Area of $R=15-10.5=\underline{4.5}$	M1 A1 (7) (12)
	Alternative for (b)	
	$(6x-x^2-3)-(x+1)=5x-x^2-4$	M1 A1
	$\int(5x-x^2-4)\,dx = \frac{5x^2}{2}-\frac{x^3}{3}-4x$	M1 A1ft
	Limits x_A and x_B: $\left(40-\frac{64}{3}-16\right)-\left(\frac{5}{2}-\frac{1}{3}-4\right), =4.5$	M1 A1, A1

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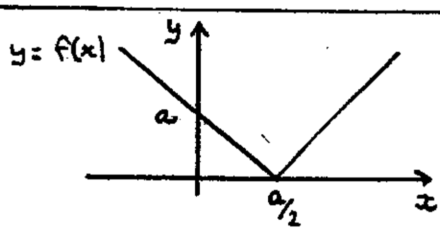
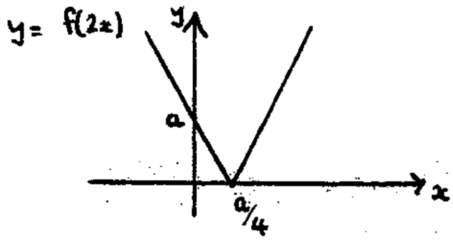
Subject **PURE MATHEMATICS 6672** Paper No. **P2**

Question number	Scheme	Marks
1. (a)	$p = 1.357$; $q = 1.382$ AWRT	B1 ; B1 (2)
(b)	$I \approx \frac{0.5}{2}$, $[1 + 2(1.216 + 1.357 + 1.413) + 1.382]$	B1, [M1 A1ft]
	$= 2.589$ or 2.59 only	A1 (4) ⑥
2. (a)	$\log_q 16 = \log_q 2^4$, $\therefore p = 4\log_q 2$ i.e. $\log_q 2 = \frac{p}{4}$	M1, A1 (2)
(b)	$\log_q(8q) = \log_q 8 + \log_q q$	M1
	$= \cdots + 1$ $\log_q q = 1$	B1
	$= 3\log_q 2 + \cdots$ $\log_q 8$ in terms of $\log_q 16$ or $\log_q 2$	M1
	$\therefore \log_q(8q) = \frac{3}{4}p + 1$	A1 (4) ⑥
3. (a)	 Fairly even V, vertex on the x axis	B1
	ONLY $(a/2, 0)$ and $(0, a)$ on graph or in text [Clearly read off graph paper is OK]	B1 (2)
(b)	 Steeper, even V and 1 correct intersection	B1 [ft $\frac{a}{2}$ from (a)]
	ONLY both $(a/4, 0)$ and $(0, a)$ on graph or in text	B1 (2)
(c)	$-(2x - a) = \frac{1}{2}x$ when $x = 4$, $\Rightarrow a - 8 = 2$ $\therefore a = 10$	M1, A1
	$2x - a = \frac{1}{2}x$ when $x = 4$, $\Rightarrow 8 - a = 2$ $\therefore a = 6$	M1, A1 (4) ⑧

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Subject **PURE MATHEMATICS 6672** Paper No. **P2**

Question number	Scheme	Marks
4.	$[f(x)]^2 = x^2 + \frac{4}{x^4} + \frac{4}{x}$ ⩾ 2 terms correct	M1
	$\int [f(x)]^2 dx = \left[\frac{x^3}{3} - \frac{4}{3x^3}, + 4\ln x \right]$	M1 A1, B1
	$\int_1^2 [f(x)]^2 dx = \left(\frac{8}{3} - \frac{4}{24} + 4\ln 2\right) - \left(\frac{1}{3} - \frac{4}{3} + 4\ln 1\right)$	M1
	$\left(= \frac{7}{2} + 4\ln 2\right)$	
	$V = \pi \int_1^2 [f(x)]^2 dx$, $\Rightarrow V = \pi\left(\frac{7}{2} + \ln 16\right)$ or $a = \underline{7/2}$, $b = \underline{16}$	M1, A1 A1 (8)
S.C.	$\int y\,dx$ can score M1 for integration and M1 for Limits only	i.e. max of 2/8
5. (a)	$u_1 = 1.05 \times 500\,000 - 15000 = 510\,000$ ⩾1 correct iteration	M1
	$u_2 = 520\,500$	
	$u_3 = 531\,525$	A1 (all 3)
	the population is <u>increasing</u>	B1 (3)
(b)	$\begin{pmatrix} u_1 = 425\,000 \\ u_2 = 346\,250 \\ u_3 = 263\,562.5 \\ u_4 = 176\,740.625 \end{pmatrix}$ Attempt upto u_5 and u_6	M1
	$u_5 = 85\,577.64...$ $u_6 = -10\,143.46...$	A1
	$u_5 > 0$, $u_6 < 0$ so population died out during 6^{th} year	B1 (3)
(c)	Require $u_1 = u_0$ i.e. $1.05 \times 500\,000 - d = 500\,000$	M1
	i.e. $d = 0.05 \times 500\,000$	
	i.e. $d = \underline{\underline{25\,000}}$	A1 (2)
		(8)

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Subject **PURE MATHEMATICS 6672** Paper No. **P2**

Question number	Scheme	Marks
6 (a)	LHS $= \dfrac{2\sin^2\theta}{2\sin\theta\cos\theta}$ Attempt $\cos 2\theta$ or $\sin 2\theta$; both correct	M1 A1
	$= \dfrac{\sin\theta}{\cos\theta} = \tan\theta = \text{RHS}$	A1 c.s.o. (3)
(b)	From (a) $\dfrac{1-\cos 2\theta}{\tan\theta} = \dfrac{1}{2}$ **OR** $4\sin^2\theta = \dfrac{\sin\theta}{\cos\theta}$	M1 Steps to show $\sin 2\theta = \alpha$
	$\Rightarrow \sin 2\theta = \frac{1}{2}$; $(\sin\theta)(4\sin\theta\cos\theta - 1) = 0$ $\Rightarrow \sin 2\theta = \frac{1}{2}$	A1 $\alpha = \frac{1}{2}$
	$\Rightarrow 2\theta = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$ one sol^n for 2θ (β); $\pi - \beta$; $\div 2$	B1 (accept degrees) M1 M1
	$\therefore \theta = \dfrac{\pi}{12}, \dfrac{5\pi}{12}$ both	A1 c.a.o. (6) (9)
7 (a)	$f'(x) = 0.5e^x - 2x$ diff^n	M1
	$f'(0) = 0.5$	A1 c.s.o.
	Equation of tangent at A is: $y = f'(0)x + f(0)$, i.e. $y = 0.5x + 0.5$	M1, A1 (4)
(b)	$f'(x) = 0 \Rightarrow 2x = \frac{1}{2}e^x$ $f'(x)=0$	M1 } (dependent)
	i.e. $4x = e^x$	M1
	$\Rightarrow x = \ln(4x)$ (*)	A1 c.s.o. (3)
(c)	$x_1 = \ln 8.6 = 2.1517622$ 3 iterations	M1
	$x_2 = 2.1525814$	
	$x_3 = 2.152962\ldots = \underline{2.1530}$ (4dp) ONLY	A1 c.a.o. (2) (9)

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Question number	Scheme		Marks
8. (a)	$f(x) = \frac{2(2x+1)-6}{(x-1)(2x+1)}$, $= \frac{4x-4}{(x-1)(2x+1)}$	[M for Attempt same denominator]	M1, A1
	i.e. $f(x) = \frac{4(x-1)}{(x-1)(2x+1)}$, $= \frac{4}{(2x+1)}$ ⊛	[M for factorising]	M1, A1 c.s.o. (4)
(b)	$0 < f < 4/3$ or $0 < y < 4/3$	$\alpha < f < \beta$, $\alpha = 0$ or $\beta = 4/3$	B1
		both	B1 (2)
(c)	$y = \frac{4}{2x+1} \Rightarrow y(2x+1) = 4$		M1
			M1
	i.e. $x = \frac{4-y}{2y}$	must be $f^{-1}(x)$	A1 (3)
	$\therefore f^{-1}(x) = \frac{4-x}{2x}$ (o.e.)		
(d)	Range of f^{-1} = domain of f $\therefore$ $f^{-1} > 1$	or $y > 1$ or > 1	B1 (1)
	[B0 for $f > 1$ or $x > 1$ or $f^{-1} \geq 1$]		(10)
9. (a)	$f(x) = \dots + \binom{n}{2}\frac{x^2}{k^2} + \binom{n}{3}\frac{x^3}{k^3} \dots$	Attempt both terms	M1
	$\frac{2 \times n(n-1)}{2k^2} = \frac{n(n-1)(n-2)}{6k^3}$	Correct eqn. No $\binom{n}{r}$ (factorials O.K.)	M1
	$\Rightarrow 6k = n-2$ or $n = 6k+2$ ⊛		A1 c.s.o. (3)
(b)	$\frac{n(n-1)(n-2)(n-3)}{4!\ k^4} = \frac{n(n-1)(n-2)(n-3)(n-4)}{5!\ k^5}$, $\Rightarrow 5k = n-4$ (o.e.)		M1, A1
	Solving: $5k = 6k+2-4$, $\Rightarrow$ $k = 2$ and $n = 14$ ⊛		M1, A1 cso (4)
(c)	$\left(1+\frac{x}{2}\right)^{14} = 1 + 7x + \binom{14}{2}\frac{x^2}{4} + \binom{14}{3}\frac{x^3}{8} + \binom{14}{4}\frac{x^4}{16} + \binom{14}{5}\frac{x^5}{32} \dots$		M1 (≥ 3 correct)
	$= 1 + 7x, + \frac{91}{4}x^2 + \frac{91}{2}x^3, + \frac{1001}{16}x^4 + \frac{1001}{16}x^5 \dots$		B1, A1, A1 (4)
			(11)

GBA 26.1.02

FINAL
MARK
SCHEME

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H.M.K.
17/1/2002

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Question number		Scheme	Marks
1.		Try to use remainder theorem i.e. evaluate $f(-\frac{1}{2})$ or $f(+\frac{1}{2})$	**M1**
		Uses correct substitution to give $4(-\frac{1}{2})^3+3(-\frac{1}{2})^2-2(-\frac{1}{2})-6=-4\frac{3}{4}$	**M1 A1** (3)
		Alternative : Uses long division dividing by (2x+1), obtaining $2x^2+\ldots$	**M1**
		Continues as far as remainder, to get $2x^2+\frac{1}{2}x-\frac{5}{4}$ *rem* $-4\frac{3}{4}$	**M1,** **A1** (3)
2.		$y=\tan x=\dfrac{\sin x}{\cos x}$	**M1**
		$\dfrac{dy}{dx}=\dfrac{\cos x\cos x-\sin x(-\sin x)}{\cos^2 x}$ (use of quotient rule) (or of appropriate product rule)	**M1 A1**
		$=\dfrac{1}{\cos^2 x} = \sec^2 x.$ *	**A1** (4)
3.	**(a)**	$A=2,\ B=16$ (complete method to find A and B)	**M1 A1 A1** (3)
	(b)	$A(1-2x)^{-1}+B(2+x)^{-1}$ and attempt at expansion	**M1**
		$A(1+2x+4x^2+8x^3+\ldots)$	**A1**
		$+\dfrac{B}{2}(1-\dfrac{x}{2}+\dfrac{x^2}{4}-\dfrac{x^3}{8}+\ldots)$	**M1 A1**
		$=10+10x^2+15x^3+\ldots$ (final M mark needs correct powers of 2)	**A1** (5)

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Question number		Scheme	Marks
4.	(a)	$(x-3)^2+(y-4)^2=18$ OR EQUIVALENT $(accept\ (3\sqrt{2})^2)$	**M1 A1** (2)
	(b)	Use $y=x+3$ to obtain $(x-3)^2+(x-1)^2=18$	**M1**
		And thus $2x^2-8x=8$	**A1**
		Solve quadratic, to obtain $x=2\pm\sqrt{8},\ y=5\pm\sqrt{8}$	**M1, A1✓, A1✓** (5)
	(c)	Distance $=\sqrt{((2\sqrt{8})^2+(2\sqrt{8})^2}=8$	**M1A1** cso (2)
5.	(a)	$\frac{dN}{dt}=-kN$ -sign, or k negative needed for A1	**M1 A1** (2)
	(b)	$\int\frac{dN}{N}=\int-kdt$ (✓ on sign error only)	**B1✓**
		$\ln N=-kt+c$ (✓ on sign error only)	**M1 A1✓**
		$N=e^{-kt+c}=Ae^{-kt}$ *	**M1 A1** (5)
	(c)	$3\times10^{17}=7\times10^{18}e^{-8k}$	**M1**
		$e^{-k}=\sqrt[8]{\frac{3}{70}}=.6745$ or $k=\frac{1}{8}\ln\frac{70}{3}$ or equivalent	**M1**
		$k=.3937$	**A1** (3)
	(d)	$N=7\times10^{18}e^{-0.3937\times16}$ or $\frac{3}{70}\times3\times10^{17}$	**M1**
		$=1.286\times10^{16}$	**A1** (2)

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Question number	Scheme	Marks
6. (a)	$\overrightarrow{AB} = 3\mathbf{i} + 6\mathbf{j} + 6\mathbf{k}$	**B1** (1)
(b)	$\cos A = \frac{-12-48+6}{\sqrt{81}\sqrt{81}} = -\frac{2}{3}$ OR $\cos A = \frac{81+81-270}{162} = -\frac{2}{3}$	**M1 A1 A1** (3)
(c)	$\lambda = 4$ at point A and $\lambda = 7$ at point B.	**B1 B1**
	$\mathbf{r} = -9\mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$ represents a line.	**B1** (3)
(d)	$(\lambda\mathbf{i} + 2\lambda\mathbf{j} + (2\lambda - 9)\mathbf{k}) \bullet (\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}) = 0$	**M1**
	$\lambda + 4\lambda + 4\lambda - 18 = 0$. Therefore $\lambda = 2$	**M1 A1** (3)
(e)	The point is (2, 4, -5)	**M1 A1** (2)
7. (i)	$\int_1^3 x^2 \ln x \, dx = \left[\frac{x^3}{3}\ln x\right] - \int \frac{x^3}{3}\frac{1}{x} dx$	**M1 A1**
	$= \left[\frac{x^3}{3}\ln x\right] - \int \frac{x^2}{3} dx$	**DM1**
	$= \left[\frac{x^3}{3}\ln x\right] - \frac{1}{9}x^3$	**A1**
	$= 9\ln 3 - 3 + 1/9 = 9\ln 3 - 2\frac{8}{9}$ or any equivalent	**D DM1 A1** (6)
(ii)	$\int \frac{\cos\theta}{\cos^3\theta} d\theta, = \int \sec^2\theta \, d\theta$	**M1A1, M1**
	$= \tan\theta \; (+c)$	**B1**
	$= \frac{\sin\theta}{\cos\theta} (+c) = \frac{x}{\sqrt{(1-x^2)}} (+c)$ *	**M1 A1** (6)

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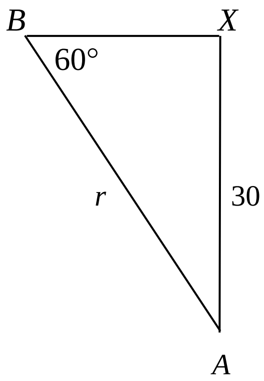
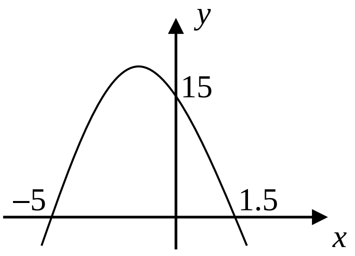
January 2002

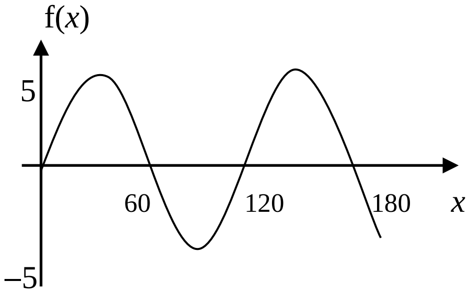
Advanced Supplementary/ Advanced Level
General Certificate of Education

Subject **PURE MATHEMATICS 6673** Paper no. **P3**

Question number	Scheme	Marks	
8. (a)	$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{4\cos\theta}{-5\sin\theta}$	**M1 A1**	
	Equation of tangent is $y - 4\sin\alpha = \frac{4\cos\alpha}{-5\sin\alpha}(x - 5\cos\alpha)$	**M1**	
	$\therefore 5y\sin\alpha + 4x\cos\alpha = 20(\cos^2\alpha + \sin^2\alpha) = 20$ *	**A1**	(4)
(b)	$\int y \frac{dx}{d\theta}\, d\theta = -\int 4\sin\theta\, 5\sin\theta\, d\theta$	**M1**	
	$= 10\int(\cos 2\theta - 1)\, d\theta$	**M1**	
	$= [5\sin 2\theta - 10\theta]$	**M1**	
	Area $= 20\pi$ (appropriate limits used correctly)	**A1 cso**	(4)
(c)	When $x = 0, y = \frac{4}{\sin\alpha}$, OR when $y = 0, x = \frac{5}{\cos\alpha}$.	**B1**	
	Area of parallelogram $= 4 \times \frac{10}{\sin\alpha\cos\alpha} = \frac{80}{\sin 2\alpha}$	**M1A1**	
	$\therefore A = \frac{80}{\sin 2\alpha} - 20\pi$ *	**A1**	(4)
(d)	$\frac{80}{\sin 2\alpha} - 20\pi = 20\pi$	**M1 A1**	
	$\sin 2\alpha = \frac{2}{\pi}$		
	$\alpha = 0.345$	**A1**	(3)

Question Number	Scheme	Marks
1. (*a*)	$1 \times 7 + 2 \times 7 + \ldots \quad a = 7, d = 7, n = 142$ $n = 142$	B1
	$S_n = \frac{1}{2}n(a+b)$ or $\frac{1}{2}n(2a + (n-1)d)$ or $7 \times \frac{n(n+1)}{2}$	M1 (use of correct formula)
	$= \frac{142}{2}(7 + 994)$ or $\frac{142}{2}(14 + 141 \times 7)$ or $7 \times \frac{142 \times 143}{2} = \mathbf{71\,071}$	A1 **(3)**
(*b*)	$\sum_{r=1}^{142}(7r+2) = \sum_{r=1}^{142}7r + \sum_{r=1}^{142}2$ split	M1
	$\sum_{r=1}^{142}2 = 2 \times 142$	
	$\therefore \sum_{r=1}^{142}(7r+2) = 71\,071 + 2 \times 142 = \mathbf{71\,355}$	A1 **(3)**
		(6 marks)
2. (*a*)	 	
	$\sin 60° = \frac{3}{r}$ or $r = 2x, 4x^2 = x^2 + 3^2, x = \sqrt{3}$	M1
	$\boldsymbol{r = \frac{6}{\sqrt{3}}}$ or $\boldsymbol{r = 2\sqrt{3}}$	A1 **(2)**
(*b*)	Area $= \frac{1}{2}r^2\theta^c$ or $\frac{\theta°}{360°} \times \pi r^2 = , \frac{1}{6} \times \pi \times 12 = \mathbf{2\pi}$ (cm^2)	M1, A1 **(2)**
(*c*)	Arc $= r^2\theta^c$ or $\frac{\theta°}{360°} \times 2\pi r = , \frac{1}{6} \times 2\pi \times 2\sqrt{3}$	M1
	Perimeter = Arc + $2r$ = , $\frac{2\sqrt{3}}{3}\pi + 2 \times 2\sqrt{3} = \boldsymbol{\frac{2\sqrt{3}}{3}(\pi + 6)}$ (cm) (✱)	M1, A1 cso **(3)**
		(7 marks)

Question Number	Scheme	Marks
3. (*a*)	$f(x) = 0 \Rightarrow 2x^2 + 7x - 15 = 0$	
	$(2x - 3)(x + 5) = 0$ attempt to solve $f(x) = 0$	M1
	$\therefore$ points are $\mathbf{(\frac{3}{2}, 0), (-5, 0)\ ;\ (0, 15)}$	A1 (both); B1 **(3)**
(*b*)	 shape	B1
	vertex in correct quadrant	B1 ft **(2)**
(*c*)	Symmetry: $x = \frac{1}{2}(-5 + 1.5)$ or Calculus: $-7 - 4x = 0$ or Algebra: $-2[(x + \frac{7}{4})^2 - k]$	M1
	$\Rightarrow \boldsymbol{x = -\frac{7}{4},\ y = 21\frac{1}{8}}$	A1, A1 **(3)**
		(8 marks)
4. (*a*)	$(x + k)^2 - 7 - k^2 = 0$ $(x + k)^2$ (LHS)	M1 A1
	$\Rightarrow (x + k)^2 = 7 + k^2 = 0 \quad \therefore x + k = (\pm)\sqrt{7 + k^2}$	M1 (no need for ±)
	$\therefore \boldsymbol{x = -k \pm \sqrt{7 + k^2}}$	A1 (both) **(4)**
(*b*)	$7 + k^2 > 0$ (or discriminant > 0)	M1
	$\therefore$ roots are real and distinct	A1 **(2)**
(*c*)	$k = \sqrt{2} \quad \Rightarrow \quad x = -\sqrt{2} \pm \sqrt{7 + 2}$	M1
	$\boldsymbol{x = -\sqrt{2} + 3}$ **or** $\boldsymbol{-\sqrt{2} - 3}$	A1 (both) **(2)**
		(8 marks)

Question Number	Scheme	Marks
5. (*a*)	 shape	B1
	60, 120, 180 on x-axis	B1
	5, –5 on y-axis (may be implied by part (*b*))	B1 **(3)**
(*b*)	**(30°, 5); (150°, 5); (90°, –5)** one x-coordinate	B1
	all x-coordinates	B1
	all correct	B1 **(3)**
(*c*)	$f(x) = 2.5 \Rightarrow \sin 3x° = \frac{1}{2}$	
	$3x = 30$ (150, 390, 510) one correct value	B1
	$3x = (\alpha)$, $180 - \alpha$, $360 + \alpha$, $(540 - \alpha)$	M1, M1
	$\boldsymbol{x = 10, 50, 130, 170}$	A1 (ignore extras out of range) **(4)**
		(10 marks)
6. (*a*)	$2x^{\frac{3}{2}} - 3x^{-\frac{3}{2}} = 0$	M1
	$x^3 = \frac{3}{2}$	
	$x = \sqrt[3]{\frac{3}{2}}$ $x = \sqrt[3]{\alpha}$	M1
	$= 1.1447\ldots = \mathbf{1.14}$ (3 sf)	A1 cao **(3)**
(*b*)	$f(x) = 4x^3 + 9x^{-3} - 12 + 5$ $\boldsymbol{A = 4}$	B1
	$= 4x^3 + \dfrac{9}{x^3} - 7$ $\boldsymbol{B = 9, C = -7}$	B1, B1 **(3)**
(*c*)	$\int_1^2 f(x)\ dx = \left[x^4 - \frac{9}{2}x^{-2} - 7x\right]_1^2$ $x^n \rightarrow x^{n+1}$	M1
		A2 ft (candidate's A B, C) (–1 eeoo)
	$= (2^4 - \frac{9}{2} \times 2^{-2} - 14) - (1 - \frac{9}{2} - 7)$ [2] – [1]	M1 (use of limits)
	$= \mathbf{11\frac{3}{8}}$ or **11.375**	A1 **(5)**
		(11 marks)

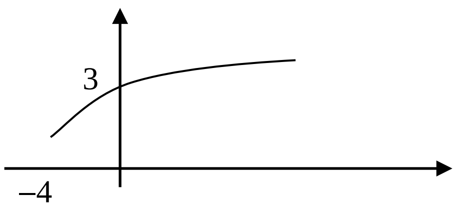
Question Number		Scheme	Marks	
7.	(a)	$l = (50 - 2x)$ $w = (40 - 2x)$	B1	
		$V = x(50 - 2x)(40 - 2x)$ $V = xlw$	M1	
		$V = x(2000 - 80x - 100x + 4x^2)$ = $\mathbf{4x(x^2 - 45x + 500)}$ (*)	A1 cso	(3)
	(b)	$\mathbf{0 < x < 20}$ (accept $\leq$)	B1	(1)
	(c)	$\frac{dV}{dx} = 12x^2 - 360x + 2000$ (accept ÷ 4)	M1, A1	
		$\frac{dV}{dx} = 0 \Rightarrow 3x^2 - 90x + 500 = 0 \Rightarrow x = \frac{90 \pm \sqrt{8100 - 6000}}{6}$	M1 (dV/dx = 0 & attempt to solve)	
		$x = (22.6)$, **required x = 7.36 or 7.4 or 7.362**	A1	(4)
	(d)	$V_{max} = 4 \times 7.36(7.36^2\ldots)$, = **6564 or 6560 or 6600**	M1, A1	(2)
	(e)	e.g. $V'' = 24x - 360 \Big\vert_{x = 7.36}$ $(= -183\ldots) < 0$, $\therefore$ maximum	M1 full method A1 full accuracy	(2)
				(12 marks)
8.	(a)	Mid-point of $AB = [\frac{1}{2}(-3 + 8), \frac{1}{2}(-2 + 4)]$, = $\mathbf{(\frac{5}{2}, 1)}$	M1, A1	(2)
	(b)	$M_{AB} = \frac{4 - (-2)}{8 - (-3)}$, $= \frac{6}{11}$	M1, A1	
		Equation of AB: $y - 4 = \frac{6}{11}(x - 8)$	M1	
		$\Rightarrow 11y - 44 = 6x - 48$, $\Rightarrow$ $\mathbf{6x - 11y - 4 = 0}$ (or equivalent)	A1	(4)
	(c)	Gradient of tangent = $-\frac{11}{6}$	B1 ft	
		Equation: $\mathbf{y - 4 = -\frac{11}{6}(x - 8)}$ (or $6y + 11x - 112 = 0$)	M1 A1	(3)
	(d)	Equation of l: $y = \frac{2}{3}x$	B1	
		Substitute into part (c): $\frac{2}{3}x - 4 = -\frac{11}{6}x + \frac{88}{6}$	M1	
		$\Rightarrow$ $\mathbf{x = 7\frac{7}{15}, y = 4\frac{44}{45}}$	A1, A1	(4)
				(13 marks)

Question Number	Scheme	Marks
1. (*a*)	$1 + n(3x), + \frac{n(n-1)}{2!}(3x)^2 + \frac{n(n-1)(n-2)}{3!}(3x)^3$	B1, B1 **(2)**
(*b*)	$\frac{n(n-1)(n-2)}{6} \times 27 = 10 \times \frac{n(n-1)}{2} \times 9$	M1
	$n = 12$	A1 **(2)**
(*c*)	$\frac{n(n-1)(n-2)(n-3)}{4!}(3x)^4$ coefficient: 40 095	M1 A1 **(2)**
		(6 marks)
2. (*a*)	$\frac{3}{x(x+2)} + \frac{x-4}{(x+2)(x-2)}$	B1 B1
	$= \frac{3(x-2)+x(x-4)}{x(x+2)(x-2)}$	M1 A1
	$= \frac{(x-3)(x+2)}{x(x+2)(x-2)}$	M1 A1 A1 **(7)**
		(7 marks)
3. (*a*)	0, 29.05, 33.46	B1 B1 B1 **(3)**
(*b*)	When $t = 24.5$, $v = 33.76$. Slower at $t = 25$	B1 **(1)**
(*c*)	$s = \frac{1}{2}(5)[2\,(1.56 + 7.23 + 17.36 + 29.05) + 33.46]$	M1 A1 A1ft
	$= 359.65$ (359.7, 360)	A1 **(4)**
		(8 marks)
4. (*a*)	Adding: $\sin(A + B) + \sin(A - B) = 2\sin A\cos B$	M1
	$\left.\begin{matrix} A+B=X \\ A-B=Y \end{matrix}\right\\}\ 2A = X + Y$	M1
	$A = \frac{1}{2}(X + Y),\ B = \frac{1}{2}(X - Y)$	A1
	$\sin X + \sin Y = 2\sin\left(\frac{X+Y}{2}\right)\cos\left(\frac{X-Y}{2}\right)$ (✱)	A1 **(4)**
(*b*)	$\sin 4\theta + \sin 2\theta = 2\sin 3\theta\cos\theta$	M1
	$\sin 3\theta = 0$ (or $\cos\theta = 0$)	M1
	$\theta = 0°, 60°, 120°, 180°, 240°, 300°$ 4 correct:	A1
	90°, 270° 6 correct:	A1
	8 correct:	A1 **(5)**
		(9 marks)

(✱) indicates final line given in the question paper; ft = follow-through mark

Question Number	Scheme	Marks
5. (*a*)	$2 \log x = \log x^2$	B1
	Combine logs, e.g. $\log_2 \left(\frac{y}{x^2}\right) = 3$	M1
	$\frac{y}{x^2} = 2^3, \quad y = 8x^2$ (✱)	A1 **(3)**
(*b*)	$14x - 3 = 8x^2$	M1
	$(4x - 1)(2x - 3) = 0$ Roots $\frac{1}{4}$ and $\frac{3}{2}$	M1 A1 **(3)**
(*c*)	$\log_2 \alpha = \log_2 \frac{1}{4} = \log_2 (2^{-2}) = -2$ (✱)	B1 **(1)**
(*d*)	$\log_2 1.5 = k \qquad 2^k = 1.5$	M1
	$k = \frac{\log 1.5}{\log 2} = 0.585$	M1 A1 **(3)**
		(10 marks)
6. (*a*)	$f(3.1) = 10 + \ln 9.3 - \frac{1}{2} e^{3.1} = 1.131$	
	$f(3.2) = 10 + \ln 9.6 - \frac{1}{2} e^{3.2} = -0.0045$	M1
	Sign change, so $3.1 < k < 3.2$	A1 **(2)**
(*b*)	$f'(x) = \frac{1}{x} - \frac{1}{2} e^x$	M1 A2 (1, 0) **(3)**
(*c*)	$f(1) = 10 + \ln 3 - \frac{1}{2} e$	B1
	$f'(x) = 1 - \frac{1}{2} e$	B1
(i)	$y - (10 + \ln 3 - \frac{1}{2} e) = (1 - \frac{1}{2} e)(x - 1)$	M1
(ii)	$x = 0$: $y = 10 + \ln 3 - \frac{1}{2} e - 1 + \frac{1}{2} e$	M1
	$= 9 + \ln 3$	A1 **(5)**
		(10 marks)

(✱) indicates final line given in the question paper

Question Number	Scheme	Marks
7. (*a*)	A: $y = 16$, B: $y = 2$	B1 **(1)**
(*b*)	$y(x - 3) = 4$, $yx - 3y = 4$	M1
	$x = \dfrac{3y+4}{y}$ (✱)	**(1)**
(*c*)	$x^2 = \left(3 + \dfrac{4}{y}\right)^2 = 9 + \dfrac{24}{y} + \dfrac{16}{y^2}$	M1 A1
	$\int x^2 \, dy = \int (9 + 24y^{-1} + 16y^{-2}) \, dy$	
	$= 9y - \dfrac{16}{y}, + 24 \ln y$	M1 A1ft, A1ft
	$\left[9y + 24 \ln y - 16y^{-1}\right]_2^{16} = (144 + 24 \ln 16 - 1) - (18 + 24 \ln 2 - 8)$	M1
	$V = \pi(133 + 24 \ln 8)$	A1 **(7)**
(*d*)	$V \times 27 \approx 15\,500$ (✱)	M1 A1 **(2)**
		(11 marks)
8. (*a*)	$f(x) \geq -4$	B1 **(1)**
(*b*)	Domain: $x \geq -4$, range: $f^{-1}(x) \geq 1$	B1, B1 **(2)**
(*c*)	 Shape:	B1
	Above x-axis, right way round:	B1
	x-scale: -4	B1
	y-intercept: 3	B1 **(4)**
(*d*)	$gf(x) = \lvert(x^2 - 2x - 3) - 4\rvert$	M1 A1 **(2)**
(*e*)	$x^2 - 2x - 7 = 8$: $x^2 - 2x - 15 = 0$	M1
	$(x - 5)(x + 3) = 0$	
	$x = 5$, $x = -3$ (reject)	A1 A1ft
	$x^2 - 2x - 7 = -8$: $x^2 - 2x + 1 = 0$	M1
	$x = 1$	A1 **(5)**
		(14 marks)

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Question number	Scheme	Marks
1. (a)	*Complete* attempt at remainder theorem, or long division	
	Either $f(3) = 27 + 9a + 3b - 10 = 14$,	**M1**
	Or complete attempt at long division by (x-3) leading to equation.	
	Either $f(-1) = -1 + a - b - 10 = -18$ or long division by (x+1) leading to equation.	
	Equation equivalent to $9a + 3b = -3$ $(3a + b = -1)$	**A1**
	Equation equivalent to $a - b = -7$	**A1**
	Solve two equations to get $a = -2,\ b = 5$	**M1, A1** **(5)**
(b)	Either $f(2) = 8 - 8 + 10 - 10 = 0$, Or complete division with no remainder.	**M1,**
	$\therefore (x-2)$ is a factor.	**A1**
	Or $f(x) = (x-2)(x^2+5)$	**(M1 A1)** **(2)**
2. (a)	$\int x\cos 2x\,dx = \frac{x\sin 2x}{2} - \int \frac{\sin 2x}{2}dx$ **(integration in correct direction)**	**M1 A1**
	$= \frac{x\sin 2x}{2} + \frac{\cos 2x}{4}(+k)$ **(second integration)**	**M1 A1** **(4)**
(b)	$x\frac{2\sin x\cos x}{2} + \frac{1-2\sin^2 x}{4}\ (+k)$ **(use of appropriate double angle formulae)**	**M1**
	$= \frac{1}{2}\sin x(2x\cos x - \sin x) + \frac{1}{4} + k$ **for** $\frac{1}{4} + k$	**A1**
	$= \frac{1}{2}\sin x(2x\cos x - \sin x) + C$ ★	**A1 c.a.o.** **(3)**

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Question number	Scheme	Marks
3. (a)	Either completion of square, $(x-4)^2+(y-8)^2 = 209+16+64$ Or use of formulae, $(-f,-g)$, $r=\sqrt{(f^2+g^2-c)}$ Centre is (4,8), radius 17.	**M1** **A1,A1** **(3)**
(b)	*Either* $2x+2y\frac{dy}{dx}-8-16\frac{dy}{dx}=0$ $\frac{dy}{dx}(2y-16)=8-2x$, so $\frac{dy}{dx}=\frac{4-x}{y-8}$. *Or* gradient of *CP* is $\frac{y-8}{x-4}$. Tangent is perpendicular to *CP* and so has gradient $\frac{4-x}{y-8}$.	**M1, A1** **A1** **(3)** **B1** **M1, A1** **(3)**
(c)	At (21,8) the tangent is vertical, so equation is $x = 21$.	**M1, A1** **(2)**
4. (a)	$x^2+1 \equiv A(1+x)(3-x)+B(3-x)+C(1+x)$ **;** *A* = -1 2=4*B*, $B=\frac{1}{2}$; 10 = 4*C*, $C=\frac{5}{2}$	**B1** **M1 A1; A1** **(4)**
(b)	$\int(-1+\frac{1}{2(1+x)}+\frac{5}{2(3-x)})dx$ $=-x+\frac{1}{2}\ln(1+x)-\frac{5}{2}\ln(3-x)$ $\int_0^2 f(x)=(-2+\frac{1}{2}\ln 3)-(-\frac{5}{2}\ln 3)=$ -2+3ln3	 **M1A1✓A1✓** **M1A1** **(5)**

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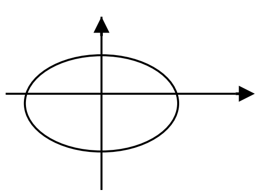
Question number	Scheme	Marks
5. **(a)**	$(1+\frac{5}{15})^{-\frac{1}{2}} = (\frac{4}{3})^{-\frac{1}{2}}$, $=(\frac{3}{4})^{\frac{1}{2}} = \dfrac{\sqrt{3}}{2}; = \sin 60\,°$	**M1,A1;A1** **(3)**
(b)	$1+5x(-\frac{1}{2})+\dfrac{(5x)^2}{2}(-\frac{1}{2})(-\frac{3}{2})+\dfrac{(5x)^3}{6}(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})\ldots$	**M1,**
	$=1-\dfrac{5}{2}x,+\dfrac{75}{8}x^2,-\dfrac{625}{16}x^3\ldots$	**B1, A1, A1** **(4)**
(c)	$=1-\dfrac{5}{2}\left(\dfrac{1}{15}\right),+\dfrac{75}{8}\left(\dfrac{1}{15}\right)^2,-\dfrac{625}{16}\left(\dfrac{1}{15}\right)^3\ldots=0.863(4259..)$	**M1 A1** **(2)**
(d)	sin 60°- (Ans) ≈ 0.0026	**A1** **(1)**
6. **(a)**	$5\cos t = 0 : t = \frac{\pi}{2},\ \ y = \ \ 2\ \ (0,\ \ 2)$	**M1A1**
	$t = \frac{3\pi}{2}, y = -6 \quad (0, -6)$	
	$4\sin t = 2 : t = \frac{\pi}{6},\ \ x = \dfrac{5\sqrt{3}}{2} \quad t = \frac{5\pi}{6},\ x = -\dfrac{5\sqrt{3}}{2} \quad (\pm\dfrac{5\sqrt{3}}{2}, 0)$	**M1A1** **(4)**
(b)	 **shape** **position**	**B1** **B1** **(2)**
(c)	$\dfrac{dx}{dt} = -5\sin t, \quad \dfrac{dy}{dt} = 4\cos t, \qquad \dfrac{dy}{dx} = \dfrac{-4\cos t}{5\sin t}$	**M1A1**
	$y = \dfrac{5\tan\dfrac{\pi}{6}}{4}(x - \dfrac{5\sqrt{3}}{2})$, i.e. $8\sqrt{3}y = 10x - 25\sqrt{3}$ ★	**M1, A1** **(4)**

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Question number	Scheme	Marks
7. **(a)**	$\frac{\mathrm{d}V}{\mathrm{d}t} = -kV$	**B1**
	$\int \frac{1}{V}\,\mathrm{d}V = -k\int \mathrm{d}t,\ \ \ln V = -kt$	**M1**
	$\ln V = -kt + C \qquad V = Ae^{-kt}\ \ *$	**A1**
		(3)
(b)	$t = 0,\ V = 20\,000: \qquad 20000 = A$	**B1**
	$t = 3,\ V = 11000: \qquad 11000 = Ae^{-3k}$	**M1,**
	$e^{-3k} = 0.55$	
	$-3k = \ln 0.55$	
	$k \approx 0.199(3) \quad (\mathit{allow}\ 0.2)$	**A1**
	$t = 10\ V = 20\,000e^{-10k}; \qquad = £2700$	**M1;A1**
		(5)
(c)	$500 = 20000e^{-kt} \qquad e^{-kt} = 0.025$	**M1**
	$-0.199t = \ln 0.025$	
	$t \approx 18.5\ (18.44)\ \ \mathit{accept}\ 18\ \mathit{or}\ 19\ \mathit{yrs}$	**A1√** **A1**
		(3)

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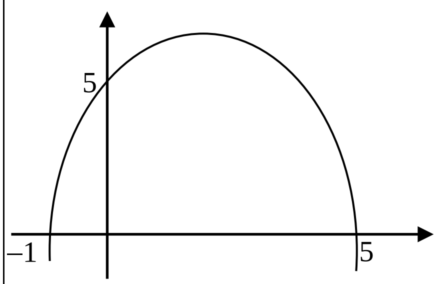
Question number	Scheme	Marks
8. **(a)**	$\mathbf{r} = (9\mathbf{i} - 2\mathbf{j} + \mathbf{k}) + \lambda(-3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k})$ (or any correct alternative)	**M1,A1** **(2)**
(b)	Uses their line equation, or recognises B is mid point of AC or merely writes down p=6, q=11	**M1, A1** **(2)**
(c)	Calculates $\overrightarrow{OC} \bullet \overrightarrow{AB}$	**M1**
	Uses $\cos\alpha = \dfrac{OC \bullet AB}{\lvert OC \rvert \lvert AB \rvert}$ to obtain α.	**M1**
	$\cos\alpha = \dfrac{70}{\sqrt{166}\sqrt{50}}$, $\alpha = 39.8^\circ$ (accept 39.79 or 40)	**A1** **(3)**
(d)	Let OD be $(9\mathbf{i} - 2\mathbf{j} + \mathbf{k}) + t(-3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k})$	**M1**
	Use scalar product OD. AB=0	**M1**
	Obtains equation in t and solves to obtain t = 0.6 (or equivalent)	**M1A1**
	Uses their t, to obtain 7.2 **i** + 0.4 **j** + 4**k**.	**M1, A1** **(6)**

EDEXCEL PURE MATHEMATICS P1 (6671) - NOVEMBER 2002 PROVISIONAL MARK SCHEME

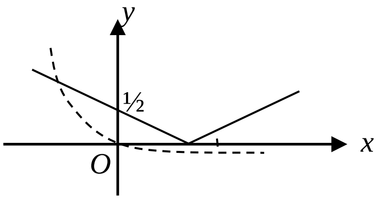
Question Number	Scheme	Marks
1. (*a*)	$3x - x > 13 + 8$ $\quad x > \frac{21}{2}$	M1, A1 **(2)**
(*b*)	$x^2 - 5x - 14 > 0$ $\quad (x-7)(x+2) > 0$ $\quad x = 7, -2$	B1
	$x < -2$ or $x > 7$	M1, A1 ft **(3)**
		(5 marks)
2. (*a*)	$f(-3) = -27 - 27 + 30 + 24 = 0 \Rightarrow (x+3)$ is factor	M1 A1 **(2)**
(*b*)	$(x+3)(x^2 - 6x + 8)$	M1 A1
	$(x+3)(x-2)(x-4)$	M1 A1 **(4)**
		(6 marks)
3. (i)	Divide: $1 + 2x^{-1}$	M1 A1
	Differentiate: $6x^2 + \frac{1}{2}x^{-\frac{1}{2}} - 2x^{-2}$	M1 A2 (1,0) **(5)**
(ii)	$\frac{x^2}{4} + \frac{x^{-1}}{-1}$	M1 A1A1
	$[\ ]^4 - [\ ]_1 = \left(4 - \frac{1}{4}\right) - \left(\frac{1}{4} - 1\right) = 4\frac{1}{2}$	M1 A1 **(5)**
		(10 marks)
4. (*a*)	$S = a + (a + d) + (a + 2d) + \dots + [a + (n-1)d]$	B1
	$S = [a + (n-1)d] + [a + (n-2)d] + \dots + a$	M1
	Add: $2S = n[2a + (n-1)d] \quad \Rightarrow \quad S = \frac{1}{2}n[2a + (n-1)d]$	M1 A1 **(4)**
(*b*)	$a = 54000$ and $n = 9$	B1
	$619200 = \frac{1}{2} \times 9 \times (2 \times 54000 + 8d)$	M1 A1ft
	$d = 3700$	A1 **(4)**
(*c*)	$a + (n-1)d = a + 10d = 54000 + 10d = £91000$	M1 A1 **(2)**
(*d*)	$ar^{n-1} = 54000 \times 1.06^{10}$ (ft their n)	M1 A1ft
	$= £96700$ (or £97000)	A1 **(3)**
		(13 marks)

Question Number	Scheme	Marks
5. (i)	arcsin 0.6 = 36.9° (awrt) α	B1
	$2x + 50 = 36.87$, $2x = -13.13° + 360° = 346.87°$	M1 M1
	$2x + 50 + 180 - 36.87$, $2x = 143.13° - 50° = 93.13°$	M1
	$x = 46.6$, 173.4	M1 A1 A1 **(7)**
(ii) (*a*)	$\sin 60° = \frac{\sqrt{3}}{2}$, $\frac{BC}{\left(\frac{1}{3}\right)} = \frac{18}{\sin 60°}$	B1, M1
	$BC = 6 \div \frac{\sqrt{3}}{2}$ $BC = \frac{12}{\sqrt{3}} = 4\sqrt{3}$ (*)	M1 A1 **(4)**
(*b*)	$\cos^2\theta = 1 - \sin^2\theta = 1 - \frac{1}{9}$	M1
	$\sin\theta = \sqrt{\frac{8}{9}}$ $\left(= \frac{\sqrt{8}}{3} = \frac{2\sqrt{2}}{3}\right)$	A1 **(2)**
		(13 marks)
6. (*a*)	9	B1 **(1)**
(*b*)	 Shape	B1
	Position of max.	B1
	5 on y-axis	B1
	–1 and 5 on x-axis	M1 A1 **(5)**
(*c*)	Gradient: $\frac{8-(-7)}{3-(-2)}$	M1 A1
	$y - 8 =$ "gradient" $(x - 3)$ $y = 3x - 1$	M1 A1 **(4)**
(*d*)	Where $y = 0$, $x = \frac{1}{3}$	M1 A1ft **(2)**
(*e*)	Mid point: $\left(\frac{-7+8}{2}, \frac{-2+3}{2}\right) = \left(\frac{1}{2}, \frac{1}{2}\right)$ $k = 1$	M1 A1 **(2)**
		(14 marks)

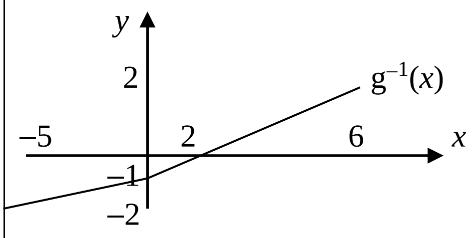
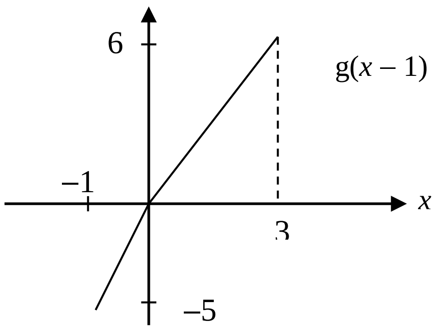
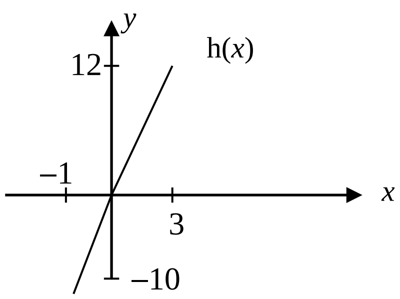
Question Number	Scheme	Marks
7. (*a*)	Integrate: $y = x^3 - 10x^2 + 29x$ $(+C)$	M1 A1
	$6 = 8 - 40 + 58 + C \Rightarrow C = -20$ $(y = x^3 - 10x^2 + 29x - 20)$	M1 A1 **(4)**
(*b*)	Substitute $x = 4$: $64 - 160 + 116 - 20 = 0$	M1 A1 **(2)**
(*c*)	At $x = 2$, $\frac{dy}{dx} = 12 - 40 + 29 = 1$	B1
	Tangent: $y - 6 = x - 2$ $(y = x + 4)$	M1 A1 **(3)**
(*d*)	$\frac{dy}{dx} = 1$	M1
	$3x^2 - 20x + 28 = 0$	M1
	$(3x - 14)(x - 2) = 0$	M1 A1
	$x = \frac{14}{3}$	A1 **(5)**
		(14 marks)

Question Number	Scheme	Marks
1.	$\frac{y+3}{(y+1)(y+2)} - \frac{y+1}{(y+2)(y+3)} \equiv \frac{(y+3)^2-(y+1)^2}{(y+1)(y+2)(y+3)}$	M1
	$\equiv \frac{(y^2+6y+9)-(y^2+2y+1)}{(y+1)(y+2)(y+3)} \equiv \frac{4y+8}{(y+1)(y+2)(y+3)}$	M1 A1
	$\equiv \frac{4(y+2)}{(y+1)(y+2)(y+3)} \equiv \underline{\underline{\frac{4}{(y+1)(y+3)}}}$ or $\frac{4}{y^2+4y+3}$	M1, A1
		(5 marks)
2. (*a*)	$4^x = (2^x)^2 = u^2$ **or** $2^{(x+1)} = 2.2^x = 2u$, $\rightarrow u^2 - 2u - 15 (= 0)$	M1, A1 c.s.o
		(2)
(*b*)	$u^2 - 2u - 15 = (u-5)(u+3)$	M1, A1
	$u = 5 \Rightarrow 2^x = 5 \Rightarrow x = \frac{\log 5}{\log 2}$, = 2.32	M1, A1 **(4)**
	[Ignore any other solution]	**(6 marks)**
3. (*a*)	1.5 sin 2x + 2 cos 2x = R sin (2x + α) = R[sin 2x cos α + cos 2x sin α]	
	$R = \sqrt{1.5^2 + 2^2} = 2.5$ Full method for R or R^2	M1, A1
	$\tan\alpha = \frac{2}{1.5}$, $\Rightarrow \alpha = 0.927$ Full method for tan α, sin α, cos α	M1, A1 **(4)**
(*b*)	3 sin x cos x = 1.5 sin 2x	M1
	4 $\cos^2 x$ = 2[2 $\cos^2 x$] = 2(cos 2x + 1)	
	$\therefore$ 3 sin x cos x + 4 $\cos^2 x$ = 1.5 sin 2x +2 cos 2x +2	A1 **(2)**
(*c*)	Maximum value of 1.5 sin 2x + 2 cos 2x = R	M1
	Maximum value of 3 sin x cos x + 4 $\cos^2 x$ = R + 2 or 4.5	A1 ft **(2)**
		(8 marks)

Question Number	Scheme	Marks
4. (*a*)	$u_2 = 2p + 5$	B1
	$u_3 = p(2p + 5) + 5$	M1
	$8 = 2p^2 + 5p + 5$ or $2p^2 + 5p - 3 = 0$	
	$(2p - 1)(p + 3) = 0$	M1
	$P = -3$, or $\frac{1}{2}$	A1, B1 cso **(5)**
(*b*)	$\log_2 \left(\frac{1}{2}\right) = \log_2 2^{-1} = -1$	B1 **(1)**
(*c*)	$\log_2 \left(\frac{p^3}{\sqrt{q}}\right) = \log_2 p^3 - \log_2 \sqrt{q}$ Use of log *a* – log *b*	M1
	$= 3\log_2 p - \frac{1}{2}\log_2 q$ Use of log a^n	M1
	$= -3 - \frac{1}{2}t$ accept $3\log_2 p - \frac{1}{2}t$	A1 ft (3)
		(9 marks)
5. (*a*)	$(0, 2)$ on $C \Rightarrow 2 = p + q$ Use of (0, 2) equation in *p* and *q*	M1
	$\frac{dy}{dx} = qe^x$, at $p \Rightarrow 5 = 2q$	M1, A1 (3)
	Solving $\Rightarrow q = 2.5, p = -0.5$ (or $2 - q$)	A1, A1 **(5)**
(*b*)	Gradient of normal at *P* is $-\frac{1}{5}$	B1
	Equation of normal at *P* is: $y - (p + 2q) = -\frac{1}{5}(x - \ln 2)$	M1
	at *L* $y = 0$ $\therefore x_L = 22.5 + \ln 2$ or $5(p + 2q) + \ln 2$ or 23.19…. at *M* $x = 0$ $\therefore y_M = 4.5 + \frac{1}{5}\ln 2$ or $p + 2q + \frac{1}{5}\ln 2$ or 4.639….	M1
	Area of triangle *OLM* is : $\frac{1}{2}x_L \times y_M = 53.792…. \approx 53.8$	M1, A1 cso **(5)**
		(10 marks)

Question Number	Scheme	Marks
6. (*a*)	 Shape  with vertex on +ve x-axis	B1
	(1, 0) and (0, ½)	B1 **(2)**
(*b*)	$x = \alpha$ given by: $e^{-x} - 1 = -\frac{1}{2}(x-1)$ Use of $-\frac{1}{2}(x-1)$	M1 A1
	$\Rightarrow 2e^{-x} - 2 = -x + 1$, i.e. $x + 2e^{-x} - 3 = 0$	A1 cso **(3)**
(*c*)	$f(x) = x + 2e^{-x} - 3$: $f(0) = 2 - 3 = -1$ 1 correct value to 1.s.f	M1
	$f(-1) = -4 + 2e^{1} = 1.43\ldots$	
	Change of sign $\therefore$ root in $-1 < \alpha < 0$ Both correct and comment	A1 **(2)**
(*d*)	$x_1 = -0.693(1\ldots)$, $x_2 = -0.613(3\ldots)$	B1, B1 **(2)**
(*e*)	$f(-0.575) = -0.0207\ldots$, $f(-0.585) = 0.00498\ldots$ } Change of sign so root is -0.58 to 2dp.	M1 A1 **(2)**
		(11 marks)
ALT (*e*)	$x_3 = 0.5914\ldots$, $x_4 = -0.5854\ldots$, $x_5 = -0.5837\ldots$, $x_6 = 0.5832\ldots$, ($x_7 = -0.5831\ldots$)	M1 A1

Question Number	Scheme		Marks
7. (*a*)	x: 0, 0.5, 1, 1.5, 2 y: 2, 2.25, 3, 4.25, 6	≥ 2 correct ys	M1
	$R \approx \frac{1}{2}\times\frac{1}{2}$, [2 + 2{2.25 + 3 + 4.25} + 6]		B1, [M1 A1 ft]
	$\frac{27}{4}$ or 6.75		A1
(*b*)	Since curve bends under straight line → overestimate		B1 M1
(*c*)	$V = \pi\int_0^2 y^2\,dx = \pi\int_0^2 (x^4 + 4x^2 + 4)\,dx$	$\pi\int y^2$, $y^2 = (\quad)$	M1 M1
	$= \pi\left[\frac{x^5}{5} + \frac{4}{3}x^3 + 4x\right]_0^2$	$x^n \to x^{n+1}$	M1 A1
	$= \pi\left[\left(\frac{32}{5} + \frac{32}{3} + 8\right) - (0)\right]$	Use of correct limits	M1
	$= \frac{\pi}{15}[96 + 160 + 120] = \frac{376}{15}\pi$ (or $25\frac{1}{15}$ or 25.1π)		A1 **(6)**
			(11 marks)

Question Number	Scheme	Marks
8. (*a*)	$y = \dfrac{3x-1}{x-3} \Rightarrow y(x-3) = 3x-1$	M1
	$yx - 3x = 3y - 1$	
	$x(y-3) = 3y - 1$ Collect *x* and factorise	M1
	$x = \dfrac{3y-1}{y-3}$ $\therefore$ $f^{-1}(x) = \dfrac{3x-1}{x-3} = f(x)$	A1 cso **(3)**
(*b*)	$ff(k) = f^{-1}f(k), = k$	M1 A1 **(2)**
(*c*)	$g(-2) = -5$	B1
	$f(-5) = \dfrac{-15-1}{-8}, = \dfrac{-16}{-8} = 2$	M1, A1 **(3)**
(*d*)	 shape	B1
	(0, –1) and (2, 0)	B1
	Domain: $-5 \le x \le 6$	B1 **(3)**
	 Translation +1 → (lines join at (0,0))	B1
	Stretch ×2↕	B1
	 Range: $-10 \le h(x) \le 12$	B1 **(3)**
		(14 marks)

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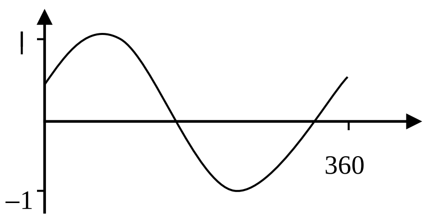
January 2003

Advanced Subsidiary / Advanced Level

General Certificate of Education

Subject **PURE MATHEMATICS 6671** Paper No. **P1**

Question number	Scheme	Marks
1.	(a) $\frac{dy}{dx} = 10 \times \frac{3}{2} x^{\frac{1}{2}}$ $\left(= 15x^{\frac{1}{2}} \right)$	M1 A1
	(b) $7x + 4x^{\frac{5}{2}} + C$	M1 A2(1,0)
2.	(a)  Scales (–1, 1 and 360)	B1
	Shape, position	B1
	(b) (0, 0.5) (150, 0) (330, 0)	B1 B1 B1
	(c) $(x + 30 =)$ $210°$ or $330°$ One of these	B1
	$x = 180°, 300°$ M: Subtract 30, A: Both	M1 A1
3.	(a) $3^x = 3^{2(y-1)}$ $x = 2(y-1)$ (*)	M1 A1
	(b) $(2y-2)^2 = y^2 + 7,$ $3y^2 - 8y - 3 = 0$	M1, A1
	$(3y+1)(y-3) = 0,$ $y = \ldots$ (or correct substitution in formula)	M1
	$y = -\frac{1}{3},$ $y = 3$	A1
	$x = -\frac{8}{3},$ $x = 4$	M1 A1ft

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Question number	Scheme	Marks
4.	(a) $\frac{a}{1-r} = \frac{1200}{1-r} = 960$	M1 A1
	$960\,(1 - r) = 1200$ $\quad r = -\frac{1}{4}$ $\quad$ (*)	A1
	(b) $T_9 = 1200 \times (-0.25)^8$ $\quad$ (or T_{10})	M1
	Difference $= T_9 - T_{10} = 0.0183105\ldots - (-0.0045776\ldots)$	M1
	$= 0.023$ $\quad$ (or -0.023)	A1
	(c) $S_n = \frac{1200\left(1-(-0.25)^n\right)}{1-(-0.25)}$	M1 A1
	(d) Since n is odd, $(-0.25)^n$ is negative,	M1
	so $S_n = 960\,(1 + 0.25^n)$ $\quad$ (*)	A1

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Question number	Scheme	Marks
5.	(a) $\frac{dC}{dv} = -160v^{-2} + \frac{2v}{100}$	M1 A1
	$-160v^{-2} + \frac{2v}{100} = 0$	M1
	$v^3 = 8\,000$ $\quad v = 20$	M1 A1
	(b) $\frac{d^2C}{dv^2} = 320v^{-3} + \frac{1}{50}$	M1
	> 0, therefore minimum	A1
	(c) $v = 20 : C = \frac{160}{20} + \frac{400}{100} = 12$	B1ft
	Cost = 250 × 12 = £30	M1 A1

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Question number	Scheme	Marks
6.	(a) P: $x = 0$ $y = -2$	B1
	Mid-point: $\left(\frac{(0+5)}{2}, \frac{(-2-3)}{2}\right) = \left(\frac{5}{2}, -\frac{5}{2}\right)$	M1 A1ft
	(b) Gradient of l_1 is $\frac{3}{2}$, so gradient of l_2 is $-\frac{2}{3}$	B1
	l_2: $y - (-3) = -\frac{2}{3}(x - 5)$	M1 A1ft
	$2x + 3y = 1$	A1
	(c) Solving: $3x - 2y = 4$	
	$2x + 3y = 1$ $x = \frac{14}{13}$	M1 A1
	$y = \frac{-5}{13}$	M1 A1ft

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Subject **PURE MATHEMATICS 6671** Paper No. **P1**

Question number	Scheme	Marks
7.	(a) $BM = \sqrt{(7^2 + 24^2)} = 25$ (*)	B1
	(b) $\tan\alpha = \frac{7}{24}$ or equiv. and $\angle BMC = 2\alpha$, or cosine rule	M1 A1
	$\angle BMC = 0.568$ radians (*)	A1
	(c) ΔABM: $\frac{1}{2}(14 \times 24)$ (= 168 mm^2) (or other appropriate Δ)	B1
	Sector: $\frac{1}{2}(25^2 \times 0.568)$	M1 A1
	Total: "168 + 168 + 177.5" = 513 mm^2 (or 514, or 510)	M1 A1
	(d) Volume = "513" × 85 mm^3 (M requires unit conversion) M1	
	= 44 cm^3 A1	

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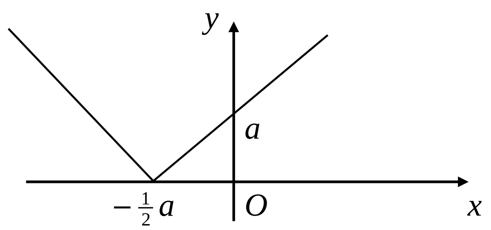
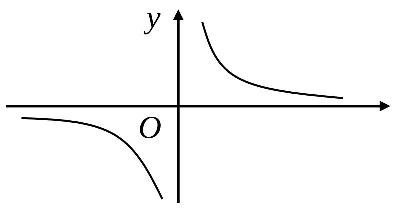
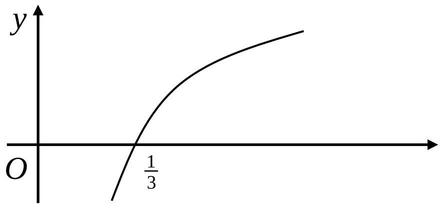
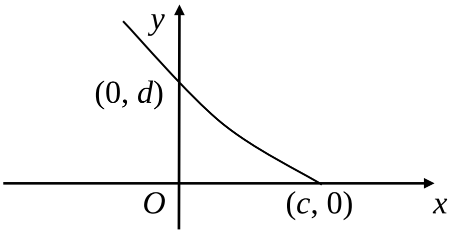
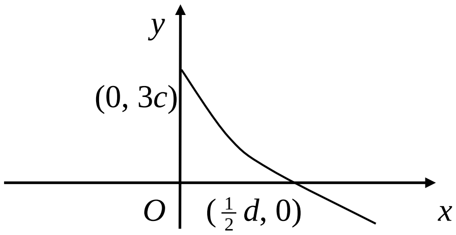
Question number	Scheme	Marks
8.	(a) A: $y = 1$ B: $y = 4$	B1
	(b) $\frac{dy}{dx} = \frac{2x}{25}$ $= \frac{2}{5}$ where $x = 5$	M1 A1
	Tangent: $y - 1 = \frac{2}{5}(x - 5)$ $(5y = 2x - 5)$	M1 A1
	(c) $x = 5y^{\frac{1}{2}}$	B1 B1
	(d) Integrate: $\frac{5y^{3/2}}{3/2}\left(= \frac{10y^{3/2}}{3}\right)$	M1 A1ft
	$[\ ]^4 - [\ ]_1 = \left(\frac{10\times 4^{3/2}}{3}\right) - \left(\frac{10\times 1^{3/2}}{3}\right)$, $= \frac{70}{3}$ $(23\frac{1}{3}, 23.3)$	M1 A1, A1
	Alternative for (d): Integrate: $\frac{x^3}{75}$	M1 A1
	Area = $(10\times4) - (5\times1) - \left(\frac{1000}{75} - \frac{125}{75}\right)$, $= \frac{70}{3}$ $(23\frac{1}{3}, 23.3)$	M1 A1, A1
	In both (d) schemes, final M is scored using candidate's "4" and "1".	

Question number	Scheme	Marks
1.	$x^2 - 9 = (x-3)(x+3)$ seen	B1
	Attempt at forming single fraction	
	$\dfrac{x(x-3)+(x+12)(x+1)}{(x+1)(x+3)(x-3)}$; $= \dfrac{2x^2+10x+12}{(x+1)(x+3)(x-3)}$	M1; A1
	Factorising numerator $= \dfrac{2(x+2)(x+3)}{(x+1)(x+3)(x-3)}$ or equivalent $= \dfrac{2(x+2)}{(x+1)(x-3)}$	M1 M1 A1 **(6)**
		(6 marks)
2.	$(1+px)^n \equiv 1 + npx, + \dfrac{n(n-1)p^2x^2}{2} + \ldots$	B1, B1
	Comparing coefficients: $np = -18$, $\dfrac{n(n-1)}{2} = 36$	M1, A1
	Solving $n(n-1) = 72$ to give $n = 9$; $p = -2$	M1 A1; A1 ft **(7)**
		(7 marks)
3. (*a*)	 V graph with 'vertex' on x-axis	M1
	$\{-\frac{1}{2}a, (0)\}$ and $\{(0), a\}$ seen	A1 **(2)**
	 Correct graph (could be separate)	B1 **(1)**
(*c*)	Meet where $\dfrac{1}{x} = \|2x + a\| \Rightarrow x\|2x+a\| - 1 = 0$; only one meet	B1 **(1)**
(*d*)	$2x^2 + x - 1$	B1
	Attempt to solve; $x = \frac{1}{2}$ (no other value)	M1; A1 **(3)**
		(7 marks)

Question number	Scheme	Marks
4.	Volume = $\pi\int_1^4 \left(1+\frac{1}{2\sqrt{x}}\right)^2 \mathrm{d}x$	M1
	$\int \left(1+\frac{1}{2\sqrt{x}}\right)^2 \mathrm{d}x = \int \left(1+\frac{1}{\sqrt{x}}+\frac{1}{4x}\right) \mathrm{d}x$	B1
	$= \left[x+2\sqrt{x}+\frac{1}{4}\ln x\right]$	M1 A1 A1ft
	Using limits correctly	M1
	Volume = $\pi\left[\left(8+\frac{1}{4}\ln 4\right)+3\right]$	A1
	$= \pi\left[5+\frac{1}{2}\ln 2\right]$	A1 **(8)**
		(8 marks)
5. (*a*)	(table below)	
	"y" = 7.80 when "x" = 4 or 6	B1
	Symmetry	B1 ft **(2)**
(*b*)	Estimate area = $\frac{2}{2}$[0 + 2(6.13 + 7.80 + 7.80 + 6.13)]	B1 M1 A1ft
	= 55.7 m^2	A1 **(4)**
(*c*)	140 – (*b*) = 84.3 m^2	A1 ft **(1)**
(*d*)	Over-estimate;	B1
	reason, e.g. area under curve is under-estimate (due to curvature)	B1 **(2)**
		(9 marks)

Distance from one side (m)	0	2	4	6	8	10
Height (m)	0	6.13	**7.80**	**7.80**	**6.13**	0

Question number	Scheme	Marks
6. (*a*)	 Shape	B1
	$p = \frac{1}{3}$ or $\{\frac{1}{3}, 0\}$ seen	B1 **(2)**
(*b*)	Gradient of tangent at $Q = \dfrac{1}{q}$	B1
	Gradient of normal $= -q$	M1
	Attempt at equation of OQ $[y = -qx]$ and substituting $x = q$, $y = \ln 3q$	
	or attempt at equation of tangent $[y - 3 \ln q = -q(x - q)]$ with $x = 0$, $y = 0$	
	or equating gradient of normal to $(\ln 3q)/q$	M1
	$q^2 + \ln 3q = 0$ (*)	A1 **(4)**
(*c*)	$\ln 3x = -x^2 \Rightarrow 3x = e^{-x^2}$; $\Rightarrow x = \frac{1}{3}e^{-x^2}$	M1; A1 **(2)**
(*d*)	$x_1 = 0.298280$; $x_2 = 0.304957$, $x_3 = 0.303731$, $x_4 = 0.303958$	M1; A1
	Root = 0.304 (3 decimal places)	A1 **(3)**
		(11 marks)
7. (*a*)	$\sin x + \sqrt{3}\cos x = R\sin(x + \alpha)$	
	$= R(\sin x\cos\alpha + \cos x\sin\alpha)$	M1
	$R\cos\alpha = 1$, $R\sin\alpha = \sqrt{3}$	A1
	Method for R or α, e.g. $R = \sqrt{(1 + 3)}$ or $\tan\alpha = \sqrt{3}$	M1
	Both $R = 2$ and $\alpha = 60$	A1 **(4)**
(*b*)	$\sec x + \sqrt{3}\,\text{cosec}\, x = 4 \Rightarrow \dfrac{1}{\cos x} + \dfrac{\sqrt{3}}{\sin x} = 4$	B1
	$\Rightarrow \sin x + \sqrt{3}\cos x = 4\sin x\cos x$	M1
	$= 2\sin 2x$ (*)	M1 **(3)**
(*c*)	Clearly producing $2\sin 2x = 2\sin(x + 60)$	A1 **(1)**
(*d*)	$\sin 2x - \sin(x + 60) = 0 \Rightarrow \cos\dfrac{3x + 60}{2}\sin\dfrac{x - 60}{2} = 0$	M1
	$\cos\dfrac{3x + 60}{2} = 0 \Rightarrow x = 40°, 160°$	M1 A1 A1 ft
	$\sin\dfrac{x - 60}{2} = 0 \Rightarrow x = 60°$	B1 **(5)**
		(13 marks)

Question number	Scheme	Marks
8. (*a*)	 shape	B1
	intersections with axes (c, 0), (0, d)	B1 **(2)**
(*b*)	 shape	B1
	x intersection ($\frac{1}{2}d$, 0)	B1
	y intersection (0, 3c)	B1 **(3)**
(*c*)(i)	$c = 2$	B1
(ii)	$-1 < \mathrm{f}(x) \le$ (candidate's) c value	B1 B1 ft **(3)**
(*d*)	$3(2^{-x}) = 1 \Rightarrow 2^{-x} = \frac{1}{3}$ and take logs; $-x = \dfrac{\ln \frac{1}{3}}{\ln 2}$	M1; A1
	d (or x) = 1.585 (3 decimal places)	A1 **(3)**
(*e*)	$\mathrm{fg}(x) = \mathrm{f}[\log_2 x] = [3(2^{-\log_2 x}) - 1]$; $= [3(2^{\log_2 \frac{1}{x}}) - 1]$ or $\dfrac{3}{2^{\log_2 x}} - 1$	M1; A1
	$= \dfrac{3}{x} - 1$	A1 **(3)**
		(14 marks)

Question Number	Scheme	Marks
1(a)	$\frac{3(x+1)}{(x+2)(x-1)} \equiv \frac{A}{x+2} + \frac{B}{x-1}$, and correct method for finding *A* or *B*	M1
	$A = 1, \quad B = 2$	A1, A1 (3)
(b)	$f'(x) = -\frac{1}{(x+2)^2} - \frac{2}{(x-1)^2}$	M1 A1
	Argument for negative, including statement that square terms are positive for all values of *x*. (f.t. on wrong values of *A* and *B*)	A1ft✓ (3)
2 (a)	$a = 4, \quad b = 5$ (both are required)	B1 (1)
(b)	$(x-4)^2 + (y-5)^2 = 25$	M1A1ft (2)
(c)	Finding the distance between centre and (8, 17), $\sqrt{[(8-a)^2 + (17-b)^2}$	M1
	Complete method to find *PT*, i.e. use Pythagoras theorem and subtraction,	M1
	PT = 11.6	A1 (3)

Question Number	Scheme	Marks
3(a)	Using $f(\pm 2) = 3$	M1
	Showing that p = 6 ✸, with no wrong working seen.	A1 (2)
	S.C. If p = 6 used and the remainder is shown to be 3 award B1	
(b)	Attempt to find quotient when dividing (n + 2) into f(n) or attempting to equate coefficients.	M1
	Quotient = $n^2 + 4n + 3$, or finding either q = 1 or r = 3	A1
	Finding both q = 1 and r = 3	A1 (3)
(c)	The product of three consecutive numbers must be divisible by 3	M1
	Complete argument	A1 (2)
4. (a)	$(1+3x)^{-2} = 1 + (-2)(3x) + \frac{(-2)(-3)}{2!}(3x)^2 + \frac{(-2)(-3)(-4)}{3!}(3x)^3 + ...$ = 1, – 6x, + 27x^2 ...(–108x^3)	M1 B1, A1, A1 (4)
(b)	Using (a) to expand $(x+4)(1+3x)^{-2}$ or complete method to find coefficients [e.g. Maclaurin or $\frac{1}{3}(1+3x)^{-1} + \frac{11}{3}(1+3x)^{-2}$]. $= 4 - 23x, +102x^2, -405x^3$ = 4, – 23x, + 102x^2 ...(– 405x^3)	M1 A1,A1ft,A1ft (4)

Question Number	Scheme	Marks
6(a)	$\mathbf{r} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k} \pm \lambda(4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k})$ or $\mathbf{r} = 5\mathbf{i} - 3\mathbf{j} \pm \lambda(4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k})$ (or any equivalent vector equation)	M1A1 (2)
(b)	Show that μ = -3	B1 (1)
(c)	Using cos θ $= \dfrac{(4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}).(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})}{\sqrt{(4^2+5^2+3^2)}\sqrt{(1^2+2^2+2^2)}}$	M1
	$= \dfrac{20}{15\sqrt{2}} = \dfrac{4}{3\sqrt{2}}$ (ft on 4i – 5j + 3k) num, denom.	A1ft A1ft
	θ = 19.5° (allow 19 or 20 if no wrong working is seen)	A1 (4)
(d)	Shortest distance = AC sin θ	M1
	AC = $\sqrt{((a-1)^2 + 2^2 + (b+3)^2)}$ (= 3)	M1A1
	Shortest distance = 1 unit	A1 (4)
	Alternatives Since $X = (1+4\lambda, 2-5\lambda, -3+3\lambda)$ $\mathbf{CX} = (-1+4\lambda)\mathbf{i} + (2-5\lambda)\mathbf{j} + (-2+3\lambda)\mathbf{k}$	M1
	Use Scalar product $\mathbf{CX}.(4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}) = 0$, OR differentiate $\lvert\mathbf{CX}\rvert$ or $\lvert\mathbf{CX}\rvert^2$ and equate to zero,	M1
	to obtain λ = 0.4	A1
	and thus $\lvert\mathbf{CX}\rvert = 1$	A1 (4)

Question Number	Scheme	Marks
5. (a)	$\frac{\mathrm{d}V}{\mathrm{d}t} = 30 - \frac{2}{15}V$	M1A1
	$\Rightarrow -15\frac{\mathrm{d}V}{\mathrm{d}t} = -450 + 2V$, no wrong working seen	A1* (3)
(b)	Separating the variables $\Rightarrow$ $-\frac{15}{2V-450}\mathrm{d}V = \mathrm{d}t$	M1
	Integrating to obtain $-\frac{15}{2}\ln\lvert 2V-450\rvert = t$ OR $-\frac{15}{2}\ln\lvert V-225\rvert = t$	dM1 A1
	Using limits correctly or finding c $(-\frac{15}{2}\ln 1550$ *OR* $-\frac{15}{2}\ln 775)$	M1
	$\ln\frac{2V-450}{1550} = -\frac{2}{15}t$, or equivalent	A1
	Rearranging to give $V = 225 + 775e^{-\frac{2}{15}t}$.	dM1A1 (7)
(c)	$V = 225$	B1 (1)

Question Number	Scheme	Marks
7(a)	$\frac{dy}{dx} = -2e^{-2x}\sqrt{x} + \frac{e^{-2x}}{2\sqrt{x}}$	M1 A1 A1
	Putting $\frac{dy}{dx} = 0$ and attempting to solve	dM1
	$x = \frac{1}{4}$	A1 (5)
(b)	Volume $= \pi\int_0^1 (\sqrt{x}e^{-2x})^2 dx \quad = \pi\int_0^1 xe^{-4x} dx$	M1 A1
	$\int xe^{-4x} dx = -\frac{1}{4}xe^{-4x} + \int \frac{1}{4}e^{-4x} dx$	M1 A1
	$= -\frac{1}{4}xe^{-4x} - \frac{1}{16}e^{-4x}$	A1 ft
	Volume = $\pi[-\frac{1}{4}e^{-4} - \frac{1}{16}e^{-4}] - [-\frac{1}{16}] = \frac{\pi}{16}[1 - 5e^{-4}]$	M1 A1 (7)

Question Number	Scheme	Marks
8 (a)	$\cos(A+A) = \cos^2 A - \sin^2 A$ $= \cos^2 A - (1-\cos^2 A) = 2\cos^2 A - 1$	M1 A1 (2)
(b)	$[x = 2, \theta = \frac{\pi}{4}; x = \sqrt{6}, \theta = \frac{\pi}{3}]$	B1
	$x = 2\sqrt{2}\sin\theta, \frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\sqrt{2}\cos\theta$	B1
	$\int\sqrt{8-x^2}\mathrm{d}x = \int 2\sqrt{2}\cos\theta 2\sqrt{2}\cos\theta\mathrm{d}\theta = \int 8\cos^2\theta\mathrm{d}\theta$	M1A1
	Using $\cos 2\theta = 2\cos^2\theta - 1$ to give $\int 4(1+\cos 2\theta)\mathrm{d}\theta$	dM1
	$= 4\theta + 2\sin 2\theta$	A1ft
	Substituting limits to give $\frac{1}{3}\pi + \sqrt{3} - 2$ or given result	A1 (7)
(c)	$\frac{dy}{d\theta} = \frac{-2\sin 2\theta}{1+\cos 2\theta}$	B1
	Using the chain rule, with $\frac{dx}{d\theta} = \sec\theta\tan\theta$ to give $\frac{\mathrm{d}y}{\mathrm{d}x}(= -2\cos\theta)$	M1
	Gradient at the point where $\theta = \frac{\pi}{3}$ is –1.	A1ft
	Equation of tangent is $y + \ln 2 = -(x-2)$ (o.a.e.)	M1A1 (5)

Question Number	Scheme	Marks
1. (*a*)	$y = 5x - x^{-1} + C$	M1 A2 (1,0)
(*b*)	$7 = 5 - 1 + C$, $C = 3$	M1 A1 ft
	$x = 2$: $y = 10 - \frac{1}{2} + 3 = 12\frac{1}{2}$	M1 A1
		(7 marks)
2. (*a*)	$6x - 2x < 3 + 7$ $x < 2\frac{1}{2}$	M1 A1
(*b*)	$(2x - 1)(x - 5)$ Critical values $\frac{1}{2}$ and 5	M1 A1
	$\frac{1}{2} < x < 5$	M1 A1 ft
(*c*)	$\frac{1}{2} < x < 2\frac{1}{2}$	B1 ft
		(7 marks)
3. (*a*)(i)	$a + (n - 1)d = 280 + (35 \times 5) = 455$	M1 A1
(ii)	$\frac{1}{2}n\,[2a + (n - 1)d\,] = 18\,[560 + (35 \times 5)] = 13\,230$	M1 A1 ft
(*b*)	$18\,[560 + (35 \times d)] = 17\,000$	M1 A1
	$d = 10.98\ldots$ $x = 11$ (allow 11.0 or 10.98 or 10.99 or $10\frac{62}{63}$)	M1 A1
		(8 marks)

(ft = follow-through mark)

Question Number	Scheme	Marks
4. (*a*)	$\frac{1}{2}r^2\theta = \frac{1}{2}r^2 \times 1.5 = 15$	M1 A1
	$r^2 = 20 = \sqrt{(4 \times 5)}$ $r = 2\sqrt{5}$ (*)	A1
(*b*)	$r\theta + 2r = 3\sqrt{5} + 4\sqrt{5} = 7\sqrt{5}$ cm (or 15.7, or a.w.r.t 15.65….)	M1 A1
(*c*)	ΔOAB: $\frac{1}{2}r^2 \sin\theta = 10 \sin 1.5$ (= 9.9749…)	M1
	Segment area = 15 – ΔOAB = 5.025 cm^2	M1 A1
		(8 marks)
	$2\cos^2\theta - \cos\theta - 1 = 1 - \cos^2\theta$	M1
	$3\cos^2\theta - \cos\theta - 2 = 0$	A1
	$(3\cos\theta + 2)(\cos\theta - 1) = 0$ $\cos\theta = -\frac{2}{3}$ or 1	M1 A1
	$\theta = 0$ $\theta = 131.8°$	B1 A1
	$\theta = (360 - \text{“}131.8\text{”})° = 228.2°$	M1 A1 ft
		(8 marks)
6. (*a*)	$m = \frac{2-6}{12-4}\left(= -\frac{1}{2}\right)$	M1 A1
	$y - 6 = (\text{their } m)(x - 4)$ $x + 2y = 16$	M1 A1
(*b*)	$y = -4x$	B1
(*c*)	$x + 2(-4x) = 16$ $-7x = 16$ $x = -\frac{16}{7}$	M1 A1
	$y = \frac{64}{7}$	A1 ft
	A (4, 6), $C\left(-\frac{16}{7}, \frac{64}{7}\right)$: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \rightarrow \left(\frac{6}{7}, \frac{53}{7}\right)$	M1 A1 ft
		(10 marks)

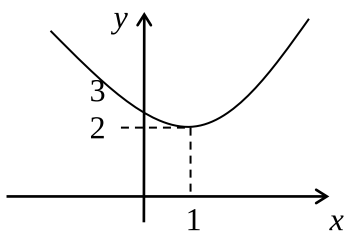
(ft = follow-through mark)

Question Number	Scheme	Marks
7. (*a*)	$x^2 - 2x + 3 = 9 - x$	M1
	$x^2 - x - 6 = 0$ $\quad (x+2)(x-3) = 0 \quad x = -2,\ 3$	M1 A1
	$y = 11,\ 6$	M1 A1 ft
(*b*)	$\int (x^2 - 2x + 3)\,dx = \frac{x^3}{3} - x^2 + 3x$	M1 A1
	$\left[\frac{x^3}{3} - x^2 + 3x\right]_{-2}^{3} = (9 - 9 + 9) - \left(\frac{-8}{3} - 4 - 6\right) \quad \left(= 21\frac{2}{3}\right)$	M1 A1
	Trapezium: $\frac{1}{2}(11 + 6) \times 5 \quad \left(= 42\frac{1}{2}\right)$	B1 ft
	Area = $42\frac{1}{2} - 21\frac{2}{3} = 20\frac{5}{6}$	M1 A1
	<u>Alternative:</u> $(9 - x) - (x^2 - 2x + 3) = 6 + x - x^2$	M1 A1
	$\int (6 + x - x^2)\,dx = 6x + \frac{x^2}{2} - \frac{x^3}{3}$	M1 A1 ft
	$\left[6x + \frac{x^2}{2} - \frac{x^3}{3}\right]_{-2}^{3} = \left(18 + \frac{9}{2} - 9\right) - \left(-12 + 2 + \frac{8}{3}\right), = 20\frac{5}{6}$	M1 A1, A1
		(12 marks)

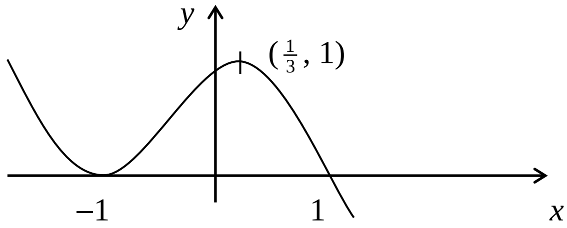
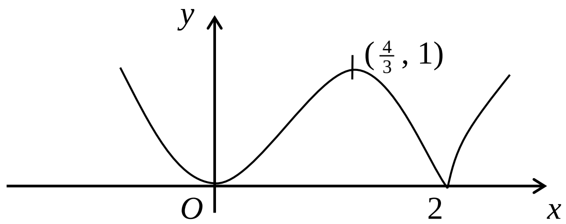
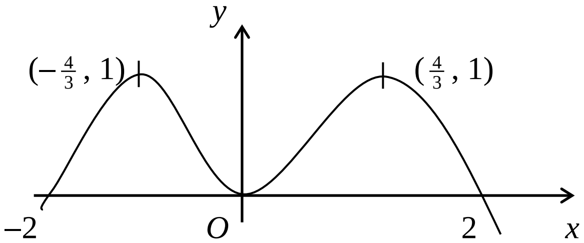
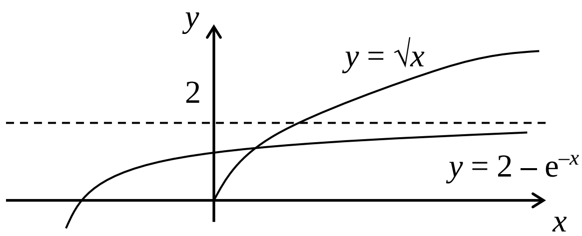
(ft = follow-through mark)

Question Number	Scheme	Marks
8. (*a*)	$\frac{dy}{dx} = 4x^3 - 16x$	M1 A1
(*b*)	$4x^3 - 16x = 0$	M1
	$4x(x^2 - 4) = 0$ $x = 0,\ 2,\ -2$	A2 (1, 0)
	$y = 3,\ -13,\ -13$	M1 A1
(*c*)	$\frac{d^2y}{dx^2} = 12x^2 - 16$	M1
	$x = 0$ Max. $x = 2$ Min. $x = -2$ Min. One of these, ft / All three	A1ft A1
(*d*)	$x = 1$: $y = 1 - 8 + 3 = -4$	B1
	At $x = 1$, $\frac{dy}{dx} = 4 - 16 = -12$ (m)	B1 ft
	Gradient of normal = $-\frac{1}{m}$ $\left(= \frac{1}{12}\right)$	M1
	$y - (-4) = \frac{1}{12}(x - 1)$ $x - 12y - 49 = 0$	M1 A1
		(15 marks)

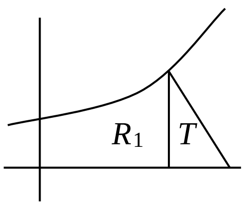
(ft = follow-through mark)

Question Number	Scheme	Marks
1. (*a*)	$\dfrac{x^2+4x+3}{x^2+x} = \dfrac{(x+3)(x+1)}{x(x+1)}$ Attempt to factorise numerator or denominator	M1
	$= \dfrac{x+3}{x}$ or $1 + \dfrac{3}{x}$	A1 (2)
(*b*)	LHS = $\log_2\left(\dfrac{x^2+4x+3}{x^2+x}\right)$ Use of log *a* – log *b*	M1
	RHS = 2^4 or 16	B1
	$x + 3 = 16x$ Linear or quadratic equation in *x*	M1
	$x = \frac{3}{15}$ or $\frac{1}{5}$ or 0.2	A1 (4)
		(6 marks)
2. (*a*)	 $x^2 - 2x + 3 = (x-1)^2 + 2$ Full method to establish min. f	M1
	$f(4) = 3^2 + 2 = 11$ $f \geq 2$	A1
	$f \leq 11$	B1 (3)
(*b*)	$f(2) = 3$; $\therefore 16 = gf(2) \Rightarrow 16 = 3\lambda + 1$ M for using their f(2) for eqn	B1; M1
	$\therefore \lambda = 5$ ft their genuine f(2)	A1 ft (3)
		(6 marks)
3.	$(2 - px)^6 = 2^6 + \binom{6}{1}2^5(-px) + \binom{6}{2}2^4(-px)^2$ Coeff. of *x* or x^2	M1 $\binom{n}{r}$ okay
	$= 64 + 6 \times 2^5(-px)$; $+ 15 \times 2^4(-px)^2$ No $\binom{n}{r}$	A1; A1
	$15 \times 16p^2 = 135 \Rightarrow p^2 = \frac{9}{16}$ or $p = \frac{3}{4}$ (only)	M1, A1
	$-6.32p = A$	M1
	$\Rightarrow A = -144$	A1 ft (their *p* (> 0))
		(7 marks)

(ft = follow-through mark)

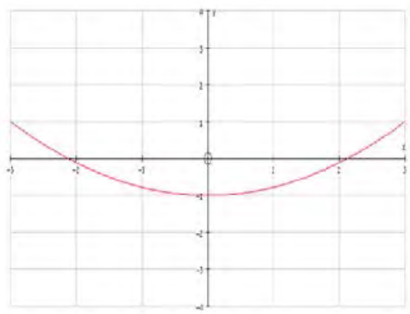
Question Number	Scheme	Marks
4. (*a*)		
	Translation in ← or →	B1
	Points correct	B2, 1, 0
		(–1eeoo) (3)
(*b*)		
	$x < 2$ including points	B1
	$x > 2$ correct reflection	B1
	cusp at (2, 0) (not ∪)	B1 (3)
(*c*)		
	correct shape $x \geq 0$	B1
	symmetry in *y*-axis	B1
	correct maxima	B1
	correct *x* intercepts	B1 (4)
		(10 marks)
5. (*a*)		
	$y = \sqrt{x}$: starting (0,0)	B1
	$y = 2 - e^{-x}$:	
	shape & int. on + *y*-axis	B1
	correct relative posns	B1 (3)
(*b*)	Where curves meet is solution to f(x) = 0; only one intersection	B1 (1)
(*c*)	f(3) = –0.218… f(4) = 0.018… — one correct value to 1 sf	M1
	change of sign ∴ root in interval — both correct (1 sf) + comment	M1 (2)
(*d*)	$x_0 = 4$ $x_1 = (2 - e^{-4})^2 = 3.92707\ldots$ — expression or x_1 to 3 dp	M1
	$x_2 = 3.92158\ldots$ — x_1, x_2 to ≥ 4 dp	A1
	$x_3 = 3.92115\ldots$ — carry on to x_4	
	$x_4 = 3.92111(9)\ldots$ — to ≥ 3 dp	M1
	Approx. solution = 3.921 (3 dp)	A1 cao (4)
		(10 marks)

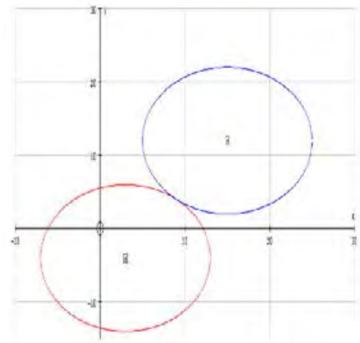
(–1eooo = minus 1 mark for each error or omission; cao = correct answer only)

Question Number	Scheme		Marks
6. (*a*)	$\frac{dy}{dx} = -\frac{c}{x^2}$	Attempt $\frac{dy}{dx}$	M1
	When $x = p \Rightarrow -4 = -\frac{c}{p^2}$ $\therefore c = 4p^2$ (*)		A1 cso (2)
(*b*)	$5 = 1 + \frac{c}{p}$ and solve with $c = 4p^2$		M1
	$5 = 1 + 4p \Rightarrow p = 1 \quad \therefore c = 4$ (*)		A1 cso (2)
(*c*)	$y^2 = 1 + \frac{8}{x} + \frac{16}{x^2}$	$y^2 =$; ≥ 2 terms correct	M1
	$\int y^2 dx = \left[x + 8\ln x - \frac{16}{x}\right]$	some correct ∫ ; all correct	M1 ; A1
	$\int_1^2 y^2 dx = (2 + 8\ln 2 - \frac{16}{2}) - (1 + 8\ln 1 - 16)$	Use of correct limits	M1
	$V = \pi \int_1^2 y^2 dx$		B1
	$\therefore V = \pi(9 + 8\ln 2)$	$k = 9$; $q = 8$	A1; A1 (7)
			(11 marks)
7. (*a*)	M is (0, 7)		B1
	$\frac{dy}{dx} = 2e^x$	Attempt $\frac{dy}{dx}$	M1
	$\therefore$ gradient of normal is $-\frac{1}{2}$	ft their $y'(0)$	M1
	$\therefore$ equation of normal is $y - 7 = -\frac{1}{2}(x - 0)$ or $x + 2y = 14$		A1 (4)
(*b*)	$y = 0, x = 14 \quad \therefore N$ is (14, 0) (*)		B1 cso (1)
(*c*)	 $\int (2e^x + 5)\,dx = [2e^x + 5x]$	some correct ∫	M1
	$R_1 = \int_0^{\ln 4} (2e^x + 5)\,dx = (2 \times 4 + 5\ln 4) - (2 + 0)$		M1 limits used
	$= 6 + 5\ln 4$		A1
	$T = \frac{1}{2} \times 13 \times (14 - \ln 4)$	Area of T	B1
	$T = 13(7 - \ln 2)$; $R_1 = 6 + 10\ln 2$	Use of ln 4 = 2ln 2	B1
	$R = T + R_1$, $\quad R = 97 - 3\ln 2$		M1, A1 (7)
			(12marks)

Question Number	Scheme	Marks
8. (i)	$\cos x \cos 30 - \sin x \sin 30 = 3(\cos x \cos 30 + \sin x \sin 30)$ Use of $\cos(x \pm 30)$	M1
	$\Rightarrow \sqrt{3}\cos x - \sin x = 3\sqrt{3}\cos x + 3\sin x$ Sub. for sin 30 etc decimals M1, surds A1	M1, A1
	i.e. $-4\sin x = 2\sqrt{3}\cos x \rightarrow \tan x = -\frac{\sqrt{3}}{2}$ (*) Use $\tan x = \frac{\sin x}{\cos x}$	M1, A1cso (5)
(ii) (*a*)	LHS = $\frac{1-(1-2\sin^2\theta)}{2\sin\theta\cos\theta}$ Use of cos 2*A* or sin 2*A*; both correct	M1; A1
	$= \frac{\sin\theta}{\cos\theta} = \tan\theta$ (*)	A1 cso (3)
(*b*)	Verifying: $0 = 2 - 2$ (since sin 360 = 0, cos 360 = 1)	B1 cso
(*c*)	Equation $\rightarrow$ $1 = \frac{2(1-\cos 2\theta)}{\sin 2\theta}$ Rearrange to form $\frac{1-\cos 2\theta}{\sin 2\theta}$	M1
	$\Rightarrow \tan\theta = \frac{1}{2}$	A1
	i.e. $\theta = 26.6°$ or $206.6°$ (Accept 27°, 207°)	M1 (any acc.)
		A1 (both) (4)
		(13 marks)
Alt 1 (*c*)	$2\sin\theta\cos\theta = 2 - 2(1 - 2\sin^2\theta)$ Use of cos 2*A* <u>and</u> sin 2*A*	M1
	$0 = 2\sin\theta(2\sin\theta - \cos\theta)$	
	$\Rightarrow$ $(\sin\theta = 0)$ $\tan\theta = \frac{1}{2}$ etc, as in scheme	A1
Alt 2 (*c*)	$2\cos 2\theta + \sin 2\theta = 2 \Rightarrow \cos(2\theta - \alpha) = \frac{2}{\sqrt{5}}$	M1
	$\alpha = 22.6$ (or 27)	A1
	$2\theta = 2\alpha$, 360, $360 + 2\alpha$ $\theta = \alpha$ or $180 + \alpha$	M1
	$\theta = \alpha$, $180 + \alpha$ i.e. $\theta = 27°$ or $207°$ (or 1 dp)	A1 both

((*) indicates final line is given on the paper; cso = correct solution only; ft = follow-through mark)

Question number	Scheme	Marks
1.	Attempt to use correctly stated double angle formula $\cos 2t = 2\cos^2 t - 1$, or complete method using other double angle formula for cos*2t* with $\cos^2 t + \sin^2 t = 1$ to eliminate *t* and obtain $y =$	**M1**
	$y = 2(\frac{x}{3})^2 - 1$ or any correct equivalent.(even y $= \cos 2(\cos^{-1}(\frac{x}{3}))$	**A1** **(2)**
	shape 	**B1**
	position including restricted domain $-3 < x < 3$	**B1** **(2)**
2. (a)	$p + 6 + 12 + q = -\frac{1}{8}p + \frac{6}{4} - 6 + q$	**M1 , M1**
	$\therefore \frac{9}{8}p = -22\frac{1}{2}$	**M1**
	$p = -20$	**A1** **(4)**
(b)	Remainder = $p + q + 18 = p + 21$ (=1)	**B1√ ft on *p*** **(1)**

Question number	Scheme	Marks
3. **(a)**	Centre is at (3,-4)	**B1**
	radius $= \sqrt{(3^2 + (-4)^2 - -75)} = 10$	**M1 A1** **(3)**
(b)	1^{st} circle	**B1**
	2^{nd} circle	**B1**
	Circles touching	**B1**
	At (9 , 4) 	**B1** **(4)**
4. **(a)**	$14x + (48x\frac{dy}{dx} + 48y) - 14y\frac{dy}{dx} = 0$	**M1 (B1) A1**
	Substitutes $\frac{dy}{dx} = \frac{2}{11}$ into derived expression to obtain	**M1**
	$14x + \frac{96}{11}x + 48y - \frac{28}{11}y = 0$	
	$\therefore 250x + 500y = 0 \quad \Rightarrow x + 2y = 0$	**A1** **(5)**
(b)	Eliminates one variable to obtain, for example, $7(2y)^2 + 48(-2y)y - 7y^2 + 75 = 0$ and obtains y (or x)	**M1**
	Substitutes y to obtain x (or y)	**M1**
	Obtains coordinates (-2,1) and (2,-1)	**A1, A1** **(4)**

Question number	Scheme	Marks
5. (a)	$\frac{dy}{dx} = \sqrt{\sin x} + \frac{x}{2}(\sin x)^{-\frac{1}{2}} \cos x$	**M1,A1**
	At A $\sqrt{\sin x} + \frac{x}{2}(\sin x)^{-\frac{1}{2}} \cos x = 0$	**dM1**
	$\therefore \sin x + \frac{x}{2}\cos x = 0$ (essential to see intermediate line before given answer)	
	$\therefore 2\tan x + x = 0$ *	**A1**
		(4)
(b)	$V = \pi\int y^2 dx = \pi\int x^2 \sin x dx$	**M1**
	$= \pi\left[-x^2 \cos x + \int 2x\cos x dx\right]_0^{\pi}$	**M1 A1**
	$= \pi\left[-x^2 \cos x + 2x\sin x - \int 2\sin x dx\right]_0^{\pi}$	**M1**
	$= \pi\left[-x^2 \cos x + 2x\sin x + 2\cos x\right]_0^{\pi}$	**A1**
	$= \pi\left[\pi^2 - 2 - 2\right]$	**M1**
	$= \pi\left[\pi^2 - 4\right]$	**A1**
		(7)

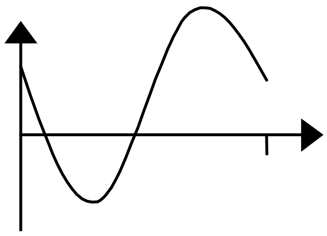
Question number	Scheme	Marks
6. **(a)**	$\overrightarrow{AB} = (2\mathbf{i} - \mathbf{j} + 2\mathbf{k}),\ \overrightarrow{CB} = (-2\mathbf{i} + 2\mathbf{j} + \mathbf{k}),$ (or $\overrightarrow{BA}, \overrightarrow{BC},$ *or* $\overrightarrow{AB}, \overrightarrow{BC},$ stated in above form or column vector form.	**M1A1**
	$\cos A\hat{B}C = \dfrac{CB \bullet AB}{\lvert CB \rvert \lvert AB \rvert} = -\dfrac{4}{9}$	**M1 A1** **(4)**
(b)	Area of $\Delta ABC = \frac{1}{2} \times 3 \times 3 \times \sin B$	**M1**
	$\sin B = \sqrt{(1 - \frac{16}{81})} = \dfrac{\sqrt{65}}{9}$	**M1**
	$\therefore$ Area $= \frac{1}{2}\sqrt{65}$	**A1** **(3)**
(c)	$\overrightarrow{AC} = \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix}$ $\overrightarrow{DC} = \begin{pmatrix} 0 \\ -1 \\ -3 \end{pmatrix}$ or given in alternative form with attempt at scalar product	**M1**
	$\overrightarrow{AC} \bullet \overrightarrow{DC} = 0$, therefore the lines are perpendicular.	**A1** **(2)**
(d)	$\overrightarrow{AD} = \begin{pmatrix} 4 \\ -2 \\ 4 \end{pmatrix}$ $\overrightarrow{DB} = \begin{pmatrix} -2 \\ 1 \\ -2 \end{pmatrix}$ and AD:DB = 2:-1 (allow 2:1)	**M1, A1** **(2)**

Question number	Scheme	Marks
7. **(a)**	$\frac{\mathrm{d}V}{\mathrm{d}t} = \pm c\sqrt{V}$ *or* $\frac{\mathrm{d}V}{\mathrm{d}t} \propto \sqrt{V}$	**M1**
	As $V = Ah$, $\frac{dV}{dh} = A$ *or* $V \propto h$	**M1**
	$\therefore$*use chain rule to obtain* $\frac{dh}{dt} = -\frac{c}{A}\sqrt{V} = \frac{-c}{\sqrt{A}}\sqrt{h} = -k\sqrt{h}$	**A1** **(3)**
(b)	$\int \frac{dh}{h} = -\int kdt$	**M1,**
	$2h^{\frac{1}{2}} = A - kt$	**M1 A1**
	$h^{\frac{1}{2}} = \frac{A}{2} - \frac{kt}{2}$	
	$h = (A - Bt)^2$ *	**A1** **(4)**
(c)	$t = 0,\ h = 1:$ $A = 1$	**B1**
	$t = 5,\ h = 0.5:$ $0.5 = (1 - 5B)^2$	
	$B = \frac{(1-\sqrt{0.5})}{5}$ $(B = 0.0586)$	**B1**
	$h = 0, t = \frac{A}{B} = \frac{5}{1-\sqrt{0.5}} = 17.1\text{min}$	**B1** **(3)**
(d)	$h = \frac{A^2}{4} = 0.25\text{m}$	**M1 A1** **(2)**

Question number	Scheme	Marks
8. **(a)**	Method using either	**M1**
	$\frac{A}{(1-x)}+\frac{B}{(2x+3)}+\frac{C}{(2x+3)^2}$ **or** $\frac{A}{1-x}+\frac{Dx+E}{(2x+3)^2}$	
	A = 1	**B1**
	C=10 , B=2 **or** D=4 and E = 16	**A1, A1**
		(4)
(b)	$\int [\frac{1}{1-x}+\frac{2}{2x+3}+10(2x+3)^{-2}]dx$ **or** $\int \frac{A}{1-x}+\frac{Dx+E}{(2x+3)^2}dx$	**M1**
	$-\ln\|1-x\|+\ln\|2x+3\|-5(2x+3)^{-1}(+c)$ **or**	**M1A1√ A1√A1√**
	$-\ln\|1-x\|+\ln\|2x+3\|-(2x+8)(2x+3)^{-1}(+c)$	**(5)**
(c)	**Either**	
	$(1-x)^{-1}+2(3+2x)^{-1}+10(3+2x)^{-2}=$	**M1**
	$1+x+x^2+...$	**A1**
	$+\frac{2}{3}(1-\frac{2x}{3}+\frac{4x^2}{9}...)$	**M1 A1**
	$+\frac{10}{9}(1+(-2)(\frac{2x}{3})+\frac{(-2)(-3)}{2}(\frac{2x}{3})^2+...)$	**A1**
	$=\frac{25}{9}-\frac{25}{27}x+\frac{25}{9}x^2...$	**M1A1**
		(7)
	Or	
	$25\left[(9+12x+4x^2)(1-x)\right]^{-1}=25\left[(9+3x-8x^2-4x^3)\right]^{-1}$	**M1 A1**
	$\frac{25}{9}\left[1+\frac{3x}{9}-\frac{8x^2}{9}-\frac{4x^3}{9}\right]^{-1}=\frac{25}{9}\left[1-(\frac{3x}{9}-\frac{8x^2}{9}-\frac{4x^3}{9})+(\frac{x^2}{9}..).\right]$	**M1 A1 A1**
	$=\frac{25}{9}-\frac{25}{27}x+\frac{25}{9}x^2...$	**M1A1**
		(7)

EDEXCEL PURE MATHEMATICS P1 (6671)
PROVISIONAL MARK SCHEME NOVEMBER 2003

Question Number	Scheme	Marks
1.	**(a)** 77 74	B1 B1 (2)
	(b) $d = 74 - 77 = -3$	B1 $\sqrt{}$ (1)
	(c) $S_{50} = \frac{1}{2}n\left[2a + (n-1)d\right] = 25\left[(2\times 77) + (49\times -3)\right]$	M1 A1 $\sqrt{}$
	$= 175$	A1 (3)
		6
2.	**(a)** $4x(x+3)$ *or* $x(4x+12)$ (or use of quadratic formula)	M1
	$x = 0$ $x = -3$	A1 A1 (3)
	(b) Using $b^2 - 4ac = 0$ $144 - 16c = 0$ $c = 9$	M1 A1
	$(2x+3)(2x+3) = 0$ $x = \ldots$ (or quadratic formula)	M1
	$x = -\frac{3}{2}$	A1 (4)
		7
3.	$x = 3y - 1$	M1
	$(3y-1)^2 - 3y(3y-1) + y^2 = 11$ $y^2 - 3y - 10 = 0$	M1 A1
	$(y-5)(y+2) = 0$ $y = 5$ $y = -2$	M1 A1
	$x = 14$ $x = -7$	M1 A1 $\sqrt{}$ (7)
		7

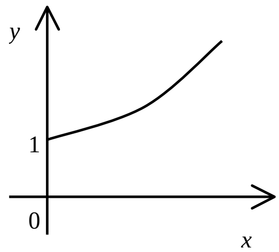
Question Number	Scheme	Marks
4.	**(a)** $4x+9, \quad +12\sqrt{x}$	B1, B1 (2)
	(b) $\int\left(4x+12x^{1/2}+9\right)dx = 2x^2+8x^{3/2}+9x$ ($\sqrt{}$ dep. on 3 terms)	M1 A1$\sqrt{}$
	(c) $[\ldots\ldots]_1^2 = \left(8+(8\times 2^{3/2})+18\right)-(2+8+9)$	M1
	$= 7+16\sqrt{2}$	M1 A1 (5)
		7
5.	**(a)**  Shape	B1
	Position	B1 (2)
	(b) $\left(0,\dfrac{1}{\sqrt{2}}\right), \quad \left(\dfrac{\pi}{4},0\right), \quad \left(\dfrac{5\pi}{4},0\right)$	B1 B1 B1 (3)
	(c) $\left(x+\dfrac{\pi}{4}=\right) \quad \dfrac{\pi}{3}$	B1
	Other value $\left(2\pi-\dfrac{\pi}{3}=\right) \quad \dfrac{5\pi}{3}$	M1
	Subtract $\dfrac{\pi}{4}$ $\quad x=\dfrac{\pi}{12},\ x=\dfrac{17\pi}{12}$	M1 A1 (4)
		9

Question Number	Scheme	Marks
6.	(a) $V = \pi r^2 h = 500, \qquad A = 2\pi rh + \pi r^2$	B1 M1
	$A = 2\pi r\left(\frac{500}{\pi r^2}\right) + \pi r^2 = \pi r^2 + \frac{1000}{r}$ *	M1 A1 (4)
	(b) $\frac{\mathrm{d}A}{\mathrm{d}r} = 2\pi r - 1000r^{-2}$	M1 A1
	$2\pi r - 1000r^{-2} = 0 \qquad r = \sqrt[3]{\frac{500}{\pi}} \qquad (\approx 5.42)$	M1 A1 (4)
	(c) $\frac{\mathrm{d}^2A}{\mathrm{d}r^2} = 2\pi + 2000r^{-3}, \qquad > 0$ therefore minimum	M1 A1√ (2)
	(d) $A = \pi r^2 + \frac{1000}{r} = 277$ (nearest integer)	M1 A1 (2)
		12

Question Number	Scheme	Marks
7.	**(a)** $\frac{5-(-3)}{8-2}=\frac{4}{3}$	M1 A1 (2)
	(b) $M:\ \left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$ (5,1)	M1 A1
	Gradient of *CM* is $-\frac{3}{4}$	B1√
	Equation of *CM*: $y-1=-\frac{3}{4}(x-5)$	M1 A1 (5)
	$(4y=-3x+19)$	
	(c) When x=4, $y=\frac{7}{4}$	M1 A1√ (2)
	(d) Radius = $\sqrt{(4-2)^2+\left(\frac{7}{4}+3\right)^2}$	M1 A1√
	$=\sqrt{4+\frac{361}{16}}=\sqrt{\frac{425}{16}}=\sqrt{\frac{25}{16}}\sqrt{17}=\frac{5\sqrt{17}}{4}$ *	M1 A1 (4)
		13

Question Number	Scheme	Marks
8.	**(a)** $x\left(x^2-6x+5\right)$	M1
	$=x\left(x-1\right)\left(x-5\right)$	M1 A1 (3)
	(b) 1 and 5	B1 √ (1)
	(c) $\frac{dy}{dx}=3x^2-12x+5$	M1 A1
	At $x=1$. $\frac{dy}{dx}=3-12+5=-4$	A1 (3)
	(d) $\int\left(x^3-6x^2+5x\right)dx=\frac{x^4}{4}-\frac{6x^3}{3}+\frac{5x^2}{2}$	M1 A1
	$\left[\ldots\ldots\right]_0^1=\frac{1}{4}-2+\frac{5}{2}$ $\left(=\frac{3}{4}\right)$ *R*	M1 A1 √
	Evaluating at 5: $\frac{625}{4}-250+\frac{125}{2}$ $\left(=-31\frac{1}{4}\right)$	A1
	To find *S*: $-31\frac{1}{4}-\frac{3}{4}=-32$	M1
	Total Area $=32+\frac{3}{4}=32\frac{3}{4}$	A1 (7)
		14

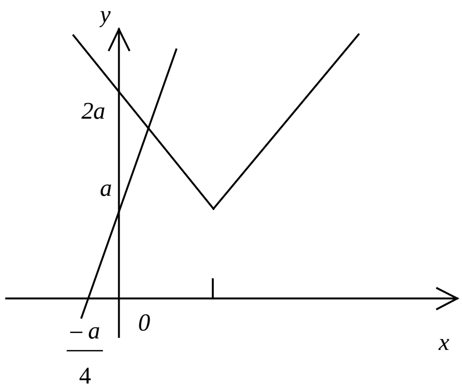
EDEXCEL PURE MATHEMATICS P2 (6672)
PROVISIONAL MARK SCHEME NOVEMBER 2003

Question Number	Scheme	Marks
1.	**(a)** $\frac{2}{x-3}+\frac{13}{(x-3)(x+7)}$	M1
	$=\frac{2(x+7)+13}{(x-3)(x+7)}=\frac{2x+27}{(x-3)(x+7)}$	M1 A1 3
	(b) $2x+27=x^2+4x-21$	M1
	$x^2+2x-48=(x+8)(x-6)=0$	
	$x=-8, 6$	M1 A1 3 **6**
2.	**(a)** 	
	Shape	B1
	domain, intercept	B1 2
	(b) $£800\times 1.04^{10}\approx £1184$ cao	M1 A1 2
	(c) $1.04^x=2$	M1
	$x=\frac{\ln 2}{\ln 1.04}\approx 18$ (years)	M1 A1 3 **7**

Question Number	Scheme	Marks
3.	**(a)** $1+nax, +\frac{n(n-1)}{2}(ax)^2+\frac{n(n-1)(n-2)}{6}(ax)^3+\ldots$ accept 2!, 3!	B1, B1 2
	(b) $na=8,\quad \frac{n(n-1)}{2}a^2=30$ both	M1
	$\frac{n(n-1)}{2}\cdot\frac{64}{n^2}=30,\quad \frac{\frac{8}{a}\left(\frac{8}{a}-1\right)a^2}{2}=30$ either	M1
	$n=16,\quad a=\frac{1}{2}$	A1, A1 4
	(c) $\frac{16.15.14}{6}\cdot\left(\frac{1}{2}\right)^3=70$	M1 A1 2 **8**
4.	**(a)** $\frac{8}{x}-x^2=0 \;\Rightarrow\; x^3=8 \;\Rightarrow\; x=2$	M1 A1 2
	(b) $\left(\frac{8}{x}-x^2\right)=x^4-16x+\frac{64}{x^2}$ M1 3(or 4) terms	M1 A1
	$\int\left(x^4-16x+64x^{-2}\right)\mathrm{d}x=\frac{x^5}{5}-8x^2-\frac{64}{x}$	M1 A1
	$\left[\frac{x^5}{5}-8x^2-\frac{64}{x}\right]_1^2=\left(\frac{32}{5}-32-32\right)-\left(\frac{1}{5}-8\right)-64$	M1 A1 ft
	Volume is $\frac{71}{5}\pi$ (units^3)	A1 7 **9**

Question Number	Scheme	Marks
5.	**(a)** $\dfrac{1-\tan^2\theta}{1+\tan^2\theta} = \dfrac{1-\dfrac{\sin^2\theta}{\cos^2\theta}}{1+\dfrac{\sin^2\theta}{\cos^2\theta}}$ $\left(\text{or } \dfrac{1-\dfrac{\sin^2\theta}{\cos^2\theta}}{\sec^2\theta} \text{ or equivalent}\right)$	M1 / M1
	$\dfrac{\cos^2\theta-\sin^2\theta}{\cos^2\theta+\sin^2\theta} = \dfrac{\cos 2\theta}{1} = \cos 2\theta$ * cso	M1 A1 4
	(b) $\theta = \dfrac{\pi}{8},\ \cos 2\theta = \dfrac{1}{\sqrt{2}}$	M1
	$\dfrac{1-t^2}{1+t^2} = \dfrac{1}{\sqrt{2}}$	M1
	$t^2 = \dfrac{\sqrt{2}-1}{\sqrt{2}+1}$	M1
	$= \dfrac{\sqrt{2}-1}{\sqrt{2}+1}\cdot\dfrac{\sqrt{2}-1}{\sqrt{2}-1} = 3-2\sqrt{2}$ * cso	M1 A1 5 **9**
	Alternative to 5(b)	
	$\dfrac{2t}{1-t^2} = \tan 2\theta = 1$	M1
	$t^2+2t-1=0$	M1
	$t=\sqrt{2}-1$	M1
	$t^2 = \left(\sqrt{2}-1\right)^2 = 3-2\sqrt{2}$ * cso	M1 A1 5

Question Number	Scheme		Marks
6.	**(a)** x: 0, $\frac{\pi}{6}$, $\frac{\pi}{3}$, $\frac{\pi}{2}$	1, 0	B1
	y: 1, 1.46, 1.42, 0	1.46, 1.42	B1, B1 <u>3</u>
	NB. Not giving 2 d.p. loses a maximum of one mark		
	(b) $I \approx \frac{1}{2}\left(\frac{\pi}{6}\right)\ldots$		B1
	$\approx \ldots(1+2(1.46+1.42)+0)$	ft their *ys*	M1 A1 ft
	≈ 1.8	accept 1.77	A1 <u>4</u>
	(c) underestimates		B1
	diagram or explanation		B1 <u>2</u> <u>**9**</u>
	NB. Exact answer is $\frac{1}{2}\left(e^{\frac{\pi}{2}}-1\right) \approx 1.905\ldots$		

Question Number	Scheme	Marks
7.	(a)	
	V shape right way up	B1
	vertex in first quadrant	B1
	g	B1
	-1 eeoo; $2a,\ a,\ -\frac{a}{4}$	B2 (1, 0) <u>5</u>
		
	(b) $4x + a = (a - x) + a$	M1
	$5x = a, \quad x = \frac{a}{5}$	M1
	$y = \frac{9a}{5}$ both correct	A1 <u>3</u>
	(c) $\text{fg}(x) = \lvert 4x + a - a \rvert + a = \lvert 4x \rvert + a$	M1 A1 <u>2</u>
	(d) $\lvert 4x \rvert + a = 3a \Rightarrow \lvert 4x \rvert = 2a$	M1 A1
	$x = \frac{a}{2}, -\frac{a}{2}$	A1, A1 <u>3</u> **<u>13</u>**

Question Number	Scheme	Marks
8.	**(a)** $\text{f}'(x) = \frac{3}{x} - \frac{1}{x^2}$	M1 A1
	$\frac{3}{x} - \frac{1}{x^2} = 0 \Rightarrow 3x^2 - x = 0 \Rightarrow x = \frac{1}{3}$	M1 A1 4
	(b) $y = 3\ln\left(\frac{1}{3}\right) + \frac{1}{\left(\frac{1}{3}\right)} = 3 - 3\ln 3 \quad (k = 3)$	M1 A1 2
	(c) $x = 1 \Rightarrow y = 1$	B1
	$\text{f}'(1) = 2 \Rightarrow m' = -\frac{1}{2}$	M1
	$y - 1 = -\frac{1}{2}(x - 1) \quad \left(y = -\frac{x}{2} + \frac{3}{2}\right)$	M1 A1 4
	(d) i $-\frac{x}{2} + \frac{3}{2} = 3\ln x + \frac{1}{x}$	M1
	leading to $6\ln x + x + \frac{2}{x} - 3 = 0$ * cso	A1
	ii g(0.13)=0.273…	
	g(0.14)=-0.370… Both, accept one d.p.	M1
	Sign change (and continuity) $\Rightarrow$ root $\in (0.13, 0.14)$	A1 4 **14**

Question	Mark Scheme	Marks
1. (a)	f (–2) = $(-2)^3-(19\times -2)-30$ M: Evaluate f (–2) or f (2)	M1
	f (–2) = 0, so (x + 2) is a factor	A1 (2)
	Alternative: $(x^3-19x-30)\div(x+2)=(x^2+ax+b)$, $a\neq 0$, $b\neq 0$ [M1]	
	$=(x^2-2x-15)$, so (x + 2) is a factor [A1]	
(b)	$(x^3-19x-30)=(x+2)(x^2-2x-15)$	M1 A1
	= (x + 2)(x + 3)(x – 5)	M1 A1 (4)
		(6)
2. (a)	$\frac{1}{2}r^2\theta=\frac{1}{2}\times 6.5^2\times 0.8=16.9$ (a.w.r.t. if changed to degrees)	M1 A1 (2)
(b)	$\sin 0.4=\frac{x}{6.5}$, x = 6.5 sin 0.4 , (where x is half of AB)	M1, A1
	(n.b. 0.8 rad = 45.8°)	
	AB = 2x = 5.06 (a.w.r.t.) (*)	A1 (3)
	Alternative: $AB^2=6.5^2+6.5^2-2\times 6.5\times 6.5\cos 0.8$ [M1]	
	$AB=\sqrt{6.5^2+6.5^2-2\times 6.5\times 6.5\cos 0.8}$ [A1]	
	AB = 5.06 [A1]	
(c)	$r\theta+5.06=(6.5\times 0.8)+5.06=10.26$ (a.w.r.t) (or 10.3)	M1 A1 (2)
		(7)
3.(a)	$(5p-8)-p=(3p+8)-(5p-8)$	M1
	Solve, showing steps, to get p = 4, or verify that p = 4 . (*)	A1 c.s.o. (2)
	Alternative: Using p = 4, finding terms (4, 12, 20), and indicating differences.[M1]	
	Equal differences + conclusion (or “common difference = 8”). [A1]	
(b)	a = 4 and d = 8 (stated or implied here or elsewhere).	B1
	$T_{40}=a+(n-1)d=4+(39\times 8)=316$	M1 A1 (3)
(c)	$S_n=\frac{1}{2}n[2a+(n-1)d]=\frac{1}{2}n[8+8(n-1)]$	M1 A1ft
	$=4n^2=(2n)^2$	A1 (3)
		(8)

Question	Mark Scheme	Marks
4.(a)	$b^2 - 4ac = (-k)^2 - 36 = k^2 - 36$	M1 A1
	Or, (completing the square), $\left(x - \frac{1}{2}k\right)^2 = \frac{1}{4}k^2 - 9$	
	Or, if b^2 and $4ac$ are compared directly, [M1] for finding both [A1] for k^2 and 36.	
	No real solutions: $k^2 - 36 < 0$, $-6 < k < 6$ (ft their "36")	M1, A1ft (4)
(b)	$x^2 - 4x + 9 = (x-2)^2$.......... ($p = 2$)	B1
	Ignore statement $p = -2$ if otherwise correct.	
	$x^2 - 4x + 9 = (x-2)^2 - 4 + 9 = (x-2)^2 + 5$ ($q = 5$) M: Attempting $(x \pm a)^2 \pm b \pm 9$, $a \neq 0$, $b \neq 0$.	M1 A1 (3)
(c)	Min value 5 (or just q), occurs where $x = 2$ (or just p)	B1ft, B1ft (2)
	<u>Alternative:</u> $f'(x) = 2x - 4$ (Min occurs where) $x = 2$ [B1]	
	Where $x = 2$, $f(x) = 5$ [B1ft]	
		(9)
5.(a)	$\sqrt{8} = 2\sqrt{2}$ seen or used somewhere (possibly implied).	B1
	$\frac{12}{\sqrt{8}} = \frac{12\sqrt{8}}{8}$ or $\frac{12}{2\sqrt{2}} = \frac{12\sqrt{2}}{4}$	
	Direct statement, e.g. $\frac{6}{\sqrt{2}} = 3\sqrt{2}$ (no indication of method) is M0.	M1
	At $x = 8$, $\frac{dy}{dx} = 3\sqrt{8} + \frac{12}{\sqrt{8}} = 6\sqrt{2} + 3\sqrt{2} = 9\sqrt{2}$ (*)	A1 (3)
(b)	Integrating: $\frac{3x^{3/2}}{\left(3/2\right)} + \frac{12x^{1/2}}{\left(1/2\right)} (+C)$ (C not required)	M1 A1 A1
	At (4, 30), $\frac{3 \times 4^{3/2}}{\left(3/2\right)} + \frac{12 \times 4^{1/2}}{\left(1/2\right)} + C = 30$ (C required)	M1
	$(f(x) =)$ $2x^{3/2} + 24x^{1/2}, -34$	A1, A1 (6)
		(9)

Question	Mark Scheme	Marks
6.(a)	(2, 0) (or $x = 2$, $y = 0$)	B1 (1)
(b)	$y^2 = 4\left(\frac{3y+12}{2} - 2\right)$ or $\left(\frac{2x-12}{3}\right)^2 = 4(x-2)$	M1
	$y^2 - 6y - 16 = 0$ or $x^2 - 21x + 54 = 0$ (or equiv. 3 terms)	A1
	$(y+2)(y-8) = 0$, $y = \ldots$ or $(x-3)(x-18) = 0$, $x = \ldots$ (3 term quad.)	M1
	$y = -2$, $y = 8$ or $x = 3$, $x = 18$	A1
	$x = 3$, $x = 18$ or $y = -2$, $y = 8$ (attempt <u>one</u> for M mark) (A1ft requires both values)	M1 A1ft (6)
(c)	Grad. of $AQ = \frac{8-0}{18-2}$, Grad. of $AP = \frac{0-(-2)}{2-3}$ (attempt <u>one</u> for M mark)	M1 A1ft
	$m_1 \times m_2 = \frac{1}{2} \times -2 = -1$, so $\angle PAQ$ is a right angle (A1 is c.s.o.)	M1 A1 (4)
	<u>Alternative:</u> Pythagoras: Find 2 lengths [M1] $AQ = \sqrt{320}$, $AP = \sqrt{5}$, $PQ = \sqrt{325}$ (O.K. unsimplified) [A1ft] (if decimal values only are given, with no working shown, require at least 1 d.p. accuracy for M1(implied) A1) $AQ^2 + AP^2 = PQ^2$, so $\angle PAQ$ is a right angle [M1, A1] M1 requires attempt to use Pythag. for right angle at A, and A1 requires correct <u>exact</u> working + conclusion.	**(11)**

Question	Mark Scheme	Marks
7.(a)	Solve $\frac{3}{2}x^2 - \frac{1}{4}x^3 = 0$ to find $p = 6$, or verify: $\frac{3}{2}\times 6^2 - \frac{1}{4}\times 6^3 = 0$ (*)	B1 (1)
(b)	$\frac{dy}{dx} = 3x - \frac{3x^2}{4}$	M1 A1
	$m = -9$, $y - 0 = -9(x-6)$ (Any correct form)	M1 A1 (4)
(c)	$3x - \frac{3x^2}{4} = 0$, $x = 4$	M1, A1ft (2)
(d)	$\int\left(\frac{3x^2}{2} - \frac{x^3}{4}\right)dx = \frac{x^3}{2} - \frac{x^4}{16}$ (Allow unsimplified versions)	M1 A1
	$[\ldots\ldots\ldots]_0^6 = \frac{6^3}{2} - \frac{6^4}{16} = 27$ M: Need 6 and 0 as limits.	M1 A1 (4)
		(11)
8.(a)	$\theta - 10 = 15$ $\theta = 25$ ($\cos(\theta - 10) = \cos\theta - \cos 10$, etc, is B0)	B1
	$\theta - 10 = 345$ $\theta = 355$ M: Using 360 – "15" (can be implied) Stating $\theta = 345$ scores M1 A0	M1 A1 (3)
	(Other methods: M1 for <u>complete</u> method, A1 for 25 and A1 for 355)	
(b)	$2\theta = 21.8\ldots$ (α) (At least 1 d.p.) (Could be implied by a correct θ).	B1
	$2\theta = \alpha + 180$ or $2\theta = \alpha + 360$ or $2\theta = \alpha + 540$ (One more solution)	M1
	$\theta = 10.9$, 100.9, 190.9, 280.9 (M1: divide by 2)	M1 A1ft A1
	(A1ft: 2 correct, ft their α) (A1: all 4 correct cao, at least 1 d .p.)	(5)
(c)	$2\sin\theta\left(\frac{\sin\theta}{\cos\theta}\right) = 3$, $2\sin^2\theta = 3\cos\theta$	M1, A1
	$2(1-\cos^2\theta) = 3\cos\theta$	M1
	$2\cos^2\theta + 3\cos\theta - 2 = 0$	
	$(2\cos\theta - 1)(\cos\theta + 2) = 0$ $\cos\theta = \frac{1}{2}$ (M: solve 3 term quadratic up to cos $\theta = \ldots$ or $x = \ldots$)	M1 A1
	$\theta = 60$, $\theta = 300$	A1 (6)
		(14)

Question number	Mark Scheme	Marks
1(a)	$2+\frac{3}{x+2}\ \left(=\frac{2(x+2)+3}{x+2}\right) \quad = \quad \underline{\underline{\frac{2x+7}{x+2}}}$ or $\frac{2(x+2)+3}{x+2}$	B1 **(1)**
(b)	$y = 2+\frac{3}{x+2}$ OR $y = \frac{2x+7}{x+2}$	
	$y-2 = \frac{3}{x+2}$ | $y(x+2) = 2x+7$, $yx - 2x = 7 - 2y$	M1
	$x+2 = \frac{3}{y-2}$ | $x(y-2) = 7-2y$	M1
	$x = \frac{3}{y-2} - 2$ | $x = \frac{7-2y}{y-2}$	
	$\therefore\ \underline{\underline{f^{-1}(x) = \frac{3}{x-2} - 2}}$ | $\underline{\underline{f^{-1}(x) = \frac{7-2x}{x-2}}}$ o.e	A1 **(3)**
(c)	Domain of $f^{-1}(x)$ is $x \in \mathbb{R}, x \neq 2$ $[\text{NB } x \neq +2]$	B1 (1) **(5)**

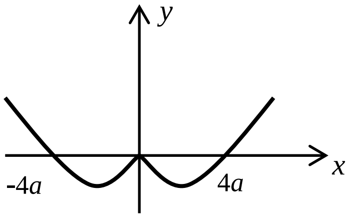
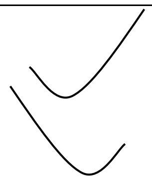
	Notes	
1(b)	M1	y = f (x) and <u>1st step</u> towards x = .
	M1	One step from x = .
	A1	y or f^{-1} (x) = in terms of x.

Question number	Mark scheme	Marks
2(a)	$u_2 = \sqrt{\left(\frac{3}{2}+\frac{20}{3}\right)}$ = 2.85773…. = $\underline{\underline{2.86}}$	M1
	$u_3 =$ 2.90300… = $\underline{\underline{2.90}}$	A1 c.a.o
	$u_4 =$ 2.88806…. = $\underline{\underline{2.89}}$	A1 c.a.o
S.C.	[If u_3 = AWRT 2.90 and u_4 = AWRT 2.89 penalise once only]	(3)
(b)	(i) $3 = \sqrt{\left(\frac{3}{2}+\frac{a}{3}\right)}$ or $9 = \frac{3}{2}+\frac{a}{3}$	M1
	$\frac{a}{3} = 9 - \frac{3}{2}$ or $a = 3\left(9-\frac{3}{2}\right)$	M1
	$\underline{\underline{a = 22.5}}$	A1 (3)
	(ii) (If $u_1 = u_2$, then $u_2 = u_3$……..) $u_5 \underline{\underline{=3}}$	B1 (1) **(7)**

	Notes	
2(a)	M1	Correct expression or AWRT 2.86
(b)(i)	M1 M1	A correct equation for a, with or without $\sqrt{\ }$. Attempt correct manipulation to $ka = \ ,(k>0)$.

Question number	Mark Scheme	Marks
3(a)	$\log_2(16x) = \log_2 16 + \log_2 x$	M1
	$= \underline{\underline{4 + a}}$	A1 c.a.o (2)
(b)	$\log_2\left(\frac{x^4}{2}\right) = \log_2 x^4 - \log_2 2$	M1
	$= 4\log_2 x - \log_2 2$	M1
	$= \underline{\underline{4a - 1}}$ (accept $\underline{\underline{4\log_2 x - 1}}$)	A1 (3)
(c)	$\frac{1}{2} = 4 + a - (4a - 1)$	M1
	$a = \frac{3}{2}$	A1
	$\log_2 x = \frac{3}{2} \Rightarrow x = 2^{\frac{3}{2}}$	M1
	$\underline{\underline{x = \sqrt{8} \text{ or } 2\sqrt{2}}}$ or $\underline{\underline{\sqrt{2^3} \text{ or } (\sqrt{2})^3}}$	A1 √ (4) **(9)**

	Notes	
3(a)	M1	Correct use of $\log(ab) = \log a + \log b$
(b)	M1	Correct use of $\log\left(\frac{a}{b}\right) = \cdots$
	M1	Use of $\log x^n = n\log x$
(c)	M1	Use their (a)&(b) to form equ in a
	M1	Out of logs: $x = 2^a$
	A1 √	Must write x in surd form, follow through their rational a.

Question number	Mark Scheme	Marks
4(a)	  $x>0$ $x<0$ $(4a,0)$ & $(-4a,0)$ and shape at $(0,0)$	B1 B1 √ B1 (3)
(b)	$f(2a)=(2a)^2-4a(2a) \quad = 4a^2-8a^2 = \underline{\underline{-4a^2}}$ $f(-2a)[= \ f(2a) \ (\because \textit{even function}) \quad] = \underline{\underline{-4a^2}}$	B1 B1 √ (2)
(c)	$a=3$ and $f(x) = 45 \Rightarrow 45 = x^2-12x \qquad (x>0)$ $0 = x^2-12x-45$ $0=(x-15)(x+3)$ $x=15 \quad (or -3)$ $\therefore$Solutions are $\underline{\underline{x=\pm 15}}$ <u>only</u>	M1 M1 A1 A1 (4) **(9)**

	Notes	
4(b)	B1 √	√ their $f(2a)$
(c)	M1 M1 A1 A1	Attempt 3TQ in x Attempt to solve At least $x=15$ can ignore $x=-3$ To get final A1 must make clear <u>only</u> answers are ±15.

Question number	Mark Scheme	Marks
5(a)(i)	$\underline{\underline{x = a^y}}$	B1 (1)
(ii)	In both sides of (i) i.e $\ln x = \ln a^y$ $= \underline{y \ln a}$ * or (y =) $\log_a x = \frac{\ln x}{\ln a}$ $\Rightarrow y \ln a = \ln x$	B1cso (1)
(b)	$y = \frac{1}{\ln a} \bullet \ln x$,$\Rightarrow$ $\frac{dy}{dx}, = \underline{\frac{1}{\ln a} \times \frac{1}{x}}$ *	M1, A1cso
ALT.	$\left[or \quad \frac{1}{x} = \frac{dy}{dx} \cdot \ln a \quad ,\Rightarrow \underline{\frac{dy}{dx} = \frac{1}{x \ln a}} \quad * \right]$	(2)
(c)	$\log_{10} 10 = 1$ $\Rightarrow$ A is $(10, \underline{1})$ $y_A = 1$	B1
	from(b) $m = \underline{\frac{1}{10 \ln a} \quad or \quad \frac{1}{10 \ln 10}}$ or 0.043 (or better)	B1
	equ of target $y - 1 = m(x - 10)$	M1
	i.e $\underline{y - 1 = \frac{1}{10 \ln 10}(x - 10) \ or \ y = \frac{1}{10 \ln 10} x + 1 - \frac{1}{\ln 10}}$ (o.e)	A1 (4)
(d)	$y = 0 \, in (c) \Rightarrow 0 = \frac{x}{10 \ln 10} + 1 - \frac{1}{\ln 10}$ $\Rightarrow x, = 10 \ln 10 \left(\frac{1}{\ln 10} - 1 \right)$	M1
	$\underline{x = 10 - 10 \ln 10 \quad or \quad 10(1 - \ln 10)}$ or $\underline{10 \ln 10 (\frac{1}{\ln 10} - 1)}$	A1 (2) **(10)**

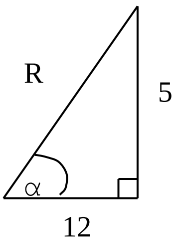
	Notes	
5(a)	B1	$x = e^{y \ln a}$ *is BO*
	B1	Must see $\ln a^y$ or use of change of base formula.
(b)	M1, A1cso	M1 needs some correct attempt at differentiating.
(c)	B1 M1	Allow either √ their y_A and m
(d)	M1	Attempt to solve correct equation. Allow if a not = 10.

Question number	Mark Scheme	Marks
6(a)	$f^1(x)=0$ for maximum (or stationary point or turning point)	B1
	$f^1(1.48) = e^{1.48} - 2\times1.48^2 = 0.0121\ldots$ $f^1(1.49) = \qquad = -0.0031\ldots$	M1
	change of sign $\therefore$ root / maximum in range	A1 (3)
(b)	$y = e^x - \frac{2}{3}x^3 \ (+c)$	M1 A1
	at $(0,5)$ $\quad 5 = e^0 - 0 + c$	M1
	c = 4 $\quad \left(y = e^x - \frac{2}{3}x^3 + 4\right)$ $\quad (c=4)$	A1 (4)
(c)	Area $= \int_0^2 \left(e^x - \frac{2}{3}x^3 + 4\right)dx$	M1
	$= \left[e^x - \frac{2}{12}x^4 + 4x\right]_0^2$	A1 √
	$= \left(e^2 - \frac{16}{6} + 8\right) - \left(e^0 - 0 + 0\right)$	M1
	$= e^2 + 4\frac{1}{3} \quad or \quad e^2 + \frac{13}{3}$	A1 cao (4) **(11)**

	Notes	
6(a)	M1 M1 A1	May be $\Rightarrow$ if maximum mentioned at A1 One value correct to 1 S.F. Both correct and comment
(b)	M1 A1 M1	Some correct $\int$ $e^x - \frac{2}{3}x^3$ Attempt to use (0,5) No + c is M0
(c)	M1 A1 √ M1	Some correct $\int$ <u>other</u> than $e^x \rightarrow e^x$. [ ] their $c(\neq 0)$. Attempt both limits

Question number	Mark Scheme	Marks
7(a)	4, 4.84, 7.06	B2/1/0 (2)
(b)	$I \approx \frac{1}{2} \times 0.25\left[6.06 + 7.06 + 2\left(4.32 + 4 + 4.84\right)\right]$ $= \frac{1}{2} \times 0.25\left[39.44\right]$ $= \underline{\underline{4.93}}$ or $4.\underline{\underline{9}}$ (AWRT 4.93 or just 4.9)	B1 [M1 A1]√ A1 (4)
(c)	$\int_{0.5}^{1.5}\left(\frac{3}{x} + x^4\right)dx = \left[3\ln x + \frac{1}{5}x^5\right]_{0.5}^{1.5}$ $= \left(3\ln 1.5 + \frac{1}{5}1.5^5\right) - \left(3\ln 0.5 + \frac{1}{5}0.5^5\right)$ $= 3\ln 3 + 1.5125$ *or* $3\ln 3 + \frac{121}{80}$	M1 A1 M1 A1 (4)
(d)	$\frac{[4.93 - (c)]}{(c)} \times 100$, = 2.53% (i.e < 3%) AWRT 2.5%	M1, A1 (2) **(12)**

	Notes	
7(b)	B1	$\frac{1}{2} \times 0.25$
	M1 A1	√[ ]
(c)	M1	Some correct $\int$
	A1	$3\ln x + \frac{1}{5}x^5$
	M1	Use of limits

Question number	Mark Scheme	Marks
8(a)(i)	$12\cos\theta - 5\sin\theta = R\cos\theta\cos\alpha - R\sin\theta\sin\alpha.$  $R^2 = 5^2 + 12^2$, $\Rightarrow \underline{R = 13}$ $\tan\alpha = \dfrac{5}{12}$, $\Rightarrow \alpha = 22.6^\circ$ (AWRT 22.6) or 0.39^c (AWRT 0.39^c)	M1, A1 M1, A1 (4)
(b)	$\cos(\theta + 22.6) = \dfrac{4}{13}$ $\theta + 22.6 = 72.1$, $\underline{\underline{\theta = 49.5}}$ (only)	M1 M1 A1 (3)
(ii)	$\dfrac{8}{\tan\theta} - 3\tan\theta = 2$ i.e. $0 = 3\tan^2\theta + 2\tan\theta - 8$ $0 = (3\tan\theta - 4)(\tan\theta + 2)$ $\tan\theta = \dfrac{4}{3}$ *or* -2 $\tan\theta = \dfrac{4}{3} \Rightarrow \underline{\underline{\theta = 53.1}}$ [*ignore* θ *not in range e.g.* $\theta = 116.6$]	M1 M1 M1 A1 A1 (5) **(12)**

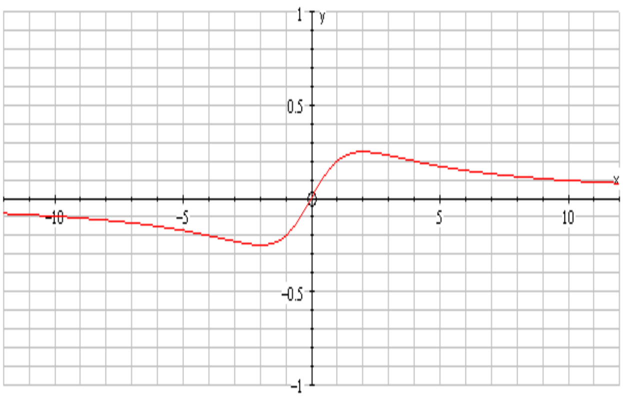
	Notes	
8(a)(i)	M1, A1 M1, A1	M1 for correct expression for R or R^2 M1 for correct trig expression for α
(b)	M1 M1	$\cos(\theta + \alpha) = \dfrac{4}{R}$ $\theta + \alpha = \cdots$ their R
(ii)	M1 M1 M1 A1	Use of cot $\theta = \dfrac{1}{\tan\theta}$ 3TQ in tan $\theta = 0$ Attempt to solve 3TQ = 0 For Final A mark must deal with tan $\theta = -2$

Question Number	Scheme	Marks
1.	**Either** Obtains centre (0, 6.5) Finds radius or diameter by Pythagoras Theorem, to obtain r = 2.5 or $r^2 = 6.25$ $x^2 + (y-6.5)^2 = 2.5^2$ or $x^2 + y^2 - 13y + 36 = 0$	B1 M1, A1 B1 (4)
	Or $\frac{y-8}{x+2} \times \frac{y-5}{x-2} = -1$ Gradients multiplied and put = to -1 $x^2 + y^2 - 13y + 36 = 0$	B1 M1A1 B1 (4)
	Or Obtains centre (0, 6.5) $x^2 + (y-6.5)^2 = r^2$ or $x^2 + y^2 - 13y + c = 0$ substitutes either (2 , 5) or (-2 , 8) $x^2 + (y-6.5)^2 = 2.5^2$ or $x^2 + y^2 - 13y + 36 = 0$	B1 B1 M1 A1 (4)
2. (a)	$na = -6,$ $\frac{n(n-1)}{2}a^2 = 27$ Attempts solution by eliminating variable e.g. $\frac{n(n-1)36}{n^2} = 54$ or $-\frac{6}{a}(-\frac{6}{a}-1)a^2 = 54$ *n* = -2, $a = 3$	B1, B1 M1 A1, A1 (5)
(b)	$\frac{(-2)(-3)(-4)3^3}{6} = -108$ for M1 allow a instead of a^3	M1 A1 (2)
(c)	$\lvert x\rvert < \frac{1}{3}$ or $-\frac{1}{3} < x < \frac{1}{3}$ f.t. on $\frac{1}{a}$	B1 f.t. (1)

Question Number	Scheme	Marks
3. (a)	$10x, +(2y + 2x\frac{dy}{dx}), -6y\frac{dy}{dx} = 0$	M1,(B1),A1
	At (1 , 2) $10 + (4 + 2\frac{dy}{dx}) - 12\frac{dy}{dx} = 0$	M1
	$\therefore \frac{dy}{dx} = \frac{14}{10} = 1.4 \ or \ \frac{7}{5} \ or \ 1\frac{2}{5}$	A1 (5)
(b)	The gradient of the normal is $-\frac{5}{7}$	M1
	Its equation is $y - 2 = -\frac{5}{7}(x - 1)$ (allow tangent)	M1
	$y = -\frac{5}{7}x + 2\frac{5}{7}$ or $y = -\frac{5}{7}x + \frac{19}{7}$	A1cao (3)
4. (a)	Uses the remainder theorem with $x = \frac{1}{2}$, or long division, and puts remainder = 0	M1
	To obtain $p + 2q = -35$ or any correct equivalent (allow more than 3 terms)	A1
	Uses the remainder theorem with $x = 1$, or long division, and puts remainder = ± 7	M1
	To obtain $p + q = -21$ or any correct equivalent (allow more than 3 terms)	A1
	Solves simultaneous equations to give *p* = -7, and *q* = - 14	M1 A1 (6)
(b)	Then $6x^3 - 7x^2 - 14x + 8 = (2x - 1)\left(3x^2 - 2x - 8\right)$	M1 A1 ft
	So f(x) = (2x – 1)(3x + 4)(x – 2)	B1 (3)

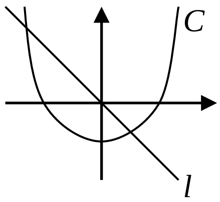
Question Number	Scheme	Marks
5. (a)	Area of triangle = $\frac{1}{2}\times 30\times 3\pi^2$ (= 444.132) Accept 440 or 450	B1 (1)
(b)	**Either** Area shaded = $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 30\sin 2t.32t\mathrm{d}t$	M1 A1
	$=\left[-480t\cos 2t+\int 480\cos 2t\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$	M1 A1
	$=\left[-480t\cos 2t+240\sin 2t\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$	A1 ft
	$=240(\pi-1)$	M1A1 (7)
	or $\int_{\frac{\pi}{2}}^{\frac{\pi}{4}} 60\cos 2t.(16t^2\text{-}\pi^2)\mathrm{d}t$	M1 A1
	$=\left[(30\sin 2t(\pi^2-16t^2))-480t\cos 2t+\int 480\cos 2t\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$	M1 A1
	$=\left[-480t\cos 2t+240\sin 2t\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$	A1 ft
	$=240(\pi-1)$	M1 A1 (7)
(c)	Percentage error = $\dfrac{240(\pi-1)-estimate}{240(\pi-1)}\times 100$ = 13.6% (Accept answers in the range 12.4% to 14.4%)	M1 A1 (2)

Question Number	Scheme	Marks
6. (a)	Uses $\frac{A}{(2x-3)}+\frac{B}{(x+1)}$	M1
	Considers –2x + 13 = A (x + 1) + B (2x –3) and substitutes x = -1 **or** x = 1.5, or compares coefficients and solves simultaneous equations	M1
	To obtain A = 4 and B = -3.	A1, A1 (4)
(b)	Separates variables $\int \frac{1}{y}\,\mathrm{d}y = \int \frac{4}{2x-3}-\frac{3}{x+1}\,\mathrm{d}x$	M1
	$\ln y = 2\ln(2x-3) - 3\ln(x+1) + C$	A1, B1 ft
	Substitutes to give $\ln 4 = 2\ln 1 - 3\ln 3 + C$ and finds C (ln 108)	M1
	$\ln y = \ln(2x-3)^2 - \ln(x+1)^3\ (+\ln 108)$	M1
	$= \ln\frac{C(2x-3)^2}{(x+1)^3}$	A1
	$\therefore y = \frac{108(2x-3)^2}{(x+1)^3}$	A1 cso (7)
	Or $y = e^{2\ln(2x-3)-3\ln(x+)+\ln 108}$ special case M1 A2	

Question Number	Scheme	Marks
7. (a)	$\frac{dy}{dx} = \frac{(4+x^2) - x(2x)}{(4+x^2)^2}$ or (fr Need numerical answers for M1 $x^2)^{-2}$	M1 A1
	Solve $\frac{dy}{dx}$=0 to obtain (2, ¼) , and (-2, -¼) or (2 and –2 A1, full solution A1)	M1 A1, A1
		(5)
(b)	When x = 2, $\frac{d^2y}{dx^2}$=-0.0625 <0 thus maximum	B1
		M1
	When x = -2, $\frac{d^2y}{dx^2}$=0.0625 >0 thus minimum.	B1
		(3)
(c)		
	Shape for $-2 \leq x \leq 2$	B1
	Shape for $x > 2$	B1
	Shape for $x < 2$	B1
		(3)

Question Number	Scheme	Marks
8. (a)	Any two of $1+\lambda=-2+$ … ; $3+2\lambda=3+$ … ; $5-\lambda=-4+4\mu$ [Need two of these for M1]	M1
	Solve simultaneous equations to obtain μ = 2, **or** λ = 1	M1 A1
	$\therefore$ intersect at (2 , 5 , 4)	M1 A1
	Check in the third equation or on second line	B1 (6)
(b)	$1\times2+2\times1+(-1)\times4=0$ $\therefore$ perpendicular	M1 A1 (2)
(c)	P is the point (3, 7, 3) [i.e. λ = 2] and R is the point (4, 6, 8) [i.e. μ = 3]	M1 A1
	$PQ=\sqrt{1^2+2^2+(-1)^2}=\sqrt{6}$ $RQ=\sqrt{2^2+1^2+4^2}=\sqrt{21}$ $PR=\sqrt{27}$	M1 A1 ft
	The area of the triangle = $\frac{1}{2}\times\sqrt{6}\times\sqrt{21}=\dfrac{3\sqrt{14}}{2}$	M1 A1 (6)
	Or area = $\frac{1}{2}\times\sqrt{6}\times\sqrt{27}\sin P$ where sin *P* = $\dfrac{\sqrt{7}}{3}$ = $\dfrac{3\sqrt{14}}{2}$	
	Or area = $\frac{1}{2}\times\sqrt{21}\times\sqrt{27}\sin R$ where sin *R* = $\dfrac{\sqrt{2}}{3}$ = $\dfrac{3\sqrt{14}}{2}$ *(must be simplified)*	

Question number	Scheme	Marks
1.	(a) $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{1+5}{2},\frac{2+8}{2}\right)=(3,5)$	M1 A1 (2)
	(b) Gradient = $\frac{y_2-y_1}{x_2-x_1}=\frac{8-2}{5-1}$	M1 A1
	$y-2=m(x-1)$ $y=\frac{3}{2}x+\frac{1}{2}$	M1 A1 (4)
	Allow $y=\frac{3x+1}{2}$ or $y=\frac{1}{2}(3x+1)$	**6**
2.	(a) $\sqrt{8}=\sqrt{4}\sqrt{2}=2\sqrt{2}$ (seen or implied)	B1
	$(3-\sqrt{8})(3-\sqrt{8})=9-6\sqrt{8}+8=17-12\sqrt{2}$	M1 A1 (3)
	(b) $\frac{1}{4-\sqrt{8}}\times\frac{4+\sqrt{8}}{4+\sqrt{8}}$, $=\frac{4+\sqrt{8}}{16-8}=\frac{1}{2}+\frac{1}{4}\sqrt{2}$	M1, M1 A1 (3)
	Allow $\frac{1}{4}(2+\sqrt{2})$ or equiv. (in terms of $\sqrt{2}$)	**6**
3.	(a) $2x+2(x+20)<300$ (Using x – 20 is A0)	M1 A1 (2)
	(b) $x(x+20)>4800$ (Using x – 20 is A0)	M1 A1 (2)
	(c) 65 (i.e. Allow wrong inequality sign or x = 65).	B1ft
	Solving 3 term quadratic, $(x+80)(x-60)=0$ $x=\ldots$	M1
	$x>60$ ($x<-80$ may be included here, but there must be no other wrong solution to the quadratic inequality such as $x>-80$)	A1
	$60<x<65$	A1 (4)
		8

Question number	Scheme	Marks
4.	(a)  C : "U" shape	B1
	C : Position	B1
	l : Straight line through origin with negative gradient	B1 (3)
	(b) (2, 0), (–2, 0), (0, – 4) 2 of these correct:	B1
	All 3 correct:	B1 (2)
	(c) $x^2 - 4 = -3x$	
	$x^2 + 3x - 4 = 0$ $(x+4)(x-1) = 0$ $x = ...$	M1
	$x = -4$ $x = 1$	A1
	$y = 12$ $y = -3$ M: Attempt one y value	M1 A1 (4)
		9
5.	(a) $\tan x = \frac{8}{3}$ (or exact equivalent, or 3 s.f. or better)	B1 (1)
	(b) $\tan x = \frac{8}{3}$ $x = 69.4°$ (α), $x = 249.4°$ ($180 + \alpha$)	M1 A1, A1ft (3)
	(c) $3(1 - \cos^2 y) - 8\cos y = 0$ $3\cos^2 y + 8\cos y - 3 = 0$	M1 A1
	$(3\cos y - 1)(\cos y + 3) = 0$ $\cos y = ...,$ $\frac{1}{3}$ (or –3)	M1 A1
	$y = 70.5°$ (β), $x = 289.5°$ ($360 - \beta$)	A1 A1ft (6)
		10

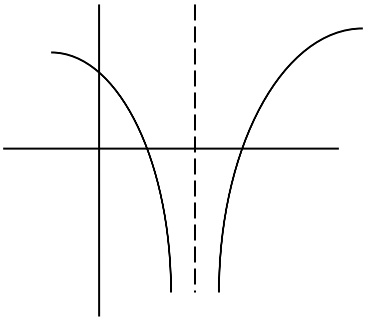
Question number	Scheme	Marks	
6.	(a) $(x^4 - 6x^2 + 9)$	M1	
	$(x^4 - 6x^2 + 9) \div x^3 = x - 6x^{-1} + 9x^{-3}$ (*)	A1	(2)
	(b) $f'(x) = 1 + 6x^{-2} - 27x^{-4}$ First A1: 2 terms correct (unsimplified)	M1 A1 A1	(3)
	Second A1: all 3 correct (simplified)		
	(c) When $x = \pm\sqrt{3}$, $f'(x) = 1 + \frac{6}{(\sqrt{3})^2} - \frac{27}{(\sqrt{3})^4}$	M1	
	$\left(= 1 + \frac{6}{3} - \frac{27}{9}\right)$ $= 0$, $\therefore$ Stationary	A1	(2)
	(d) $f''(x) = -12x^{-3} + 108x^{-5}$ M: Attempt to diff. $f'(x)$, <u>not</u> $g(x)\,f'(x)$.	M1	
	$f''(\sqrt{3}) = -\frac{12}{(\sqrt{3})^3} + \frac{108}{(\sqrt{3})^5}$ $(\approx -2.309 + 6.928 = 4.619)\left(= \frac{8}{\sqrt{3}}\right)$	A1	
	> 0, $\therefore$ Minimum (not dependent on a numerical version of $f''(x)$)	A1ft	(3)
			10
7.	(a) $(S =)\ a + ar + ... + ar^{n-1}$ "$S =$" not required. Addition required.	B1	
	$(rS =)\ ar + ar^2 + ... + ar^n$ "$rS =$" not required (M: Multiply by r)	M1	
	$S(1 - r) = a(1 - r^n)$ $S = \frac{a(1 - r^n)}{1 - r}$ (M: Subtract and factorise each side) (*)	M1 A1	(4)
	(b) $r = 0.9$	B1	
	$S_{20} = \frac{10(1 - 0.9^{20})}{1 - 0.9} = 87.8$	M1 A1	(3)
	(c) Sum to infinity $= \frac{a}{1 - r} = \frac{10}{1 - 0.9} = 100$ (ft only for $\lvert r \rvert < 1$)	M1 A1ft	(2)
	(d) $\frac{a}{1 - r} = \frac{r}{1 - r} = 10$ (Put $a = r$ in the formula from (c), and equate to 10)	M1	
	$r = 10(1 - r)$ $r = ...,$ $\frac{10}{11}$ (or exact equivalent)	M1, A1	(3)
			12

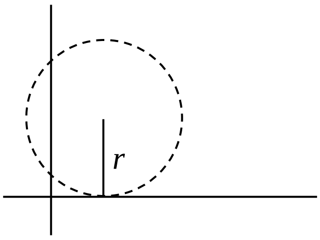
Question number	Scheme	Marks
8.	(a) $\frac{dy}{dx} = 3x^2 - 14x + 15$	M1 A1 (2)
	(b) $3x^2 - 14x + 15 = 0$	M1
	$(3x-5)(x-3) = 0$ $x = ...,\ 3$ (A1 requires <u>correct</u> quadratic factors).	M1, A1
	$y = 12$ (Following from $x = 3$)	A1 (4)
	(c) P: $x = 1$ $y = 12$	B1
	Same y-coord. as Q (or "zero gradient"), so PQ is parallel to the x-axis	B1 (2)
	(d) $\int (x^3 - 7x^2 + 15x + 3)dx = \frac{x^4}{4} - \frac{7x^3}{3} + \frac{15x^2}{2} + 3x$	M1 A1 A1
	(First A1: 3 terms correct, Second A1: all correct)	
	$\left[\frac{x^4}{4} - \frac{7x^3}{3} + \frac{15x^2}{2} + 3x\right]_1^3 = \left(\frac{81}{4} - 63 + \frac{135}{2} + 9\right) - \left(\frac{1}{4} - \frac{7}{3} + \frac{15}{2} + 3\right)$	M1
	$\left(33\frac{3}{4} - 8\frac{5}{12}\right) - 24 = 25\frac{1}{3} - (2 \times 12) = 1\frac{1}{3}$ (or equiv. or 3 s.f or better)	M1 A1 (6)
		14

Question Number	Scheme	Marks
1.	$\dfrac{(x-3)(x-5)}{(x-3)(x+3)} \times \dfrac{2x(x+3)}{(x-5)^2}$ (3 × factorising)	B1 B1 B1
	$= \dfrac{2x}{x-5}$	B1
		(4 marks)
2. (i)	A correct form of $\cos 2x$ used	M1
	$1-2\left(\frac{3}{5}\right)^2$ or $\left(\frac{4}{5}\right)^2-\left(\frac{3}{5}\right)^2$ or $2\left(\frac{4}{5}\right)^2-1$ $\left\{\frac{7}{25}\right\}$	A1
	$\sec 2x = \dfrac{1}{\cos 2x}$; $= \frac{25}{7}$ or $3\frac{4}{7}$	M1A1 (4)
(ii)	(*a*) $\dfrac{\cos 2x}{\sin 2x} + \dfrac{1}{\sin 2x}$ or (*b*) $\dfrac{1}{\tan 2x} + \dfrac{1}{\sin 2x}$	M1
	Forming single fraction (or multiplying both sides by $\sin 2x$)	M1
	Use of correct trig. formulae throughout and producing expression in terms of $\sin x$ and $\cos x$	M1
	Completion (cso) e.g. $\dfrac{2\cos^2 x}{2\sin x\cos x} = \dfrac{\cos x}{\sin x} = \cot x$ **(*)**	A1 (4)
		(8 marks)
3. (*a*)	$(x^3)^{12}$; ... $+\binom{12}{1}(x^3)^{11}\left((-)\frac{1}{2x}\right) + \binom{12}{2}(x^3)^{10}\left((-)\frac{1}{2x}\right)^2 + ...$	B1; M1
	[For M1, needs binomial coefficients, ${}^nC_r$ form OK, at least as far as shown]	
	Correct values for ${}^nC_r\,s$: 12, 66, 220 used (may be implied)	B1
	$(x^3)^{12} + 12(x^3)^{11}\left(-\frac{1}{2x}\right) + 66(x^3)^{10}\left(-\frac{1}{2x}\right)^2 + 220(x^3)^9\left(-\frac{1}{2x}\right)^3 ...$	
	$x^{36} - 6x^{32} + \frac{33}{2}x^{28} - \frac{55}{2}x^{24}$	A2(1,0) (5)
(*b*)	Term involving $(x^3)^3\left((-)\frac{1}{2x}\right)^9$;	M1
	coeff $= \dfrac{12.11.10}{3.2.1}\left((-)\frac{1}{2}\right)^9$	A1
	$= -\dfrac{55}{128}$ (or –0.4296875)	A1 (3)
		(8 marks)

Question Number	Scheme	Marks
4. (*a*)	$y^2 = \left(\dfrac{x+2}{\sqrt{x}}\right)^2 = \dfrac{x^2+4x+4}{x} = x + 4 + \dfrac{4}{x}$	M1A1
	$\pi\int y^2 \mathrm{d}x$ [dependent on attempt at squaring y]	B1
	$\int y^2 \mathrm{d}x = \int (\dfrac{x^2+4x+4}{x})\mathrm{d}x;\ = \dfrac{x^2}{2} + 4x + 4\ln x$	M1;A1 ft
	[A1√ must have ln x term]	
	Correct use of limits: $[\ ]_1^4 = [\ ]^4 - [\ ]_1$	M1
	[M dependent on prev. M1]	
	Volume = $\left(\dfrac{39}{2} + 4\ln 4\right)\pi$ or equivalent exact	A1 (7)
(*b*)	Showing that $y = 3$ at $x = 1$ and $x = 4$	B1 (1)
(*c*)	Volume = $2^3 \times$ answer to (a) ; = 629.5 cm^3 $\approx$ 630 cm^3 (*)	M1;A1 (2)
	[allow 629 – 630]	**(10 marks)**
5. (*a*)	Attempting to reach at least the stage $x^2(x+1) = 4x+1$	M1
	Conclusion (no errors seen) $x = \sqrt{\dfrac{4x+1}{x+1}}$ (*)	A1 (2)
	[Reverse process: need to square and clear fractions for M1]	
(*b*)	$x_2 = \sqrt{\dfrac{4+1}{1+1}} =$ 1.58…	M1
	$x_3 = 1.68,\quad x_4 = 1.70$	A1A1 (3)
	[Max. deduction of 1 for more than 2 d.p.]	
(*c*)	Suitable interval; e.g. [1.695, 1.705] (or “tighter”)	M1
	f(1.695) = –0.037… , f(1.705) = +0.0435…	Dep. M1
	Change of sign, no errors seen, so root = 1.70 (correct to 2 d.p.)	A1 (3)
(*d*)	$x = -1$, “division by zero not possible”, or equivalent	B1,B1 (2)
	or **any number in interval –1 < *x* < –¼,** “square root of neg. no.”	
		(10 marks)

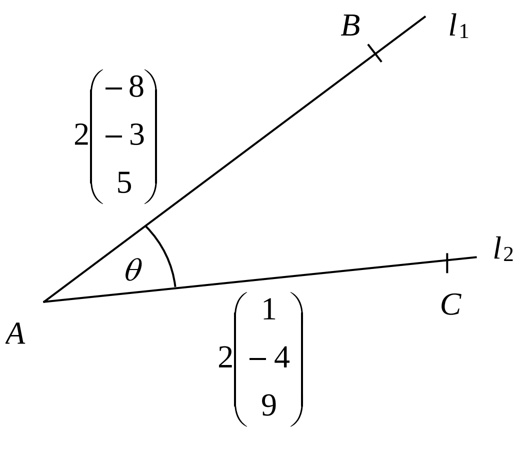
Question Number	Scheme	Marks
6. (*a*)	$\log_5 x^2 - \log_5 y$; $= 2\log_5 x - \log_5 y = 2a - b$	M1A1 (2)
(*b*)	$\log_5 25 = 2$ or $\log_5 y$	B1
	$\log_5 25 + \log_5 x + \log_5 y^{\frac{1}{2}}$; $= 2 + a + ½ b$	M1;A1 (3)
(*c*)	$2a - b = 1$, $2 + a + ½ b = 1$ (must be in *a* and *b*)	B1 ft (1)
(*d*)	Using both **correct** equations to show that $a = –0.25$ (*)	M1
	$b = –1.5$	B1 (2)
	[Mark for (c) can be gained in (d)]	
(*e*)	Using correct method to find a value for *x* or a value of *y*:	M1
	$x = 5^{-0.25} = 0.669$, $y = 5^{-1.5} = 0.089$	A1 A1 ft (3)
	[max. penalty –1 for more than 3 d.p.]	**(11 marks)**
7. (*a*)	Differentiating; $\text{f}'(x) = 1 + \frac{e^x}{5}$	M1;A1 (2)
(*b*)	*A*: $\left(0, \frac{1}{5}\right)$	B1
	Attempt at $y - \text{f}(0) = \text{f}'(0)x$;	M1
	$y - \frac{1}{5} = \frac{6}{5}x$ or equivalent "one line" 3 termed equation	A1 ft (3)
(*c*)	1.24, 1.55, 1.86	B2(1,0) (2)
(*d*)	Estimate = $\frac{0.5}{2}$; (×) [(0.45 + 1.86) + 2(0.91 + 1.24 + 1.55)]	B1 M1 A1 ft
	= 2.4275 $\left(\begin{matrix}2.428\\2.429\end{matrix}, 2.43\right)$	A1 (4)
		(11 marks)

Question Number	Scheme	Marks
8. (*a*)	$y = \ln(3x - 6) \quad \Rightarrow \ 3x - 6 = e^{y}$	M1
	$\Rightarrow x = \dfrac{e^{y} + 6}{3}; \qquad \{f^{-1}(x)\} = \dfrac{e^{x} + 6}{3}$	M1;A1 (3)
(*b*)	Domain: $x \in \Re$	B1
	Range: $f^{-1}(x) > 2$	B1 (2)
(*c*)	Attempting to find $f^{-1}(3)$ $[= \dfrac{e^{3} + 6}{3}]$; $= 8.70$	M1;A1 (2)
(*d*)		
	ln curve passing through $y = 0$	B1
	Symmetry in $x = k$, $k > 0$	M1
	All correct and asymptote at $x = 2$ labelled	A1
(*e*)	Meets *y*-axis: $(x = 0)$, $y = \ln 6$	B1
	Meets *x*-axis: $x = \frac{5}{3}$, (0); $x = \frac{7}{3}$, (0)	B1B1 (3)
	[May be seen on graph]	
		(13 marks)

Question Number	Scheme		Marks
1.	$\frac{dy}{dx} = \frac{1}{\operatorname{cosec} x + \cot x}(-\operatorname{cosec} x \cot x + -\operatorname{cosec}^2 x)$	Full attempt at chain rule	M1
	$= -\operatorname{cosec} x \frac{(\cot x + \operatorname{cosec} x)}{\operatorname{cosec} x + \cot x}$	Factorise cosec x	M1
	$= -\operatorname{cosec} x$ (*)		A1 cso (3)
			(3 marks)
2. (*a*)	3		B1 (1)
(*b*)	$f(2) = 24 \Rightarrow \quad 24 = (4 + p) \times 7 + 3$	Attempt f(±2)	M1
	$\Rightarrow \quad p = -1$ (*)		A1 cso (2)
(*c*)	$f(x) = (x^2 - 1)(2x + 3) + 3$	Attempt to multiply out	M1
	$= 2x^3 + 3x^2 - 2x - 3 + 3$		
	$= x(2x^2 + 3x - 2)$	Factor of x	M1
	$= x(2x - 1)(x + 2)$	Attempt to factorise 3 term quadratic	M1, A1 (4)
			(7 marks)
3. (*a*)	 Eqn: $(x - 5)^2 + (y - 13)^2 = r^2$		M1
	$r = 13 \quad (x - 5)^2 + (y - 13)^2 = 13^2$		A1 (2)
(*b*)	Differentiate: $2(x - 5) + 2(y - 13)\frac{dy}{dx} = 0$	Attempt to diff.	M1
	At (10, 1) $\quad (2 \times 5) + 2 \times -12\frac{dy}{dx} = 0$	Use of (10, 1)	M1
	$\frac{dy}{dx} = \frac{10}{24}$ or $\frac{5}{12}$		A1
	Eqn. of tangent $\quad y - 1 = \frac{5}{12}(x - 10)$	f.t. on their m	M1
	$5x - 12y - 38 = 0$		A1 (5)
			(7 marks)

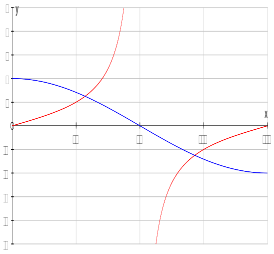
Question Number	Scheme	Marks
4.	$u = 1 + \sin x \quad \Rightarrow \quad \frac{\mathrm{d}u}{\mathrm{d}x} = \cos x$ or $\mathrm{d}u = \cos x\,\mathrm{d}x$ or $\mathrm{d}x = \frac{\mathrm{d}u}{\cos x}$	M1
	$I = \int (u-1)u^5\,\mathrm{d}u$ Full sub. to I in terms of u, correct	M1, A1
	$= \int (u^6 - u^5)\,\mathrm{d}u$ Correct split	M1
	$= \frac{u^7}{7} - \frac{u^6}{6}$ $(+\,c)$ M1 for $u^n \to u^{n+1}$	M1, A1
	$= \frac{u^6}{42}(6u - 7)$ $(+\,c)$ Attempt to factorise	M1
	$= \frac{(1+\sin x)^6}{42}(6\sin x + 6 - 7)$ $(+\,c)$ $= \frac{(1+\sin x)^6}{42}(6\sin x - 1)$ $(+\,c)$ (*)	A1 cso
		(8 marks)
Alt	Integration by parts	M1
	$I = (u-1)\frac{u^6}{6} - \frac{1}{6}\int u^6\,\mathrm{d}u$ Attempt first stage	M1
	$= (u-1)\frac{u^6}{6} - \frac{u^7}{42}$ Full integration	A1
	$= \frac{u^7}{6} - \frac{u^6}{6} - \frac{u^7}{42}$ or $\frac{6u^7 - 7u^6}{42}$	rest as scheme
5. (*a*)	$3 + 5x \equiv A(1 - x) + B(1 + 3x)$ Method for A or B	M1
	$(x = 1) \Rightarrow \quad 8 = 4B \qquad B = 2$	A1
	$(x = -\frac{1}{3}) \Rightarrow \frac{4}{3} = \frac{4}{3}A \qquad A = 1$	A1 (3)
(*b*)	$2(1 - x)^{-1} = 2[1 + x + x^2 + \ldots]$ Use of binomial with $n = -1$ scores M1(×2)	M1 [A1]
	$(1 + 3x)^{-1} = [1 - 3x + \frac{(-1)(-2)}{2!}(3x)^2 + \ldots]$	M1 [A1]
	$\therefore \frac{3+5x}{(1-x)(1+3x)} = 2 + 2x + 2x^2 + 1 - 3x + 9x^2 = 3 - x + 11x^2$	A1 (5)
(*c*)	$(1 + 3x)^{-1}$ requires $\lvert x \rvert < \frac{1}{3}$, so expansion is *not* valid.	M1, A1 (2)
		(10 marks)

Question Number	Scheme	Marks
6. (*a*)	$4 = 2\sec t \;\Rightarrow\; \cos t = \frac{1}{2}, \Rightarrow t = \frac{\pi}{3}$	M1, A1
	$\therefore\; a = 3\times\frac{\pi}{3}\times\sin\frac{\pi}{3} = \frac{\pi\sqrt{3}}{2}$	B1 (3)
(*b*)	$A = \int_0^a y\,dx = \int y\frac{dx}{dt}\,dt$ Change of variable	M1
	$= \int 2\sec t\times[3\sin t + 3t\cos t]\,dt$ Attempt $\frac{dx}{dt}$	M1
	$= \int_0^{\frac{\pi}{3}}(6\tan t, + 6t)\,dt$ (*) Final A1 requires limit stated	A1, A1cso (4)
(*c*)	$A = [6\ln\sec t + 3t^2]_0^{\frac{\pi}{3}}$ Some integration (M1) both correct (A1) ignore lim.	M1, A1
	$= (6\ln 2 + 3\times\frac{\pi^2}{9}) - (0)$ Use of $\frac{\pi}{3}$	M1
	$= 6\ln 2 + \frac{\pi^2}{3}$	A1 (4)
		(11 marks)
7. (*a*)	$A = \pi r^2,\; \frac{dr}{dt} = 4\lambda e^{-\lambda t}$	B1, B1
	$\frac{dA}{dt} = 2\pi r\frac{dr}{dt},\Rightarrow \frac{dA}{dt} = 2\pi\times 4(1 - e^{-\lambda t})\times 4\lambda e^{-\lambda t}$	M1, M1
	$\frac{dA}{dt} = 32\pi\lambda(e^{-\lambda t} - e^{-2\lambda t})$	A1cso (5)
(*b*)	$\int A^{-\frac{3}{2}}dA = \int t^{-2}dt$ Separation	M1
	$\frac{A^{-\frac{1}{2}}}{-\frac{1}{2}} = \frac{t^{-1}}{-1}$ $(+\,c)$	M1, A1
	$-2 = -1 + c$ Use of (1, 1)	M1
	$c = -1$	A1
	So $2A^{-\frac{1}{2}} = \frac{1}{t} + 1 \;\Rightarrow\; \sqrt{A} = \frac{2t}{1+t}$ Attempt $\sqrt{A}$ = or A =	M1
	i.e. $A = \frac{4t^2}{(1+t)^2}$	A1 (7)
(*c*)	Because $\frac{t^2}{(1+t)^2} < 1$ or $t^2 < (1+t)^2$ $(\Rightarrow A < 4)$	B1 (1)
		(13 marks)

Question Number	Scheme		Marks	
8. (*a*)	$9 - 8t = -16 + s$	Attempt a correct equation	M1	
	$4 + 5t = 10 + 9s$	Both correct	A1	
	Sub. $s = 25 - 8t \quad \Rightarrow \quad 5t = 6 + 225 - 72t$	Solving either	M1	
	$77t = 231 \qquad$ or $t = 3$, $s = 1$		A1	
	Sub. into 'j' $\quad 2 - 3t = \alpha - 4s$	Use of 3rd equation	M1	
	$\Rightarrow \qquad \alpha = -3$		A1	(6)
(*b*)	$\overrightarrow{OA} = \begin{pmatrix} -15 \\ -7 \\ 19 \end{pmatrix}$		B1	(1)
(*c*)	$\begin{pmatrix} -8 \\ -3 \\ 5 \end{pmatrix} . \begin{pmatrix} 1 \\ -4 \\ 9 \end{pmatrix} = -8 + 12 + 45 \ (= 49)$	Attempt scalar product	M1	
	$\cos\theta = \dfrac{49}{\sqrt{8^2 + 3^2 + 5^2}\sqrt{1^2 + 4^2 + 9^2}} = \dfrac{49}{\sqrt{98}\sqrt{98}} = \dfrac{1}{2}$	Use of $\frac{\mathbf{a.b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$, $\lvert\mathbf{a}\rvert$ or $\lvert\mathbf{b}\rvert$	M1, M1	
	$\cos\theta = \dfrac{1}{2}$		A1	
	$\theta = 60°\quad (*)$		A1 cso	(5)
	 $14\text{m} = 2\times 7\sqrt{2} = 2\sqrt{98}$		B1	
	$\overrightarrow{OB} = \overrightarrow{OA} \pm 2\begin{pmatrix} -8 \\ -3 \\ 5 \end{pmatrix} = \begin{pmatrix} -31 \\ -13 \\ 29 \end{pmatrix}$ or $\begin{pmatrix} 1 \\ -1 \\ 9 \end{pmatrix}$	M1: $\mathbf{a} \pm 2(\ )$, A1: any one	M1, A1	
	$\overrightarrow{OC} = \overrightarrow{OA} \pm 2\begin{pmatrix} 1 \\ -4 \\ 9 \end{pmatrix} = \begin{pmatrix} -13 \\ -15 \\ 37 \end{pmatrix}$ or $\begin{pmatrix} -17 \\ 1 \\ 1 \end{pmatrix}$	any correct pair	A1	(4)
				(16 marks)

Question number	Scheme	Marks
1.	$4 = 2^2$ (or log 4 = 2log 2)	B1
	Linear equation in x: $1 - x = 2x$	M1
	$x = \frac{1}{3}$	A1
		3
2.	(a) $ar^3 = 12$ $ar^4 = -8$ $r = \ldots, -\frac{2}{3}$ (or exact equivalent)	M1, A1 (2)
	(b) Using r with $ar^3 = 12$ or $ar^4 = -8$ to find $a = \ldots$	M1
	$a = -40\frac{1}{2}$ (*)	A1 (2)
	(c) $\frac{a}{1-r} = \frac{-40\frac{1}{2}}{1-\left(-\frac{2}{3}\right)}$, $= -24.3$ $\left(-24\frac{3}{10} \text{ or } -\frac{243}{10}\right)$ A1ft requires $\lvert r \rvert < 1$	M1 A1ft, A1 (3)
		7
3.	(a) $y = 4x - x^2$ $\frac{dy}{dx} = 4 - 2x$	M1 A1
	$"4-2x" = -2,$ $x = \ldots$	M1
	$x = 3,$ $y = 3$	A1 (4)
	(b) x-coordinate of A is 4	B1
	$\int (4x - x^2)\,dx = \left[\frac{4x^2}{2} - \frac{x^3}{3}\right]$	M1 A1ft
	$\left[\frac{4x^2}{2} - \frac{x^3}{3}\right]_0^4 = 32 - \frac{64}{3} = \frac{32}{3}$ $\left(= 10\frac{2}{3}\right)$ (or exact equivalent)	M1 A1 (5)
		9

Question number	Scheme	Marks	
4.	(a) $\cos A = \dfrac{6^2 + 2^2 - \left(2\sqrt{7}\right)^2}{2 \times 6 \times 2}$	M1 A1	
	$\cos A = \dfrac{1}{2}$ $A = \dfrac{\pi}{3}$ radians (*)	A1	(3)
	(b) $r\theta = \dfrac{2\pi}{3}$ $(= 2.09)$ (Exact or at least 3 s.f.)	M1 A1	(2)
	(c) Sector ABD: $\dfrac{1}{2}r^2\theta = \dfrac{1}{2} \times 2^2 \times \dfrac{\pi}{3}$ $\left(= \dfrac{2\pi}{3} \approx 2.094...\right)$	M1	
	Triangle ACB: $\dfrac{1}{2} \times 2 \times 6 \times \sin\dfrac{\pi}{3}$ $\left(= 3\sqrt{3} \approx 5.196...\right)$	M1	
	Triangle – Sector = $3\sqrt{3} - \dfrac{2\pi}{3}$ (= 3.10175…) Allow 3.1 or a.w.r.t. 3.10	M1 A1	(4)
			9
5.	(a) Gradient of AB = $\dfrac{12-4}{11-(-1)} = \dfrac{2}{3}$ (or equiv.)	M1 A1	(2)
	(b) Using $m_1 m_2 = -1$, gradient of BC = $-\dfrac{3}{2}$	B1ft	
	Equation of BC: $y - 12 = -\dfrac{3}{2}(x - 11)$	M1 A1ft	
	$3x + 2y - 57 = 0$ (Allow rearranged versions, e.g. $2y = 57 - 3x$)	A1	(4)
	(c) D: $y = 0$ in equation of BC: $x = 19$	B1ft	
	Coordinates of C: (3, 24)	B1, B1	(3)
	(d) $AC = \sqrt{("3"-(-1))^2 + ("24"-4)^2} = \sqrt{416} = 4\sqrt{26}$ (*)	M1 A1	(2)
			11

Question number	Scheme		Marks	
6.	(a) 	Tangent graph shape 180 indicated Cosine graph shape 2 and 90 indicated Allow separate sketches.	M1 A1 M1 A1	(4)
	(b) Using $\tan x = \frac{\sin x}{\cos x}$ and multiplying both sides by cos *x*.	$(\sin x = 2\cos^2 x)$	M1	
	Using $\sin^2 x + \cos^2 x = 1$		M1	
	$2\sin^2 x + \sin x - 2 = 0$	(*)	A1	(3)
	(c) Solving quadratic: $\sin x = \frac{-1 \pm \sqrt{17}}{4}$ (or equiv.)		M1 A1	
	$x = 51.3$ (3 s.f. or better, 51.33…)	α	A1	
	$x = 128.7$ (accept 129) (3 s.f. or better)	$180 - \alpha$ $(\alpha \neq 90n)$	B1ft	(4)
				11

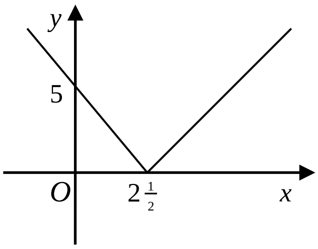
Question number	Scheme	Marks
7.	(a) $\pi r^2 h = 780$, $h = \dfrac{780}{\pi r^2}$	M1, A1 (2)
	(b) $A = 2\pi r^2 + 2\pi rh$ and substitute for h.	M1
	$A = 2\pi r^2 + \dfrac{1560}{r}$ (*)	A1 (2)
	(c) $\dfrac{dA}{dr} = 4\pi r - 1560r^{-2}$	M1 A1
	Equate to zero and proceed to $r^3 = ...$ or $r = …$, coping with indices.	M1
	$r = \sqrt[3]{\dfrac{1560}{4\pi}}$ $\left(= \sqrt[3]{\dfrac{390}{\pi}} \approx 4.99 \approx 5.0 \right)$	A1 (4)
	(d) Attempt second derivative and consider its sign/value.	M1
	$\dfrac{d^2A}{dr^2} = 4\pi + 3120r^{-3}$ Correct second derivative, > 0, $\therefore$ minimum.	A1 (2)
	(e) Substitute value of r (or values of r and h) into their A formula.	M1
	469 (or a.w.r.t.) or 470 (2 s.f.)	A1cso (2) **12**

Question number	Scheme		Marks	
8.	(a) f(4) = 0 $\Rightarrow$ $64 + 16(p + 1) - 72 + q = 0$	M1: f(4) or f(–4)	M1 A1	
	$16p + q + 8 = 0$	(*)	A1	(3)
	(b) f(–p) = 0 $\Rightarrow$ $-p^3 + p^2(p+1) + 18p + q = 0$	M1: f(–p) or f(p)	M1 A1	
	$p^2 + 18p + q = 0$	(*)	A1	(3)
	(c) Combine to form a quadratic equation in one unknown.		M1	
	$p^2 + 18p - (16p + 8) = 0$ $\quad p^2 + 2p - 8 = 0$		A1	
	$(p + 4)(p - 2) = 0$ $\quad p = \ldots,\quad 2$		M1, A1cso	
	$q = -40$	(ft only for a positive p)	B1ft	(5)
	(d) Complete method to find third factor.		M1	
	$x^3 + 3x - 18x - 40 = (x - 4)(x + 2)(x + 5)$		A1	(2)
				13

Question Number	Scheme	Marks
1.	$\int\left(1+\frac{5}{x}\right)\mathrm{d}x = x+5\ln x$	M1 A1
	$\left[x+5\ln x\right]_1^{\mathrm{e}} = (\mathrm{e}+5)-1 = \mathrm{e}+4$ M1 Correct use of limits	M1 A1 (4) **[4]**
2.	(a) $\left(k+\frac{x}{2}\right)^5 = k^5+5k^4\frac{x}{2}+10k^3\left(\frac{x}{2}\right)^2+10k^2\left(\frac{x}{2}\right)^3+\dots$	M1 A1 (2)
	Need not be simplified. Accept ${}^5\mathrm{C}_1$, ${}^5\mathrm{C}_2$, ${}^5\mathrm{C}_3$	
	(b) $10k^3\left(\frac{x}{2}\right)^2 = 540x^2$ with or without x^2	M1
	Leading to $k=6$ * cso	A1 (2)
	Or substituting k = 6 into ${}^5\mathrm{C}_2\,k^3\left(\frac{x}{2}\right)^2$ and simplifying to $540x^2$.	
	(c) Coefficient of x^3 is $10\times 6^2\times\frac{1}{2^3}=45$	M1 A1 (2) **[6]**

Question Number	Scheme	Marks
3.	Use of $V = \pi \int y^2 \, dx$	M1
	$y^2 = 16x^2 + \frac{36}{x^2}; -48$	B1; B1
	Integrating to obtain $(\pi)\left[\frac{16x^3}{3} - \frac{36}{x}; -48x\right]$ ft constants only	M1 A1ft; A1ft
	$(\pi)\left[\frac{16x^3}{3} - \frac{36}{x} - 48x\right]_2^4 = (\pi)\left[140\tfrac{1}{3} - \left(-71\tfrac{1}{3}\right)\right]$ correct use of limits	M1
	$V = 211\frac{2}{3}\pi$ $\left(\text{units}^3\right)$	A1 (8)
		[8]
4.	(a) $\text{f}(1) = -2$, $\text{f}(2) = 5\tfrac{1}{2}$	M1
	Change of sign (and continuity) $\Rightarrow$ root $\in (1, 2)$	A1 (2)
	(b) $x_1 = 1.38672...$, $x_2 = 1.39609...$ awrt 4dp	B1, B1
	$x_3 = 1.39527...$, $x_4 = 1.39534...$ same to 3dp	M1
	Root is 1.395 (to 3dp) cao	A1 (4)
	(c) Choosing a suitable interval, $(1.3945, 1.3955)$ or tighter.	M1
	$\text{f}(1.3945) \approx -0.005$, $\text{f}(1.3955) \approx +0.001$	
	Change of sign (and continuity) $\Rightarrow$ root $\in (1.3945, 1.3955)$	
	$\Rightarrow$ root is 1.395 correct to 3dp	A1 (2)
		[8]

Question Number	Scheme	Marks
5.	(a) $\dfrac{2x+5}{x+3}-\dfrac{1}{(x+3)(x+2)}=\dfrac{(2x+5)(x+2)-1}{(x+3)(x+2)}$	M1
	$=\dfrac{2x^2+9x+9}{(x+3)(x+2)}$	A1
	$=\dfrac{(2x+3)(x+3)}{(x+3)(x+2)}$	M1 A1
	$=\dfrac{2x+3}{x+2}$	A1 (5)
	(b) $2-\dfrac{1}{x+2}=\dfrac{2(x+2)-1}{x+2}=\dfrac{2x+3}{x+2}$ or the reverse	M1 A1 (2)
	(c) T_1: Translation of -2 in x direction	B1
	T_2: Reflection in the x-axis	B1
	T_3: Translation of (+)2 in y direction	B1
	All three fully correct	B1 (4)
		[11]
	One alternative is T_1: Translation of -2 in x direction T_2: Rotation of 90° clockwise about O T_3: Translation of -2 in x direction	

Question Number	Scheme	Marks
6.	(a) 	
	Correct shape, vertex on x-axis	B1
	$(0, 5)$ or 5 on y-axis	B1
	$\left(2\frac{1}{2}, 0\right)$ or $2\frac{1}{2}$ on x-axis	B1 (3)
	(b) $2x-5=x \Rightarrow x=5$ accept stated	M1 A1
	$2x-5=-x$ or equivalent	M1
	$x=1\frac{2}{3}$ accept exact equivalents	A1 (4)
	(c) Method for finding either coordinate of the lowest point (differentiating and equating to zero, completing the square, using symmetry).	M1
	$x=3$ or $\mathrm{g}(x)=-9$	A1
	$\mathrm{g}(x) \geq -9$	A1 (3)
	(d) $\mathrm{fg}(1)=\mathrm{f}(-5)$	M1
	$=15$	A1 (2)
		[12]

Question Number	Scheme	Marks
7.	(a) $0 = k + \ln 2\left(\frac{1}{2\text{e}}\right) \Rightarrow 0 = k - 1 \Rightarrow k = 1$ ∗	M1 A1 (2)
	(Allow also substituting $k = 1$ and $x = \frac{1}{2\text{e}}$ into equation and showing $y = 0$ and substituting $k = 1$ and $y = 0$ and showing $x = \frac{1}{2\text{e}}$.)	
	(b) $\frac{\text{d}y}{\text{d}x} = \frac{1}{x}$	B1
	At A gradient of tangent is $\frac{1}{1/2\text{e}} = 2\text{e}$	M1
	Equations of tangent: $y = 2\text{e}\left(x - \frac{1}{2\text{e}}\right)$	M1
	Simplifying to $y = 2\text{e}x - 1$ ∗ cso	A1 (4)
	(c) $y_1 = 1.69,\ y_2 = 2.39$	B1, B1 (2)
	(d) $\int_1^3 (1 + \ln 2x)\,\text{d}x \approx \frac{1}{2} \times \frac{1}{2} \times (\ \dots\ )$	B1
	$\approx \dots \times (1.69 + 2.79 + 2(2.10 + 2.39 + 2.61))$ ft their (c)	M1 A1ft
	≈ 4.7 accept 4.67	A1 (4) **[12]**

Question Number	Scheme	Marks
8.	(i) Using $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$ *or* $\cot\theta = \frac{1}{\tan\theta}$	M1
	Forming a single fraction LHS $= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$ *or* LHS $= \frac{1+\tan^2\theta}{\tan\theta}$	M1
	Reaching the expression $\frac{1}{\sin\theta\cos\theta}$	A1
	Using $\sin 2\theta = 2\sin\theta\cos\theta$	M1
	LHS = $\frac{2}{2\sin\theta\cos\theta} = \frac{2}{\sin 2\theta} = 2\operatorname{cosec} 2\theta =$ RHS * cso	A1 (5)
	(ii)(a) $\cos\alpha = \frac{12}{13}$ M1 Use of $\sin^2\alpha + \cos^2\alpha = 1$ or right angled triangle but accept stated	M1 A1
	(b) Use of $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$ $\left(\text{or } 1 - 2\sin^2\alpha \text{ or } 2\cos^2\alpha - 1\right)$	M1
	$\cos 2\alpha = \frac{119}{169}$	A1 (4)
	(c) Use of $\cos(x+\alpha) = \cos x\cos\alpha - \sin x\sin\alpha$	M1
	Substituting for $\sin\alpha$ and $\cos\alpha$	M1
	$12\cos x - 5\sin x + 5\sin x = 6$ $(12\cos x = 6)$	A1
	$x = \frac{\pi}{3}$ awrt 1.05	M1 A1 (5)
		[14]

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January 2005

Advanced Subsidiary/Advanced Level

General Certificate of Education

Subject: **Pure Mathematics** **Paper: P1**

Question Number	Scheme	Marks
1.	Forming equation in x or y by attempt to eliminate one variable $(3-y)^2+y=15$ *or* $x^2+(3-x)=15$	M1 A1
	<u>Attempt at solution</u> i.e. solving 3 term quadratic: $(y-6)(y+1)=0$, $y=\ldots$ or $(x-4)(x+3)=0$, $x=\ldots$ or correct use of formula or correct use of completing the square	M1
	$x=4$ and $x=-3$ or $y=-1$ and $y=6$	A1
	Finding the values of the other coordinates	M1 A1 ft (6)
2.	Using $\sin^2\theta+\cos^2\theta=1$ to give a quadratic in $\cos\theta$.	M1
	Attempt to solve $\cos^2\theta+\cos\theta=0$	M1
	$(\cos\theta=0)\Rightarrow\theta=\frac{\pi}{2},\frac{3\pi}{2}$	B1, B1
	$(\cos\theta=-1)\Rightarrow\theta=\pi$ (Candidate who writes down 3 answers only with no working scores a maximum of 3)	B1 (5)
3. (a)	Attempt f (2) ; $=16-4+4-16=0\Rightarrow(x-2)$ is a factor	M1; A1 (2)
(b)	<u>$c=8$</u>	B1
	<u>A complete method to find b</u> Either compare coefficients of x or x^2: $-2b+8=2$, or $-4+b=-1$ Or substitute value of x (may be implied) : e.g.$(x=1)\Rightarrow -13=(-1)(10+b)$	M1
	<u>$b=3$</u>	A1 (3)
(c)	Checking b^2-8c; $<0\Rightarrow$ no real roots to the quadratic $\Rightarrow$ <u>$x=2$ is the only solution</u>	M1; A1 A1 (3)

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General Certificate of Education

Subject: **Pure Mathematics** **Paper: P1**

Question Number	Scheme	Marks
4 (a)	Correct strategy for differentiation e.g. $y = 4x^2 + 5 - \frac{1}{x}$, or product or quotient rules applied correctly to $\frac{5x-1}{x}$.	M1
	$\frac{dy}{dx} = 8x, \quad + \frac{1}{x^2}$ B1 for 8x seen anywhere.	B1, A1 (3)
(b)	Using $\frac{dy}{dx} = 0$	M1
	So $8x^3 + 1 = 0 \quad \Rightarrow x = -\frac{1}{2}$.	M1 A1
	M1 requires multiplication by denominator and use of a root in the solution	(3)
(c)	Complete method: Either $\frac{d^2y}{dx^2} = 8 - \frac{2}{x^3}$, with x value substituted, or gradient either side checked	M1
	Completely correct argument, either > 0 with no error seen, or –ve to +ve gradient, then minimum stated	A1 (2)
5(a)	$p = 15, \; q = -3$	B1, B1
	Special case if B0 B0, allow M1 for method, e.g. $8 = \frac{1+p}{2}$	(2)
(b)	Gradient of line *ADC* = $-\frac{5}{7}$, gradient of perpendicular line = $-\frac{1}{\text{gradient } ADC}$ $\left(\frac{7}{5}\right)$	B1, M1
	Equation of *l*: $y - 2 = (\frac{7}{5})(x - 8)$	M1 A1ft
	$\Rightarrow 7x - 5y - 46 = 0$	A1cao (5)
(c)	Substituting $y = 7$ and finding value for *x*,	M1
	$x = \frac{81}{7}$ or $11\frac{4}{7}$	A1 (2)

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Subject: **Pure Mathematics** **Paper: P1**

Question Number	Scheme	Marks
6 (a)	$P = r\theta + 2r$, $A = \frac{1}{2}r^2\theta$	B1, B1 (2)
(b)	Substituting value for r and equating P to A. $[2\sqrt{2}(2+\theta) = \frac{1}{2}(2\sqrt{2})^2\theta]$	M1
	Correct process to find θ $[\theta(\sqrt{2}-1) = 2]$	M1
	$\theta = \dfrac{2}{\sqrt{2}-1}$ *	A1 c.s.o. (3)
(c)	Multiply numerator and denominator by $(\sqrt{2}+1)$	M1
	$2, +2\sqrt{2}$	A1, A1 (3)
7 (a)	Applying correct formula [325 = 120 +5(n-1)]	M1
	Solving to give n =42 * (or verifying in correct equation)	A1 (2)
(b)	Using formula for sum of AP: S = $\frac{42}{2}\{240+5(42-1)\}$	M1 A1
	$= 9345$	A1 (3)
(c)	Recognising GP with $r = 0.98$	M1
	Value (in £) = 7200 (0.98) ^{24}	M1
	= 4434 (only this value)	A1 (3)

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Subject: **Pure Mathematics** **Paper: P1**

Question Number	Scheme	Marks
8 (a)	Substitute $x=0, y=\sqrt{3}$ to give $\sqrt{3} = k\frac{\sqrt{3}}{2} \Rightarrow k=2$ (or verify result)	B1 (1)
(b)	$p = 120$, $q = 300$ (f.t. on $p+180$)	B1, B1ft (2)
(c)	$arc\sin(-0.8) = -53.1$ or $arc\sin(0.8) = 53.1$	B1
	$(x+60) = 180 - \arcsin(-0.8)$ OR $360 + \arcsin(-0.8)$ (only need one) or equivalent [e.g. $180 + \arcsin 0.8$ OR $360 - \arcsin 0.8$]	M1
	First value of $x = 233.1 - 60$, i.e. $x = 173.1$	A1
	Second value of $x = 306.9 - 60$, i.e. $x = 246.9$ (special case ft on $p+q-1^{st}$ value)	M1, A1 (5)
9 (a)	$(x-3)^2, +9$ Value $a = 3$ and $b = 9$ may just be written down with no method shown.	B1, M1 A1 (3)
(b)	P is (3, 9)	B1 ft, B1ft (2)
(c)	$A = (0, 18)$	B1
	$\frac{dy}{dx} = 2x - 6$, at A $m = -6$	M1 A1
	Equation of tangent is $y - 18 = -6x$ (in any form)	A1ft (4)
(d)	Showing that line meets x axis directly below P, i.e. at $x = 3$.	A1cso (1)
(e)	$A = \int x^2 - 6x + 18 dx = [\frac{1}{3}x^3 - 3x^2 + 18x]$	M1 A1
	Substituting $x = 3$ to find area A under curve A [=36]	M1
	Area of $R = A -$ area of triangle$= A - \frac{1}{2}\times 18 \times 3$, $= 9$	M1 A1 (5)
	Alternative: $\int x^2 - 6x + 18 - (18 - 6x) dx$ M1 $= \frac{1}{3}x^3$ M1 A1 ft Use $x = 3$ to give answer 9 M1 A1	

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GENERAL CERTIFICATE OF EDUCATION

Advanced Subsidiary/Advanced Level

Pure Mathematics P2

MARKING SCHEME

January 2005

<u>Principal Examiner:</u>

Miss Jane Dyer
10 The Crofts
St Bees
Cumbria
CA27 0BH

Tel.: 01946 822508

Marking should be completed by **16th February 2005.**

General Instructions

1. The total number of marks for the paper is 75.

2. Method (M) marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.

3. Accuracy (A) marks can only be awarded if the relevant method (M) marks have been earned.

4. (B) marks are independent of method marks.

5. Method marks should not be subdivided.

6. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. Indicate this action by 'MR' in the body of the script (but see also note 10).

7. If a candidate makes more than one attempt at any question:

   (a) If all but one attempt is crossed out, mark the attempt which is NOT crossed out.

   (b) If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

8. Marks for each question, or part of a question, must appear in the right-hand margin and, in addition, total marks for each question, even where zero, must be ringed and appear in the right-hand margin and on the grid on the front of the answer book. It is important that a check is made to ensure that the totals in the right-hand margin of the ringed marks and of the unringed marks are equal. The total mark for the paper must be put on the top right-hand corner of the front cover of the answer book.

9. For methods of solution not in the mark scheme, allocate the available M and A marks in as closely equivalent a way as possible, and indicate this by the letters 'OS' (outside scheme) put alongside in the body of the script.

10. All A marks are 'correct answer only' (c.a.o.) unless shown, for example, as A1 f.t. to indicate that previous wrong working is to be followed through. In the body of the script the symbol √^ should be used for correct f.t. and ✗^ for incorrect f.t. After a misread, however, the subsequent A marks affected are treated as A f.t., but manifestly absurd answers should never be awarded A marks.

11. Ignore wrong working or incorrect statements following a correct answer.

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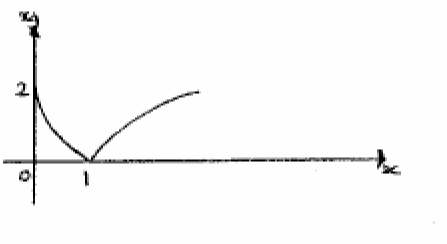
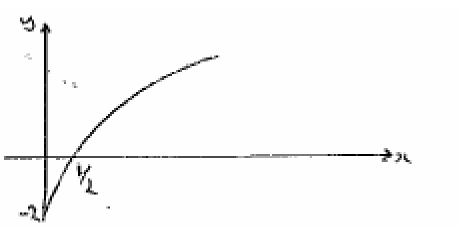
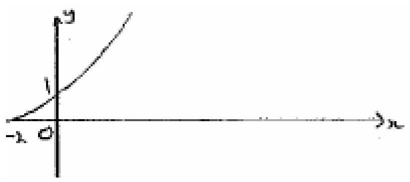
Subject: **Pure Mathematics** Paper: **P2**

Question Number	Scheme	Marks
1a)	$\frac{(x-3)(x+2)}{x(x-3)}$; $= \frac{(x+2)}{x}$ or $1+\frac{2}{x}$ B1 numerator, B1 denominator ; B1 either form of answer	B1,B1,B1 (3)
1b)	$\frac{(x+2)}{x} = x+1 \Rightarrow x^2 = 2$ M1 for equating f(x) to x + 1 and forming quadratic. A1 candidate's correct quadratic	M1 A1√
	$x = \pm\sqrt{2}$	A1 (3)
2a)	 Shape reflected 0< x < 1 Cusp + coords	M1 A1 (2)
	 General shaped -2 (1/2, 0)	B1 B1 (2)
	 Rough reflection $y = x$ (0,1) or 1 on $y - ax$ is (-2, 0) or -2 on $x - ax$ is	B1 B1 B1 (3)

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Subject: **Pure Mathematics** Paper: **P2**

Question Number	Scheme		Marks
3a)	$u_2 = (-1)(2) + d = -2 + d$		B1
	$u_3 = (-1)^2(-2 + d) + d = -2 + 2d$	Attempting to find u_3 and u_4	M1
	$u_4 = (-1)^3(-2 + 2d) + d = 2 - d$	either u_3 or u_4 correct	A1
	$u_5 = (-1)^4(2 - d) + d = 2$ * cao		A1*
			(4)
b)	$u_{10} = u_1 = d - 2$		B1√
			(1)
c)	$-2 + 2d = 3(-2 + d) \Rightarrow d = 4$	M1 equating their u_3 to their u_2	M1 A1
			(2)
4a)	(0,4), or 4 on y-axis		B1
			(1)
b)	$V = \pi \int x^2 dy$	attempt use of	M1
	$x^2 = y - 4$		B1
	$V = (\pi)\int (y-4)dy$	attempt to integrate	M1
	$= (\pi)\left[\frac{y^2}{2} - 4y\right]$		A1
	using limits to give		M1
	$\pi[(32-32)-(8-16)] = 8\pi$. (c.a.o)		A1
			(6)

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Advanced Subsidiary/Advanced Level

General Certificate of Education

Subject: **Pure Mathematics** Paper: **P2**

Question Number	Scheme		Marks
5a)	$3^x = 5 \Rightarrow \ln 3^x = \ln 5 \quad [x \ln 3 = \ln 5]$	taking logs	M1
	$x = \dfrac{\ln 5}{\ln 3}$		A1
	$= 1.46$ cao		A1 (3)
b)	$\log_2 (2x+1) - \log_2 x = \log_2 \dfrac{2x+1}{x}$		M1
	$2 = \log_2 4$		B1
	Forming non-log equation		M1
	$\dfrac{2x+1}{x} = 4$ or equivalent; $x = \dfrac{1}{2}$		A1 (4)
c)	$-\ln \sec x = \ln \cos x$		B1
	$\sin x = \cos x \Rightarrow \tan x = 1 \quad x = 45$	use of tan x	M1, A1 (3)

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Subject: **Pure Mathematics** Paper: **P2**

Question Number	Scheme		Marks
6a)	I = $3x + 2e^x$		B1
	Using limits correctly to give 1 + 2e. (c.a.o.)		M1 A1 (3)
b)	$A = (0, 5)$;		B1
	$\frac{dy}{dx} = 2e^x$		B1
	Equation of tangent: $y = 2x + 5$; c = –2.5	attempting to find eq. of tangent and subst in $y = 0$	M1; A1 (4)
c)	$y = \frac{5x+2}{x+4} \Rightarrow yx + 4y = 5x + 2 \quad \Rightarrow 4y - 2 = 5x - xy$	putting $y =$ and att. to rearrange to find x	M1; A1
	$g^{-1}(x) = \frac{4x-2}{5-x}$ or equivalent		A1 (3)
d)	gf(0) = g(5); =3	att to put 0 into f and then their answer into g	M1; A1 (2)

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Subject: **Pure Mathematics** Paper: **P2**

Question Number	Scheme	Marks
7a)	Complete method for DE [e.g. split triangle ADE and sin, or sine or cos rule]	M1
	$DE = 4 \sin \theta$ * (c.s.o.)	A1* (2)
b)	$P = 2\,DE + 2EF$ or equivalent. With attempt at EF	M1
	$= 8\sin \theta + 4\cos \theta$ * (c.s.o.)	A1* (2)
c)	$8\sin \theta + 4\cos \theta = R \sin (\theta + \alpha)$	
	$= R \sin \theta \cos \alpha + \text{R} \cos \theta \sin \alpha$	
	Method for R, method for α	M1 M1
	[$R \cos \alpha = 8$, R $\sin \alpha = 4$ $\tan \alpha = 0.5$, R = $\sqrt{(8^2 + 4^2}$]	
	$\alpha = 0.464$ (allow 26.6), $R = 4\sqrt{5}$ or 8.94	A1 A1 (4)
d)	Using candidate's $R \sin (\theta + \alpha) = 8.5$ to give $(\theta + \alpha) = \sin^{-1} \frac{8.5}{R}$	M1
	Solving to give $\theta = \sin^{-1} \frac{8.5}{R} - \alpha$, $\theta = 0.791$ (allow 45.3)	M1 A1
	Considering second angle: $\theta + \alpha = \pi$ (or 180) $- \sin^{-1} \frac{8.5}{R}$;	M1
	$\theta = 1.42$ (allow 81.6)	A1 (5)

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Subject: **Pure Mathematics** Paper: **P2**

Question Number	Scheme		Marks	
8a)	$f'(x) = -\frac{1}{2x^2};+\frac{1}{x}$	M1 for evidence of differentiation	M1A1;A1	
	$f'(0)=0 \Rightarrow \frac{-1+x}{2x^2}=0; \Rightarrow x=0.5$	(or subst x = 0.5)	M1A1 *	(5)
b)	$y = 1+\ln\left(\frac{1}{4}\right)-1; = -2\ln 2$	Subst their value for x in	M1;A1	(2)
c)	f(4.905) = < 0 (–0.000955), f(4.915) = > 0 (+ 0.000874)		M1	
	Change of sign indicates root between (accept correct values above)		A1	(2)
d)	$\frac{1}{2x}+\ln\left(\frac{x}{2}\right)-1=0; \quad \Rightarrow 1-\frac{1}{2x}=\ln\left(\frac{x}{2}\right)$			
	$\Rightarrow \frac{x}{2}=e^{\left(1-\frac{1}{2x}\right)} \Rightarrow x=2e^{\left(1-\frac{1}{2x}\right)}$ * (c.s.o.)	M1 for use of e	M1A1	(2)
e)	$x_1 = 4.9192$		B1	
	$x_2 = 4.9111$, $x_3 = 4.9103$,	both	B1	(2)

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Subject: **Pure Mathematics** **Paper: P3**

Question Number	Scheme		Marks	
1. (a)	$f(-2) = -16a - 4a + 6 + 7$	Attempt $f(\pm 2)$	M1	
	$f(-2) = -3 \Rightarrow -20a + 13 = -3$ i.e. $20a = 16$	Solve eqn. $f(\pm 2) = -3 \rightarrow Ka = L$	M1	
	$\underline{a = \frac{4}{5}}$	o.e.	A1	(3)
(b)	$f(\frac{1}{2}) = \frac{a}{4} - \frac{a}{4} - \frac{3}{2} + 7 = \underline{\frac{11}{2}}$ (o.e.)	Attempt $f(\pm\frac{1}{2})$	M1	
		$\frac{11}{2}$ or exact equiv.	A1	(2) ⑤
2. (a)	$\sin(3x + x) = \sin 3x \cos x + \cos 3x \sin x$	Use of a correct formula $\cdots\pm\cdots=\cdots\pm\cdots$	M1	
	$\sin(3x - x) = \sin 3x \cos x - \cos 3x \sin x$	Both correct	A1	
	(Subtract) $\Rightarrow \underline{\sin 4x - \sin 2x = 2\sin x \cos 3x}$	$p = 4, q = 2$	A1 cso.	(3)
(b)	$\int 2\sin x \cos 3x \, dx = \int (\sin 4x - \sin 2x)\, dx$	Attempt $\int$ using a $\sin px \rightarrow \pm \frac{\cos px}{p}$	M1	
	$\underline{= -\frac{\cos 4x}{4} + \frac{\cos 2x}{2} + c}$	$\int$ their p, q	A1 $\int$	(2)
(c)	$\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} 2\sin x \cos 3x \, dx = \left(-\frac{1}{4}\cos\frac{10\pi}{3} + \frac{1}{2}\cos\frac{5\pi}{3}\right) - \left(-\frac{1}{4}\cos 2\pi + \frac{1}{2}\cos \pi\right)$		M1	
	$= \underline{\frac{9}{8}}$		A1	(2) ⑦
3. (a)	$x^2 + y^2 - 12x + 4y + 20 = 0$			
	$(x-6)^2 + (y+2)^2 + k = 0$	Attempt to complete square	M1	
		Centre $\underline{(6, -2)}$	A1	(2)
(b)	$(x-6)^2 + (y+2)^2 = 20$	$k = -36 - 4 + 20$ o.e.	M1	
		Radius $= \underline{\sqrt{20}}$	A1	(2)
Use of Formulae (a)	$2g = -12,\ 2f = 4,\ c = 20$ Centre $(-g, -f)$	$(\pm 6, \pm 2)$	M1	
		Centre $\underline{(6, -2)}$	A1	(2)
(b)	Radius $= \sqrt{36 + 4 - 20}$	36, 4, 20 and $\sqrt{\ }$	M1	
		Radius $\underline{\sqrt{20}}$	A1	(2)
(c)				

P.T.O.

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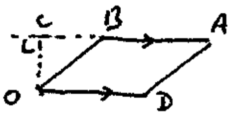
Subject: **Pure Mathematics** **Paper: P3**

Question Number	Scheme		Marks
3. (c)	$x^2 - 12x + 20 = 0$	Sub $y=0$ in $C_1 \rightarrow$ 3TQ $=0$	M1
	$\Rightarrow \quad x = 2, 10$		A1
	Centre of C_2 is $\underline{(6,0)}$		B1
	Radius of C_2 is $6-2$ or $10-6$ $\underline{=4}$	∫ their centre, A or B	B1∫
	Equation of C_2 is $\underline{(x-6)^2 + y^2 = 4^2}$ o.e.		M1 ∫ Centre and radius
			A1 (6) (10)
4. (a)	$(1-x)^{-\frac{1}{2}} = 1 + \frac{1}{2}x + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-x)^2 + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3!}(-x)^3$	Ms need attempt at binomial → at least x^2 term.	M1 A1
	$\left(= 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3 \ldots\right)$		
	$(1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x + \frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2 + \frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})}{3!}x^3$		M1 A1
	$\left(= 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 \ldots\right)$		
		Subtract	M1
	$\therefore \; f(x) = \underline{\frac{1}{2}x^2 + \frac{1}{4}x^3}$	c.a.o.	A1 (6)
(b)	$f'(x) = x + \frac{3}{4}x^2 \ldots, \quad f''(x) = 1 + \frac{3}{2}x \ldots$	Attempt f′ & f″	M1
	$f(0) = 0$ and $f'(0) = 0$		B1
	$f''(0) > 0$, $\therefore$ Minimum at origin (*)		M1, A1 c.s.o. (4) (10)
5. (a)	$\overrightarrow{AB} = \underset{\sim}{b} - \underset{\sim}{a} = \begin{pmatrix}3\\-3\\-6\end{pmatrix}$; $\therefore$ Equation of L $\underline{\underset{\sim}{r} = \begin{pmatrix}0\\5\\5\end{pmatrix} + t\begin{pmatrix}3\\-3\\-6\end{pmatrix}}$ (o.e.)		M1 ; A1 (2)
(b)	$\begin{pmatrix}3t\\5-3t\\5-6t\end{pmatrix} \cdot \begin{pmatrix}3\\-3\\-6\end{pmatrix} = 0$	Attempt $\overrightarrow{OC}$ in terms of t	M1
		$\overrightarrow{OC} \cdot \overrightarrow{AB}$ attempt	M1
		Evaluation of • product	M1
	$\Rightarrow 9t - 15 + 9t - 30 + 36t = 0$	$t =$	A1
	i.e. $t = \frac{5}{6}$	$\overrightarrow{OC} =$	A1
	$\therefore \quad \underline{\overrightarrow{OC} = \begin{pmatrix}2.5\\2.5\\0\end{pmatrix}}$		(5)
(c)	 $\overrightarrow{OD} = \overrightarrow{BA} = \underset{\sim}{a} - \underset{\sim}{b}$ or $-\overrightarrow{AB}$, $= \underline{\begin{pmatrix}-3\\3\\6\end{pmatrix}}$		M1, A1 (2)
(d)	$\lvert\overrightarrow{OC}\rvert = 2.5\sqrt{2}$; $\lvert\overrightarrow{OD}\rvert = 3\sqrt{1^2+1^2+2^2} = 3\sqrt{6}$	Attempt at least one relevant length	M1
		both correct	A1
	Area $= \lvert\overrightarrow{OC}\rvert \times \lvert\overrightarrow{OD}\rvert$ (o.e.), $= 7.5\sqrt{12}$ or $\underline{15\sqrt{3}}$ or $\underline{\text{AWRT } 26.0}$		M1, A1 (4) (13)

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Question Number	Scheme		Marks	
6. (a)	$\frac{dy}{dx} = \frac{\dot{y}}{\dot{x}} = \frac{3}{2t}$	Attempt $\frac{dy}{dx}$	M1	
	Gradient of normal is $-\frac{2t}{3}$	Use of $\perp^r$ rule	M1	
	At P $t = 2$	$t = 2$ @ P	B1	
	$\therefore$ Gradient of normal @ P is $-\frac{4}{3}$		A1	
	Equation of normal @ P is $y - 9 = -\frac{4}{3}(x - 5)$		M1	
	Q is where $y = 0$ $\therefore$ $x = \frac{27}{4} + 5 = \frac{47}{4}$ (o.e)		A1	(6)
(b)	Curved Area $= \int y\,dx = \int y \frac{dx}{dt}\,dt$	Attempt change of var.	M1	
	$= \int 3(1+t).2t\,dt$		A1	
	$= [3t^2 + 2t^3]$	$\int x^n \to x^{n+1}$	M1 A1	
	Curve cuts x-axis when $t = -1$	$t = -1$	B1	
	Curved Area $= [3t^2+2t^3]_{-1}^{2} = (12+16)-(3-2)$ $(= 27)$	Use of their $t=-1, t=2$	M1	
	Area of $\triangle$ PQ triangle $= \frac{1}{2}((a) - 5) \times 9$ $(= 30.375)$		M1	
	Total area of R = Curved Area + Δ		M1	
	$= 57.375$ or AWRT 57.4		A1	(9)
				(15)

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Question Number	Scheme		Marks	
7. (a)	$I = \int x \operatorname{cosec}^2(x+\frac{\pi}{6})\,dx = \int x\, d(-\cot(x+\frac{\pi}{6}))$	Attempt Parts	M1	
	$= -x\cot(x+\frac{\pi}{6}) + \int \cot(x+\frac{\pi}{6})\,dx$	.... $+\int \cot(x+\frac{\pi}{6})dx$	A1	
	$= \underline{-x\cot(x+\frac{\pi}{6}) + \ln\lvert\sin(x+\frac{\pi}{6})\rvert + c}$ (*)		A1 c.s.o.	(3)
(b)	$\int \frac{1}{y(1+y)}\,dy = \int 2x \operatorname{cosec}^2(x+\frac{\pi}{6})\,dx$	Attempt to separate	M1	
	LHS $= \int \left(\frac{1}{y} - \frac{1}{1+y}\right)dy$	Attempt PIF on LHS	M1	
		Correct PIF	A1	
	$\therefore \ln y - \ln\lvert 1+y\rvert$ or $\ln\left\lvert\frac{y}{1+y}\right\rvert = 2$ (a)	LHS some correct $\ln\int$	M1	
		Use of (a) in RHS	M1	
	$\therefore \underline{\frac{1}{2}\ln\left\lvert\frac{y}{1+y}\right\rvert = -x\cot(x+\frac{\pi}{6}) + \ln\lvert\sin(x+\frac{\pi}{6})\rvert + c}$ (*)		A1 c.s.o.	(6)
(c)	$y=1, x=0 \Rightarrow \frac{1}{2}\ln\frac{1}{2} = \ln(\sin\frac{\pi}{6}) + c$	Use of $y=1, x=0$	M1	
	$\therefore c = -\frac{1}{2}\ln\frac{1}{2}$		A1	
		Sub $x=\frac{\pi}{12}$	M1	
	$x=\frac{\pi}{12} \Rightarrow \frac{1}{2}\ln\left\lvert\frac{y}{1+y}\right\rvert = -\frac{\pi}{12}\cdot 1 + \ln\frac{1}{\sqrt{2}} - \frac{1}{2}\ln\frac{1}{2}$	$\sqrt{\text{"c"}}$	A1	
	(i.e. $\ln\left\lvert\frac{y}{1+y}\right\rvert = -\frac{\pi}{6}$)			
	$\frac{y+1}{y} = e^{\frac{\pi}{6}}$ (o.e.)	Out of ln	M1	
	$1 = y(e^{\frac{\pi}{6}} - 1)$			
	$\therefore y = \underline{\frac{1}{e^{\frac{\pi}{6}} - 1}}$ (o.e.)	A correct y = expression	A1	(6)
				(15)

G.B. Attwood. 29/10/04.

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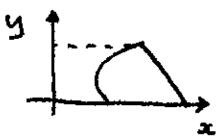
Subject: **Pure Mathematics** **Paper: P3**

Question Number	Scheme	Marks
	ALTERNATIVE SOLUTIONS	
4 (a)	$f(x) = \dfrac{1-\sqrt{1-x^2}}{\sqrt{1-x}}$	M1
	$(1-x)^{-\frac{1}{2}} = 1 + \frac{1}{2}x + \dots$	A1
	$1-(1-x^2)^{\frac{1}{2}} = 1 - \left(1 - \frac{1}{2}x^2 \dots\right) = \frac{x^2}{2} \dots$ Use of binomial to attempt both	M1 A1
	$f(x) = \frac{x^2}{2}\left\{1 + \frac{x}{2} + \dots\right\}$ Multiply	M1
	$= \frac{x^2}{2} + \frac{x^3}{4}$	A1 (6)
6. (a).	Cartesian Equation: $(y-3)^2 = 9(x-1)$ or $y = 3 + 3\sqrt{x-1}$	B1
	$\frac{dy}{dx} = \frac{3}{2\sqrt{x-1}}$	M1
	[Rest as in scheme]	
(b)	$x = 2\ (y=0)$, $x = 1\ (y=3)$	B1
	(Two functions) ±	M1
	Curved Area $= \int (3+3\sqrt{x-1})\,dx - \int (3-3\sqrt{x-1})\,dx$	A1
	[Rest as in scheme]	
ALT	 Trapezium $- \int x\,dy$ Trapezium $= \frac{1}{2}\,9(5+11.75)$	M1
	Limits $\begin{smallmatrix}9\\0\end{smallmatrix}$	B1
	$\int x\,dy = \frac{1}{9}\int [(y-3)^2+1]\,dy$ Attempt $x = x(y)$	M1
	correct integral	A1
	[Rest as in scheme + M1 for $\int + \Delta$ is for Trap $-\int$]	

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6671 Pure P1
Mark Scheme

Question Number	Scheme	Marks
1	(a) $\frac{dy}{dx} = 6 + 8x^{-3}$ or equiv.	M1 A1
		(2)
	[M1 is for correct power of x in at least one term , 6 or x^{-3} is sufficient.]	
	(b) $\int y \, dx = \frac{6x^2}{2} + 4x^{-1} + C$ or equiv.	M1 A1A1
		(3)
	[A1: $\frac{6x^2}{2} + C$; A1: $+4x^{-1}$]	
		[5]
2	(a) $a = -4$ or $(x-4)^2$	B1
	$x^2 - 8x - 29 \equiv (x \pm 4)^2 - 16$ (-29), $b = -45$	M1A1
		(3)
	[Comparing coefficients: M1 is for $a^2 + b = -29$, and comparing x coefficient]	
	(b) Method to find x:	M1
	[$x + "a" = \sqrt{\ldots\ldots}$ *or* x using the quadratic formula	
	$x = 4 \pm 3\sqrt{5}$ or $c = 4$, $d = 3$	A1 A1
		(3)
		[6]

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Question Number	Scheme	Marks
3	(a) $r\theta = 45\theta = 63$, $\theta = 1.4$ (*)	M1A1 (2)
	(b) Area of sector $OAB = \frac{1}{2}r^2\theta = \frac{1}{2}45^2 \times 1.4$ (= 1417.5)	M1A1
	Complete method for area of triangle OCD	M1
	Correct numerical expression for area : e.g. $\frac{1}{2}30^2 \times \sin 1.4$ (= 443.45...)	A1
	Shaded area = 1417.5 – 443.45…= 974 m^2 cao	A1 (5) **[7]**
4	(a) Complete method for equation of line e.g. $y - (-4) = \frac{1}{3}(x-9)$	M1A1
	$x - 3y - 21 = 0$ or $3y - x + 21 = 0$	A1 (3)
	(b) Equation of l_2: $y = -2x$	B1
	Solve l_1 *and* l_2 simultaneously to find P:	M1
	$x = 3$, $y = -6$	A1A1√
	[Follow through on first co-ord substituted in $y = -2x$]	(4)
	(c) C: $(0, -7)$	B1√
	Complete method for area of triangle OCP	M1
	Area $= 10\frac{1}{2}$ (must be exact)	A1 (3) **[10]**

5	(a) $\arctan\frac{3}{2} = 56.3°\ (= \alpha)$ seen anywhere	B1
	$\alpha - 20°$, $(\alpha - 20°) \div 3$ (that order)	M1M1
	$\alpha + 180°\ (= 236.3°)$, $\alpha - 180°\ (= -123.7°)$ (Third quadrant)	M1
	$x = -47.9°,\ 12.1°,\ 72.1°$	A1A1
	[First A1 for two correct solutions, second A1 for third]	(6)
	(b) Equation in one trig. function, using correct identities	M1
	[e.g. $2\sin^2 x + (1 - \sin^2 x) = \frac{10}{9}$ or $2(1 - \cos^2 x) + \cos^2 x = \frac{10}{9}$]	
	$\sin^2 x = \frac{1}{9}$ or $\cos^2 x = \frac{8}{9}$ or $\tan^2 x = \frac{1}{8}$ or $\cos 2x = \frac{7}{9}$	A1
	$x = 19.5°,\ -19.5°$	A1A1√ (4) **[10]**
	Notes : Max. deduction of 1 overall for not correcting to 1 dec. place. Answers outside given interval, ignore Extra answers in range, max. deduction of 1 in each part (i.e. 4 or more answers within interval in (a), –1 from any gained A marks; 3 or more answers within interval in (b), –1 from any gained A marks	

6	(a) $S = a + (a+d) + (a+2d) + \ldots + [a+(n-1)d]$ or equiv. form	B1
	$S = [a+(n-1)d] + [a+(n-2)d] + \ldots + a$ or equiv.	M1
	Add: $2S = n[2a+(n-1)d] \Rightarrow S = \frac{1}{2}n[2a+(n-1)d]$ cso (*)	M1 A1 (4)
	[If using "l", second M not gained until "l = a + (n – 1)d" substituted.]	
	(b) 3, 8, 13	B1 (1)
	(c) $a = 3$ $d = 5$ $[a = 3, l = 5n - 2]$	B1√
	Sum $= \frac{1}{2}n[(2 \times 3) + 5(n-1)]$ *or* $\frac{1}{2}n[3 + 5n - 2] = \frac{1}{2}n(5n+1)$ (*)	M1 A1 (3)
	Alt: $5\sum r - \sum 2$ B1, $= \frac{5n(n+1)}{2} - 2n$ M1, $= \frac{n(5n+1)}{2}$ A1	
	(d) Finding $\sum_{1}^{200}$ e.g. $\sum_{r=1}^{200}(5r-2) = \frac{1}{2} \times 200 \times 1001$ (= 100100)	M1
	Sum of first 4 terms: $\sum_{r=1}^{4}(5r-2) = \frac{1}{2} \times 4 \times 21$ or 42 stated	B1
	$\sum_{r=5}^{200}(5r-2) = \text{S}(200) - \text{S}(4) = 100100 - 42 = 100058$	M1 A1 (4) **[12]**
	[Allow S(200) - S(5) for second M1]	
	ALT: Working with 23, 28, 33, …………	
	$a = 23$ B1; Finding "n" and d, or equiv. M1	
	Applying $S = \frac{1}{2}n[2a+(n-1)d]$, or equivalent, with 23, $n =$ 196 , $d = 5$ M1	
	Answer: 100058 A1	

7	(a) $\frac{dy}{dx} = 3x^{\frac{1}{2}} - 6$	M1 A1
	Setting = 0 and solving , $x = 4$ (*)	M1 A1 (4)
	(b) $\int\left(2x^{\frac{3}{2}} - 6x + 10\right) dx = \left[\frac{4x^{\frac{5}{2}}}{5} - 3x^2 + 10x\right]$	M1 A1 A1
	[$\frac{4x^{5/2}}{5}$ A1, $-3x^2 + 10x$ A1]	
	$\left[\frac{4x^{\frac{5}{2}}}{5} - 3x^2 + 10x\right]_1^4 = \left(\frac{4 \times 4^{\frac{5}{2}}}{5} - (3 \times 16) + 40\right) - \left(\frac{4}{5} - 3 + 10\right)$	M1 A1√
	(= 17.6 – 7.8 = 9.8)	
	[A1√ requires 1 and 4 substituted in candidate's 3-termed integrand (unsimplified)]	
	Correct method for finding area under line	M1
	Correct unsimplified form e.g $= \frac{1}{2}(6 + 2) \times 3$ (=12)	A1
	Area of R (=12 – 9.8) = 2.2	A1 (8) **[12]**
	Alt: Working with "line – curve"	
	Area = $\int \lvert (-\frac{8}{3} + \frac{14}{3}x - 2x^{\frac{3}{2}}) \rvert\, dx$ M1A1	
	$= \left[\frac{4x^{\frac{5}{2}}}{5}, \quad \frac{7}{3}x^2 - \frac{8}{3}x\right]$ A1 A1 f.t.	
	Use of correct limits, as in main scheme M1A1 f.t. 2.2 A1	

8	(a) Substitution of $x=3$ in f(x) $\quad f(3)=27-117+165-75$	M1
	$=0$, so $(x-3)$ is a factor of $f(x)$	A1 (2)
	(b) Finding quadratic factor: $(x-3)(x^2-10x+25)$	M1 A1
	$(x-3)(x-5)(x-5)$	A1 (3)
	[S.C.: Allow M1 if just a second linear factor found]	
	(c) 3 and 5	B1 (1)
	(d) $f'(x)=3x^2-26x+55$	M1 A1
	$f'(3) = 27 - 78 + 55 = 4$	A1 (3)
	(e) "$3x^2-26x+55$" = "4"	M1
	$3x^2-26x+51=0 \Rightarrow (3x-17)(x-3)=0$ or $x = \ldots$.if using "formula"	M1 A1√
	x-coordinate of S is $\frac{17}{3}$ $\left(\frac{34}{6} \text{ or } 5\frac{2}{3} \text{ or } 5.\dot{6} \text{ or } 5.67\right)$	A1 (4) **[13]**

June 2005
6672 Pure P2
Mark Scheme (Final)

Question Number	Scheme	Marks
1	(a) $\log 5^x = \log 8$ or $x = \log_5 8$	M1
	Complete method for finding x: $x = \dfrac{\log 8}{\log 5}$ or $\dfrac{\ln 8}{\ln 5}$	M1
	$= 1.29$ only	A1 (3)
	(b) Combining two logs: $\log_2 \dfrac{(x+1)}{x}$ or $\log_2 7x$	M1
	Forming equation in x (eliminating logs) legitimately	M1
	$x = \dfrac{1}{6}$ or $0.1\dot{6}$	A1 (3)
		[6]
2	(a) $1 + 12px$, $+ 66p^2x^2$ (accept any correct equivalent)	B1,B1 (2)
	(b) $12p = -q$, $66p^2 = 11q$ Forming 2 equations by comparing coefficients	M1
	Solving for p or q	M1
	$p = -2$, $q = 24$	A1A1 (4)
		[6]

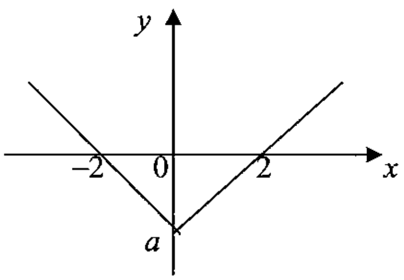
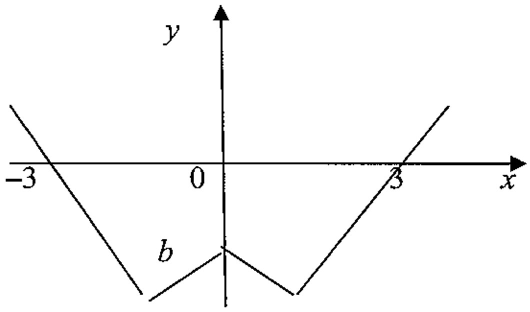
3	(a)	
	x: 0, 4, 8, 12, 16, 20; y: 0, **1.6(00)**, 2.771, **3.394**, **3.2(00)**, 0 — **1.6(00), 3.2(00)**	B1
	3.394	B1 (2)
	(b) A ≈ ½ x 4, x [(0 + 0) + 2{1.60 + 2.771 + 3.394 + 3.20}] follow through on candidate's y values	B1, [M1A1√]
	≈ 43.8(6), 43.9 or 44 m^2	A1 (4)
	(c) Vol/min ≈ [answer to (b) x 2] x 60 = 5260, 5270 or 5280 (m^3 per min)	M1 A1 (2)
		[8]
4	(a) $f(x) = \frac{5x+1}{(x+2)(x-1)} - \frac{3}{x+2}$ factors of quadratic denominator	B1
	$= \frac{5x+1-3(x-1)}{(x+2)(x-1)}$ common denominator	M1
	simplify to linear numerator	M1
	$= \frac{2x+4}{(x+2)(x-1)} = \frac{2(x+2)}{(x+2)(x-1)} = \frac{2}{x-1}$ AG	A1 (4) (cso)
	(b) $y = \frac{2}{x-1} \Rightarrow xy - y = 2 \Rightarrow$	M1
	$xy = 2 + y$ or $x - 1 = \frac{2}{y}$	A1
	$f^{-1}(x) = \frac{2+x}{x}$ or equiv.	A1 (3)
	(c) $fg(x) = \frac{2}{x^2+4}$ (attempt) $[\frac{2}{"g"-1}]$	M1
	Setting $\frac{2}{x^2+4} = \frac{1}{4}$ and finding $x^2 = \ldots$; $x = \pm 2$	DM1; A1 (3)
		[10]

Table in 3(a):

x	0	4	8	12	16	20
y	0	**1.6(00)**	2.771	**3.394**	**3.2(00)**	0

Question Number	Scheme	Marks
5	(a) $\left(\frac{x+1}{x}\right)^2 = 1 + \frac{2}{x} + \frac{1}{x^2}$ anywhere	B1
	$V = \pi \int \left(\frac{x+1}{x}\right)^2 dx$	M1
	$\int \left(\frac{x+1}{x}\right)^2 dx = x - \frac{1}{x}$, $+ 2\ln x$ [M1 attempt to $\int$]	M1A1,A1
	Using limits correctly in their integral: $(\pi)\{[x + 2\ln x - \frac{1}{x}]^3 - [x + 2\ln x - \frac{1}{x}]_1\}$	M1
	V = π [2⅔ + 2ln3] (must be exact)	A1 (7)
	(b) Volume of cone (or vol. generated by line) = $\frac{1}{3}\pi \times 2^2 \times 2$	B1
	$V_R = V_S -$ volume of cone $= V_S - \frac{1}{3}\pi \times 2^2 \times 2$	M1
	$= 2\pi \ln 3$ or $\pi \ln 9$	A1 (3)
		[10]

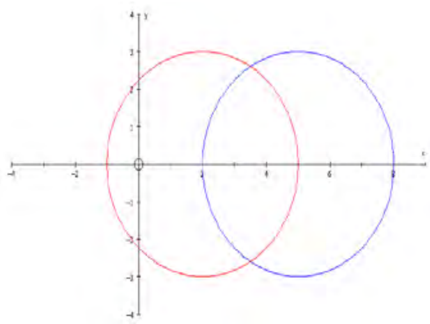
6	(a) $f'(x) = 3e^x - \frac{1}{2x}$	M1A1A1 (3)
	[M1: any evidence to suggest that tried to differentiate]	
	(b) $3e^\alpha - \frac{1}{2\alpha} = 0$ [Equating $f'(x)$ to zero]	M1
	$\Rightarrow 6\alpha e^\alpha = 1 \Rightarrow \alpha = \frac{1}{6}e^{-\alpha}$ AG	A1 (cso) (2)
	(c) $x_1 = 0.0613...$, $x_2 = 0.1568..$, $x_3 = 0.1425...$, $x_4 = 0.1445....$	M1A1 (2)
	[M1 at least x_1 correct, A1 all correct to 4 d.p.]	
	(d) Using $f'(x)$ $\{= 3e^x - \frac{1}{2x}\}$ with suitable interval	M1
	[e.g. f(0.14425) = – 0.0007, f(0.14435) = + 0.002(1)]	
	Both correct with concluding statement.	A1 (2)
		[9]

7	(a) 	
	Translation ← by 1	M1
	Intercepts correct	A1 (2)
	(b) 	
	$x \geq 0$, correct "shape"	B1
	[provided not just original]	
	Reflection in y-axis	B1√
	Intercepts correct	B1 (3)
	(c) $a = -2$, $b = -1$	B1B1(2)
	(d) Intersection of $y = 5x$ with $y = -x - 1$	M1A1
	Solving to give $x = -\frac{1}{6}$	M1A1 (4)
		[11]

8	(a) $2\sin(\theta+30)^\circ = \cos(\theta+60)^\circ$	
	$2\sin\theta^\circ\cos 30^\circ + 2\cos\theta^\circ\sin 30^\circ = \cos\theta^\circ\cos 60^\circ - \sin\theta^\circ\sin 60^\circ$	B1B1
	$\frac{2\sqrt{3}}{2}\sin\theta^\circ + \frac{2}{2}\cos\theta^\circ = \frac{1}{2}\cos\theta^\circ - \frac{\sqrt{3}}{2}\sin\theta^\circ$	M1
	Finding $\tan\theta^\circ$, $\tan\theta^\circ = -\frac{1}{3\sqrt{3}}$ or equiv. exact	M1,A1 (5)
	(b) (i) Setting $A = B$ to give $\cos 2A = \cos^2 A - \sin^2 A$	M1
	Correct completion: $= (1 - \sin^2 A) - \sin^2 A = 1 - 2\sin^2 A$	A1 (2)
	[Need to see intermediate step above for A1]	
	(ii) Forming quadratic in $\sin x$ $[2\sin^2 x + \sin x - 1 = 0]$	M1
	Solving $[(2\sin x - 1)(\sin x + 1) = 0$ or formula] $[\sin\theta = ½$ or $\sin\theta = -1]$	M1
	$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$; [A1√ for $\pi - "\alpha"$]	A1,A1√
	$\theta = \frac{3\pi}{2}$	A1 (5)
	(iii) LHS = $2\sin y\cos y\frac{\sin y}{\cos y} + (1 - 2\sin^2 y)$	
	[B1 use of $\tan y = \frac{\sin y}{\cos y}$, M1 forming expression in $\sin y$, $\cos y$ only]	B1M1
	Completion: $= 2\sin^2 y + (1 - 2\sin^2 y) = 1$ AG	
	[*Alternative*: LHS = $\frac{\sin 2y\sin y + \cos 2y\cos y}{\cos y}$ B1M1 $= \frac{\cos(2y - y)}{\cos y} = 1$ A1]	A1 (3) **[15]**

June 2005
6673 Pure P3
Mark Scheme

Question Number	Scheme	Marks
1. (a)	Finding f (± 2), and obtaining 16 – 32 + 10 + 6 = 0 **Or** uses division and obtains $2x^2 - kx...$, obtaining $2x^2 - 4x - 3$ and concluding remainder = 0	M1, A1 M1 A1
(b)	Finding f ($\pm\frac{1}{2}$), and obtaining $-\frac{1}{4} - 2 - \frac{5}{2} + 6 = 1\frac{1}{4}$ **Or** uses division and obtains $x^2 - kx...$, obtaining $x^2 - \frac{9}{2}x + \frac{19}{4}$ and concluding remainder = $\frac{5}{4}$	M1, A1
(c)	$x = 2$ (also allow $\frac{2 \pm \sqrt{10}}{2}$ or $\frac{4 \pm \sqrt{40}}{4}$)	B1 (5)
2. (a)	Writes down binomial expansion up to and including term in x^3, allow ${}^nC_r$ notation $1 + nax + n(n-1)\frac{a^2x^2}{2} + \frac{n(n-1)(n-2)}{6}a^3x^3$ (condone errors in powers of a)	M1
	States na =15	B1
	Puts $\frac{n(n-1)a^2}{2} = \frac{n(n-1)(n-2)a^3}{6}$ (condone errors in powers of a)	dM1
	$3 = (n - 2)a$	
	Solves simultaneous equations in n and a to obtain a = 6, and n = 2.5	M1 A1 A1 (6)
	[n.b. Just writes a = 6, and n = 2.5 following no working or following errors allow the last M1 A1 A1]	
(b)	Coefficient of $x^3 = 2.5 \times 1.5 \times 0.5 \times 6^3 \div 6 = 67.5$ (or equals coefficient of $x^2 = 2.5 \times 1.5 \times 6^2 \div 2 = 67.5$)	B1 (1) [7]

Question Number	Scheme	Marks
3. (a)	Attempt at integration by parts, i.e. $kx\sin 2x \pm \int k\sin 2x\,dx$, with k = 2 or ½	M1
	$= \frac{1}{2}x\sin 2x - \int \frac{1}{2}\sin 2x\,dx$	A1
	Integrates sin 2*x* correctly, to obtain $\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$ (penalise lack of constant of integration first time only)	M1, A1 (4)
(b)	Hence method : Uses cos 2*x* = $2\cos^2 x - 1$ to connect integrals	B1
	Obtains $\int x\cos^2 x\,dx = \frac{1}{2}\{\frac{x^2}{2} + \text{answer to part}(a)\} = \frac{x^2}{4} + \frac{x}{4}\sin 2x + \frac{1}{8}\cos 2x + k$	M1 A1 (3)
	Otherwise method	
	$\int x\cos^2 x\,dx = x(\frac{1}{4}\sin 2x + \frac{x}{2}) - \int \frac{1}{4}\sin 2x + \frac{x}{2}\,dx$ B1 for $(\frac{1}{4}\sin 2x + \frac{x}{2})$	B1, M1
	$= \frac{x^2}{4} + \frac{x}{4}\sin 2x + \frac{1}{8}\cos 2x + k$	A1 (3)
4 (a)	*r* = 3 (both circles)	B1
	Centres are at (2, 0) and (5, 0)	B1, B1 (3)
(b)	 1st circle correct quadrants centre on *x* axis	B1
	2nd circle correct quadrants centre on *x* axis	B1
	circles same size and passing through centres of other circle	B1 (3)
(c)	Finds circles meet at *x* = 3.5, by mid point of centres or by solving algebraically	M1
	Establishes $y = \pm\frac{3\sqrt{3}}{2}$, and thus distance is $3\sqrt{3}$.	M1, A1 (3)
	Or uses trig or Pythagoras with lengths 3, angles 60 degrees, or 120 degrees.	M1
	Complete and accurate method to find required distance	M1
	Establishes distance is $3\sqrt{3}$.	A1 (3)

Question Number	Scheme	Marks
5. (a)	Substitutes t = 4 to give V, = 1975.31 or 1975.30 or 1975 or 1980 (3 s.f)	M1 , A1 (2)
(b)	$\frac{dV}{dt} = -\ln 1.5 \times V$;= -800.92 or –800.9 or –801 M1 needs ln 1.5 term	M1 A1; A1 (3)
(c)	rate of decrease in value on 1st January 2005	B1 (1)
6, (a)	$\overrightarrow{AB} = \begin{pmatrix} c \\ d-5 \\ 10 \end{pmatrix} = k\begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$ or $11 + 5\lambda = 21, \Rightarrow \lambda = 2$, $\therefore c = 4$ $d = 7$	M1 , A1 A1 (3)
(b)	$\begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} \bullet \begin{pmatrix} 2\lambda \\ 5+\lambda \\ 11+5\lambda \end{pmatrix} = 0$	M1 A1
	$\therefore 4\lambda + 5 + \lambda + 55 + 25\lambda = 0$ $\therefore \lambda = -2$	M1 A1
	Substitutes to give the point P, $-4\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ (Accept (-4, 3, 1)	M1, A1 (6)
(c)	Finds the length of OA, or OB or OP or AB as $\sqrt{146}$ or $\sqrt{506}$ or $\sqrt{26}$ or $\sqrt{120}$ resp.	M1
	Uses area formula- either Area = $\frac{1}{2}\lvert\mathbf{AB}\rvert \times \lvert\mathbf{OP}\rvert$ or $= \frac{1}{2}\lvert\mathbf{OA}\rvert \times \lvert\mathbf{OB}\rvert \sin\angle AOB$ or = $\frac{1}{2}\lvert\mathbf{OA}\rvert \times \lvert A\mathbf{B}\rvert \sin\angle OAB$ or $= \frac{1}{2}\lvert AB\rvert \times \lvert\mathbf{OB}\rvert \sin\angle ABO$	M1
	= $\frac{1}{2}\sqrt{120}\sqrt{26}$ or $\frac{1}{2}\sqrt{146}\sqrt{506}\sin 11.86$ or $\frac{1}{2}\sqrt{146}\sqrt{120}\sin 155.04$ or $\frac{1}{2}\sqrt{120}\sqrt{506}\sin 13.10$	M1
	= 27.9	A1 (4)

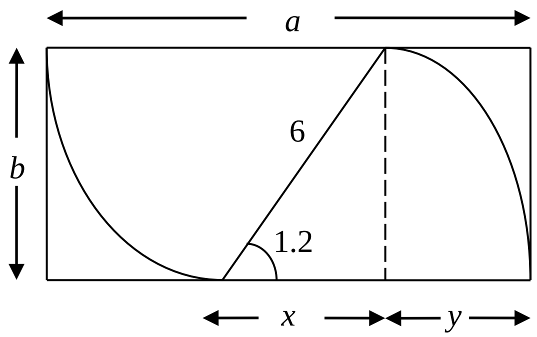
Question Number	Scheme	Marks
7 (a)	As $V = \frac{4}{3}\pi r^3$, then $\frac{\mathrm{d}V}{\mathrm{d}r} = 4\pi r^2$	M1
	Using chain rule $\frac{dr}{dt} = \frac{dV}{dt} \div \frac{dV}{dr}; = \frac{k}{\frac{4}{3}\pi r^3} \times \frac{1}{4\pi r^2}$	M1 A1
	$= \frac{B}{r^5}$ *	A1 (4)
(b)	$\int r^5 dr = \int B dt$	B1
	$\therefore \frac{r^6}{6} = Bt + c$ (allow mark at this stage, does not need r =)	M1 A1 (3)
(c)	Use r =5 at t =0 to give c= $\frac{5^6}{6}$ or 2604 or 2600	M1
	Use r = 6 at t = 2 to give B = $\frac{6^5}{2} - \frac{5^6}{12}$ or 2586 or 2588 or 2590	M1
	Put t = 4 to obtain r^6 (approx 78000)	M1 A1
	Then take sixth root to obtain r = 6.53 (cm)	A1 (5)

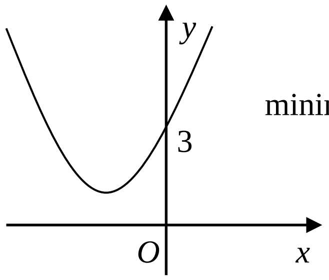
Question Number	Scheme	Marks
8. (a)	$\frac{dx}{dt} = -\frac{1}{(1+t)^2}$ and $\frac{dy}{dt} = \frac{1}{(1-t)^2}$	B1, B1
	$\therefore \frac{dy}{dx} = \frac{-(1+t)^2}{(1-t)^2}$ and at t = ½, gradient is –9 M1 requires their dy/dt / their dx/dt and substitution of t.	M1 A1cao
	At the point of contact $x = \frac{2}{3}$ and y =2	B1
	Equation is $y - 2 = -9\,(x - \frac{2}{3})$	M1 A1 (7)
(b)	**Either** obtain t in terms of x and y i,e, $t = \frac{1}{x} - 1$ *or* $t = 1 - \frac{1}{y}$ (or both)	M1
	Then substitute into other expression y = f(x) or x = g(y) and rearrange	M1
	(or put $\frac{1}{x} - 1 = 1 - \frac{1}{y}$ and rearrange)	
	To obtain $y = \frac{x}{2x-1}$ *	A1 (3)
	Or Substitute into $\frac{x}{2x-1} = \frac{\frac{1}{(1+t)}}{\frac{2}{1+t} - 1}$	M1
	$= \frac{1}{2-(1+t)} = \frac{1}{1-t}$	A1
	$= y$ *	M1 (3)
(c)	Area = $\int_{\frac{2}{3}}^{1} \frac{x}{2x-1}\,dx$	B1
	$= \int \frac{u+1}{2u}\frac{du}{2} = \frac{1}{4}\int 1 + \frac{1}{u}\,du$ putting into a form to integrate	M1
	$= \left[\frac{1}{4}u + \frac{1}{4}\ln u\right]_{\frac{1}{3}}^{1}$	M1 A1
	$= \frac{1}{4} - (\frac{1}{12} + \frac{1}{4}\ln\frac{1}{3})$	M1
	$= \frac{1}{6} + \frac{1}{4}\ln 3$ or any correct equivalent.	A1 (6)

Question Number	Scheme	Marks
8. (c)	Or Area = $\int_{\frac{2}{3}}^{1} \frac{x}{2x-1} dx$	B1
	$= \int \frac{1}{2} + \frac{\frac{1}{2}}{2x-1} dx$ putting into a form to integrate	M1
	$= \left[\frac{1}{2}x + \frac{1}{4}\ln(2x-1)\right]_{\frac{2}{3}}^{1}$	M1A1
	$= \frac{1}{2} - \frac{1}{3} - \frac{1}{4}\ln\frac{1}{3} = \frac{1}{6} - \frac{1}{4}\ln\frac{1}{3}$	dM1 A1 (6)
	Or Area = $\int \frac{1}{1-t} \frac{-1}{(1+t)^2} dt$	B1
	$= \int \frac{A}{(1-t)} + \frac{B}{(1+t)} + \frac{C}{(1+t)^2} dt$ putting into a form to integrate	M1
	$= \left[\frac{1}{4}\ln(1-t) - \frac{1}{4}\ln(1+t) + \frac{1}{2}(1+t)^{-1}\right]_{-}$	M1 A1ft
	= Using limits 0 and ½ and subtracting (either way round)	dM1
	$= \frac{1}{6} + \frac{1}{4}\ln 3$ or any correct equivalent.	A1 (6)
	Or Area = $\int_{\frac{2}{3}}^{1} \frac{x}{2x-1} dx$ then use parts	B1
	$= \frac{1}{2}x\ln(2x-1) - \int_{\frac{2}{3}}^{1} \frac{1}{2}\ln(2x-1) dx$	M1
	$= \frac{1}{2}x\ln(2x-1) - [\frac{1}{4}(2x-1)\ln(2x-1) - \frac{1}{2}x]$	M1A1
	$= \frac{1}{2} - (\frac{1}{3}\ln\frac{1}{3} - \frac{1}{12}\ln\frac{1}{3} + \frac{1}{3})$	DM1
	$= \frac{1}{6} - \frac{1}{4}\ln\frac{1}{3}$	A1

Question Number	Scheme	Marks
1.	(a) $(y=)5-2\times3=-1$ ∗ cso	B1 **(1)**
	(b) Gradient of perpendicular line is $\frac{1}{2}$	B1
	$y-(-1)=\frac{1}{2}(x-3)$ ft their $m \neq -2$	M1 A1ft
	(or substituting (3, −1) into $y=(\text{their } m)x+c$)	
	$x-2y-5=0$	A1 **(4)**
		Total 5 marks
2.	(a) $\frac{\mathrm{d}y}{\mathrm{d}x}=4x+18x^{-4}$ $x^n \mapsto x^{n-1}$	M1 A1 **(2)**
	(b) $\int(2x^2-6x^{-3})\mathrm{d}x=\frac{2}{3}x^3+3x^{-2}$ $x^n \mapsto x^{n+1}$	M1 A1
	$\left[\ldots\right]_1^3=\frac{2}{3}\times3^3+\frac{3}{9}-\left(\frac{2}{3}+3\right)$	M1
	$=14\frac{2}{3}$ $\frac{44}{3},\frac{132}{9}$ or equivalent	A1 **(4)**
		Total 6 marks
3.	(a) $\tan\theta=\frac{3}{2}$ Use of $\tan\theta=\frac{\sin\theta}{\cos\theta}$	M1
	$\theta=56.3°$ cao	A1
	$=236.3°$ ft 180° + their principle value	A1 ft **(3)**
	Maximum of one mark is lost if answers not to 1 decimal place	
	(b) $2-\cos\theta=2(1-\cos^2\theta)$ Use of $\sin^2\theta+\cos^2\theta=1$	M1
	$2\cos^2\theta-\cos\theta=0$	A1
	Allow this A1 if both $\cos\theta=0$ and $\cos\theta=\frac{1}{2}$ are given	
	$\cos\theta=0 \Rightarrow \theta=90°,270°$ M1 one solution	M1 A1
	$\cos\theta=\frac{1}{2} \Rightarrow \theta=60°,300°$ M1 one solution	M1 A1 **(6)**
		Total 9 marks

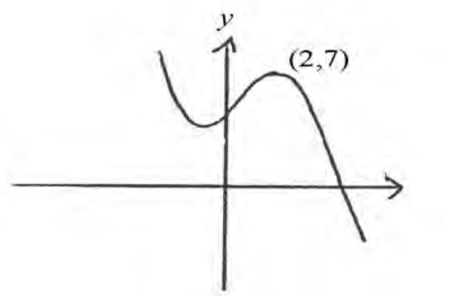
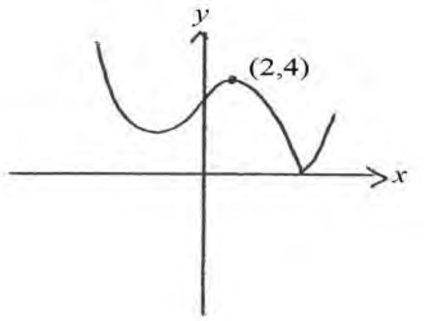
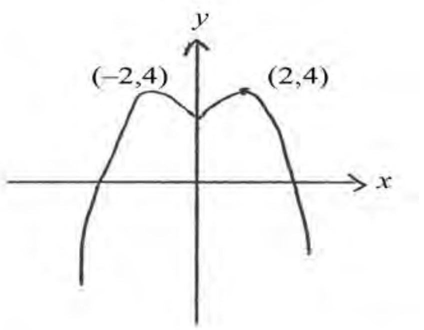
Question Number	Scheme	Marks
4.	(a) $y=0 \Rightarrow x^{\frac{1}{2}}(3-x)=0 \Rightarrow x=3$ ∗ or $3\sqrt{3}-3^{\frac{3}{2}}=3\sqrt{3}-3\sqrt{3}=0$	B1 **(1)**
	(b) $\dfrac{dy}{dx}=\dfrac{3}{2}x^{-\frac{1}{2}}-\dfrac{3}{2}x^{\frac{1}{2}}$ $\quad x^n \mapsto x^{n-1}$	M1 A1
	$\dfrac{dy}{dx}=0 \Rightarrow x^{\frac{1}{2}}=x^{-\frac{1}{2}}$ $\quad$ Use of $\dfrac{dy}{dx}=0$	M1
	$\Rightarrow x=1$	A1
	A: (1, 2)	A1 **(5)**
	(c) $\int\left(3x^{\frac{1}{2}}-x^{\frac{3}{2}}\right)dx=2x^{\frac{3}{2}}-\dfrac{2}{5}x^{\frac{5}{2}}$ $\quad$ M1 $x^n \mapsto x^{n+1}$ Accept unsimplified expressions for As	M1 A1+A1
	Area = $[...]_0^3=2\times3\sqrt{3}-\dfrac{2}{5}\times9\sqrt{3}$ $\quad$ Use of correct limits	M1
	Area is $\dfrac{12}{5}\sqrt{3}$ $\quad$ (units^2)	A1 **(5)**
	For final A1, terms must be collected together but accept exact equivalents, e.g. $\dfrac{4}{5}\sqrt{27}$	
		Total 11 marks

Question Number	Scheme		Marks
5.	(a) Arc is 6×1.2	Use of $r\theta$	M1
	Perimeter is $6\times1.2+6+6=19.2 \quad (\text{cm})$		A1 **(2)**
	(b) Area is $\frac{1}{2}\times6^2\times1.2=21.6 \quad (\text{cm}^2)$	Use of $\frac{1}{2}r^2\theta$	M1 A1 **(2)**
	(c) 		
	$b=6\sin1.2\approx5.59 \quad (\text{cm})$		B1
	$x=6\cos1.2 \quad (\approx2.174...)$		M1
	$y=6-x \quad (\approx3.825...)$		
	$a=6+y\approx9.83 \quad (\text{cm})$		M1 A1 **(4)**
			Total 8 marks

Question Number	Scheme	Marks
6.	(a) $x^2+2x+3=(x+1)^2+2$	
	$a=1, b=2$	B1, B1 **(2)**
	(b) 	
	$\cup$ shape anywhere	M1
	minimum ft their *a* and positive *b*	A1ft
	(0, 3) marked	B1 **(3)**
	(c) $\Delta=b^2-4ac=2^2-4\times 3=-8$	B1
	The negative sign implies there are no real roots and, hence, the curve in (b) does not intersect (meet, cut, …) the *x*-axis. Accept equivalent statements and the statement that the whole curve is above the *x*-axis.	B1 **(2)**
	(d) $\Delta=k^2-12$	M1
	$\Delta<0 \quad \Rightarrow \quad k^2-12<0 \quad \left(\text{or } k^2<12\right)$	A1
	$-2\sqrt{3}<k<2\sqrt{3}$ Allow $\sqrt{12}$	M1 A1 **(4)**
	If just $k<2\sqrt{3}$ allow M1 A0	
		Total 11 marks
	Alternative to (d)	
	$\frac{dy}{dx}=0 \quad \Rightarrow \quad 2x+k=0 \quad \Rightarrow \quad x=-\frac{k}{2}$	
	Minimum greater than 0 implies $\frac{k^2}{4}-\frac{k^2}{2}+3>0$	M1
	$k^2<12$	A1
	Then as before.	

Question Number	Scheme	Marks
7.	(a) x-coordinate of P is -2, x-coordinate of Q is 2.	B1, B1 **(2)**
	(b) $y = x^3 - x^2 - 4x + 4$ Multiplying out	M1
	$\frac{dy}{dx} = 3x^2 - 2x - 4$ ∗ cso	M1 A1 **(3)**
	Alternatively	
	Using product rule $\frac{dy}{dx} = 1(x^2 - 4) + (x-1)2x$	M1
	$= 3x^2 - 2x - 4$ ∗	M1 A1
	(c) $x = -1 \Rightarrow m = 3 + 2 - 4 = 1$ Substituting $x = -1$ into (b)	M1
	$y - 6 = 1(x - (-1)) \Rightarrow y = x + 7$ ∗ cso	A1 **(2)**
	(d) $x^3 - x^2 - 4x + 4 = x + 7$ line = curve	M1
	$x^3 - x^2 - 5x - 3 = 0$	
	$(x+1)(x^2 - 2x - 3) = 0$ Obtaining linear×quadratic	M1
	$(x+1)(x+1)(x-3) = 0$ Obtaining 3 linear factors	M1
	$R : (3, 10)$	A1, A1 **(5)**
		Total 12 marks
	In (d) if the correct cubic is obtained the factors can just be written down by inspection. Parts (c) and (d) can be done together. On obtaining $(x+1)^2(x-3)$, the repeated root shows that $y = x + 7$ is a tangent to the curve at $(-1, 6)$ and, if this is stated, the M1 A1 for (c) should be given at this point.	

Question Number	Scheme	Marks
8.	(a) $a+(a+d) = £\ (500 + 500 + 200) = £1200$ * cso	B1 **(1)**
	(b) $a = 500, d = 200$; $u_8 = a+(8-1)d$	M1
	$= £(500+7\times 200) = £1900$	A1 **(2)**
	(c) $S_8 = \frac{8}{2}(2\times 500+(8-1)\times 200)$	M1 A1
	$= £\ 9600$	A1 **(3)**
	(d) $\frac{n}{2}(1000+(n-1)200) = 32\,000$	M1 A1
	$n^2+4n-320=0$ M1 reducing to a 3 term quadratic A1 any multiple of the above	M1 A1
	$(n+20)(n-16)=0$	M1
	$n=16$	A1
	Age is 26	A1 **(7)**
		Total 13 marks
	In (b) if the sum is found by repeated addition, i.e. $u_1 = £500, u_2 = £700, u_3 = £900, u_4 = £1100, u_5 = £1300,$ $u_6 = £1500, u_7 = £1700, u_8 = £1900,$ allow M1 A1 at completion.	

Question Number	Scheme	Marks
1.		
(a)	$(1+6x)^4 = 1+4(6x)+6(6x)^2+4(6x)^3+(6x)^4$ $= 1+24x+216x^2+864x^3+1296x^4$	M1 A1 A1 (3)
(b)	substitute x=100 to obtain 601^4 = 1 + 2400 + 2160000 + 864000000 + 129600000000 = 130,466,162,401	M1o.e. A1 (2)
2. (a)		B1 Shape B1 Point (2)
(b)		B1 Shape B1 Point (2)
(c)		B1 Shape >0 B1 Shape x<0 B1 Point (-2,4) (3)

3.	$\dfrac{x(2x+3)}{(2x+3)(x-2)} - \dfrac{6}{(x-2)(x+1)}$	B1, B1
	$=\dfrac{x(x+1)-6}{(x-2)(x+1)}$	M1 A1ft
	$=\dfrac{(x+3)(x-2)}{(x-2)(x+1)}$	M1 A1
	$=\dfrac{x+3}{x+1}$	A1 (7)
	Alternative 1	
	$\dfrac{2x^2+3x}{(2x+3)(x-2)} - \dfrac{6}{(x-2)(x+1)}$	B1
	$=\dfrac{(2x^2+3x)(x+1)-6(2x+3)}{(2x+3)(x-2)(x+1)}$	M1 A1ft
	$=\dfrac{(2x^3+5x^2-9x-18)}{(2x+3)(x-2)(x+1)}$	A1
	$=\dfrac{(x-2)(2x^2+9x+9)}{(2x+3)(x-2)(x+1)}$	M1
	$=\dfrac{(x-2)(2x+3)(x+3)}{(2x+3)(x-2)(x+1)}, \ =\dfrac{x+3}{x+1}$	A1, A1
	Alternative 2:	
	$\dfrac{2x^2+3x}{(2x+3)(x-2)} - \dfrac{6}{x^2-x-2}$	
	$=\dfrac{x(2x+3)}{(2x+3)(x-2)} - \dfrac{6}{x^2-x-2}$	B1
	$=\dfrac{x(x^2-x-2)-6(x-2)}{(x-2)(x^2-x-2)}, = \dfrac{x^3-x^2-2x-6x+12}{(x-2)(x^2-x-2)}$	M1A1ft
	$=\dfrac{x^3-x^2-8x+12}{(x-2)(x^2-x-2)}$	A1
	$=\dfrac{(x-2)(x^2+x-6)}{(x-2)(x^2-x-2)}$	M1
	$=\dfrac{(x+3)(x-2)}{(x-2)(x+1)}, \ =\dfrac{x+3}{x+1}$	A1,A1

Question Number	Scheme	Marks
4.	$\frac{dy}{dx}=\frac{1}{x}$	M1 A1
	At x=3, gradient of normal $=\frac{-1}{\frac{1}{3}}=-3$	M1
	$y-\ln 1=-3(x-3)$	M1
	$y=-3x+9$	A1 (5)
5. (a)	$x(2x^2-1)=4$	M1
	$2x^2-1=\frac{4}{x}$	
	$2x^2=\frac{4+x}{x}$	M1
	$x^2=\frac{4+x}{2x}$	
	$x=\sqrt{\frac{2}{x}+\frac{1}{2}}$ **AG**	A1 (3)
	Alternative 1:	
	$2x^2-1-\frac{4}{x}=0$	M1
	$2x^2=1+\frac{4}{x}$	
	$x^2=\frac{1}{2}+\frac{4}{2x}$	M1
	$x\sqrt{\frac{1}{2}+\frac{2}{x}}$ **AG**	A1
	Alternative 2:	
	$x^2=\frac{2}{x}+\frac{1}{2}$	M1
	$2x^3=4+x$	M1
	$2x^2-x-4=0$	A1

(b)	1.41, 1.39, 1.39 (1.40765, 1.38593, 1.393941)	B1,B1,B1 (3)
(c)	$f\left(1.3915\right)=-3\times10^{-3}$ $f\left(1.3925\right)=7\times10^{-3}$ change in sign means root between 1.3915 & 1.3925 $\therefore 1.392$ to 3dp	M1 A1 B1 (3)
6. (a)	$-2x+4=\dfrac{3}{2x}$ $4x^2-8x+3=0$ $\left(2x-3\right)\left(2x-1\right)=0$ $x=0.5,1.5$	M1 A1 M1 A1 (4)
(b)	$\int_{0.5}^{1.5}-2x+4dx=\left[-x^2+4x\right]_{0.5}^{1.5}$ or $\dfrac{1}{2}\times\left(3+1\right)\times1$ $=2$ $\int_{0.5}^{1.5}\dfrac{3}{2x}\,dx=\left[\dfrac{3}{2}\ln x\right]_{0.5}^{1.5}$ $=\dfrac{3}{2}\ln 3$ $\therefore$ Area $=2-\dfrac{3}{2}\ln 3$	M1 A1 M1 A1 A1ft A1 (6)
	Alternative solution: Area $=\int_{0.5}^{1.5}-2x+4-\dfrac{3}{2x}dx$ $=\left[-x^2+4x-\dfrac{3}{2}\ln x\right]_{0.5}^{1.5}$ $=\dfrac{-9}{4}+\dfrac{1}{4}+6-2-\dfrac{3}{2}\ln 3$ o.e. $=2-\dfrac{3}{2}\ln 3$	 M1 M1A1A1 A1ft A1

7. (a)

0	1	2	3	4	5
0	0.062	0.271	0.716	1.612	3.482

B1, B1, B1 (3)

(b)

$$1\times\frac{1}{2}\left(0+3.482+2\right)\times\left(0.062+0.271+0.716+1.612\right)$$

$$=4.402\,\text{m}^2$$

B1,M1,A1ft

A1 (4)

(c)

$$6\times 4.402=26.4\,\text{m}^3$$

B1 ft (1)

(d) trapezium rule overestimates $\therefore$ will be enough

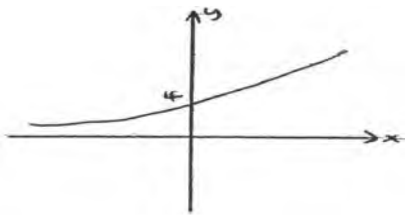
B1B1 (2)

8. (a)

$$gf(x)=e^{2(2x+\ln 2)}$$
$$=e^{4x}e^{2\ln 2}$$
$$=e^{4x}e^{\ln 4}$$
$$=4e^{4x}$$ **AG**

M1
M1
M1
A1 (4)

(b)



B1 shape & (0,4) (1)

(c) $gf\left(x\right)>0$

B1 (1)

(d)

$$\frac{d}{dx}gf\left(x\right)=16e^{4x}$$
$$e^{4x}=\frac{3}{16}$$
$$4x=\ln\frac{3}{16}$$
$$x=-0.418$$

M1
M1 attempt to solve
A1

A1 (4)

9. (a) (i)	$\dfrac{\cos 2x}{\cos x+\sin x} = \dfrac{\cos^2 x-\sin^2 x}{\cos x+\sin x}$	M1
	$= \dfrac{(\cos x+\sin x)(\cos x-\sin x)}{\cos x+\sin x}$	
	$= \cos x-\sin x$ **AG**	A1 (2)
(ii)	$\dfrac{1}{2}(\cos 2x-\sin 2x) = \dfrac{1}{2}(2\cos^2 x-1-2\sin x\cos x)$	M1, M1
	$= \cos^2 x-\dfrac{1}{2}-\sin x\cos x$ **AG**	A1 (3)
(b)	$\cos\theta\left(\dfrac{\cos 2\theta}{\cos\theta+\sin\theta}\right) = \dfrac{1}{2}$	
	$\cos\theta(\cos\theta-\sin\theta) = \dfrac{1}{2}$	M1
	$\cos^2\theta-\cos\theta\sin\theta = \dfrac{1}{2}$	
	$\dfrac{1}{2}(\cos 2\theta+1)-\dfrac{1}{2}\sin 2\theta = \dfrac{1}{2}$	M1
	$\dfrac{1}{2}(\cos 2\theta-\sin 2\theta) = 0$	
	$\sin 2\theta = \cos 2\theta$ **AG**	A1 (3)
(c)	$\sin 2\theta = \cos 2\theta$	
	$\tan 2\theta = 1$	M1
	$2\theta = \dfrac{\pi}{4}, \dfrac{5\pi}{4}, \dfrac{9\pi}{4}, \dfrac{13\pi}{4}$	A1 for 1
	$\theta = \dfrac{\pi}{8}, \dfrac{5\pi}{8}, \dfrac{9\pi}{8}, \dfrac{13\pi}{8}$	M1(4 solns) A1 (4)

Question Number	Scheme	Marks
1 (a)	$(1+\left(-\frac{1}{2}\right)(-2x)+\frac{(-\frac{1}{2})(-\frac{3}{2})}{1.2}(-2x)^2+\frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{1.2.3}(-2x)^3+\ldots.)$	M1 (corr bin coeffs) M1 (powers of $-2x$)
	$=1+x,+\frac{3}{2}x^2+\frac{5}{2}x^3+\ldots$	A1, A1 (4)
Alternative	May use McLaurin f(0)=1 and $f'(0)=1$ to obtain 1st two terms 1 + x Differentiates two further times and uses formula with correct factorials to give $\frac{3}{2}x^2+\frac{5}{2}x^3$	M1 A1 M1 A1 (4)
(b)	$(100-200x)^{-\frac{1}{2}}=100^{-\frac{1}{2}}(1-2x)^{-\frac{1}{2}}$. So series is $\frac{1}{10}$(previous series)	M1A1 ft (2)
2	Uses f(2) = 0 to give 16 – 4 + 2a + b = 0	M1 A1
	Uses f(-1) = 6 to give -2 – 1 +-a + b = 6	M1 A1
	Solves simultaneous equations to give a = -7, and b = 2	M1 A1 A1 (7)
3 (a)	Uses circle equation $(x-4)^2+(y-3)^2=(\sqrt{5})^2$	M1 A1
	Multiplies out to give $x^2-8x+16+y^2-6y+9=5$ and thus $x^2+y^2-8x-6y+20=0$ **(*)**	A1 (3)
Alternative	Or states equation of circle is $x^2+y^2+2gx+2fy+c=0$ has centre (-g, -f) and so g = - 4 and f = - 3 Uses $g^2+f^2-c=r^2$ to give $c=3^2+4^2-\sqrt{5}^2$, i.e. c = 20 $x^2+y^2-8x-6y+20=0$	M1 A1 A1 (3)
(b)	y = 2x meets the circle when $x^2+(2x)^2-8x-6(2x)+20=0$ $5x^2-20x+20=0$ Solves and substitutes to obtain x = 2 and y = 4. Coordinates are (2, 4)	M1 A1 M1 A1 (4)
	Or Implicit differentiation attempt , $2x+2y\frac{dy}{dx}-8-6\frac{dy}{dx}=0$ Uses y = 2x and $\frac{dy}{dx}$=2 to give 10x – 20 = 0. Thus x =2 and y =4	M1 A1 M1 A1 (4)

Question Number	Scheme	Marks
4.(a)	$\mathrm{f}'(x) = (x^2+1)\times\frac{1}{x} + \ln x \times 2x$	M1 A1
	$\mathrm{f}'(\mathrm{e}) = (\mathrm{e}^2+1)\times\frac{1}{\mathrm{e}} + 2\mathrm{e} = 3\mathrm{e} + \frac{1}{\mathrm{e}}$	M1 A1
		(4)
(b)	$(\frac{x^3}{3}+x)\ln x - \int(\frac{x^3}{3}+x)\frac{1}{x}dx$	M1 A1
	$= (\frac{x^3}{3}+x)\ln x - \int(\frac{x^2}{3}+1)dx$	
	$= \left[(\frac{x^3}{3}+x)\ln x - (\frac{x^3}{9}+x)\right]_1^e$	A1
	$= \frac{2}{9}e^3 + \frac{10}{9}$	M1 A1
		(5)
5. (a)	$\frac{9+4x^2}{9-4x^2} = -1 + \frac{18}{(3+2x)(3-2x)}$, so A = - 1	B1
	Uses 18 = B (3– 2x) + C (3+2x) and attempts to find B and C	M1
	B = 3 and C = 3	A1 A1
		(4)
	Or	
	Uses $9+4x^2$ = $A(9-4x^2)$+ B (3– 2x) + C (3+2x) and attempts to find A, B and C	M1
	A = -1, B = 3 and C = 3	A1, A1, A1
		(4)
(b)	Obtains $Ax + \frac{B}{2}\ln(3+2x) - \frac{C}{2}\ln(3-2x)$	M1 A1
	Substitutes limits and subtracts to give $2A + \frac{B}{2}\ln(5) - \frac{C}{2}\ln(\frac{1}{5})$	M1 A1ft
	= -2 +3ln5 or –2 +ln125	A1
		(5)

Question Number	Scheme	Marks
6 (a)	$\frac{dC}{dt} = -kC$; rate of decrease/negative sign; k constant of proportionality/positive constant	B1 (1)
(b)	$\int \frac{\mathrm{d}C}{C} = -k\int \mathrm{d}t$ $\therefore \ln C = -kt + \ln A$ $\therefore C = Ae^{-kt}$	M1 M1 A1 (3)
(c)	At $t = 0$ $C = C_0$, $\therefore A = C_0$ and at $t = 4$ $C = \frac{1}{10}C_0$, $\therefore \frac{1}{10}C_0 = C_0 e^{-4k}$, $\therefore \frac{1}{10} = e^{-4k}$ and $\therefore -4k = \ln\frac{1}{10}$, $\therefore k = \frac{1}{4}\ln 10$	B1 M1 M1, A1 (4)
7 (a)	Solves $9 + 2\lambda = 1$ *or* $7 + 2\lambda = -1$ to give $\lambda = -4$ so $p = 3$ Solves $9 + 2\lambda = 7$ *or* $7 + \lambda = 6$ to give $\lambda = -1$ so $q = 5$	M1 A1 M1 A1 (4)
(b)	$\lvert 6\mathrm{i} + 6\,\mathrm{j} + 3\mathrm{k}\rvert = 9$ so unit vector is $\frac{1}{9}(6\mathbf{i} + 6\mathbf{j} + 3\mathbf{k})$	M1 A1 (2)
(c)	$\cos\theta = \frac{2\times2 + 2\times1 + 1\times2}{3\times3}$ $\therefore \cos\theta = \frac{8}{9}$	M1 A1 A1 (3)
(d)	Write down two of $9 + 2\lambda = 3 + 2\mu$, $7 + 2\lambda = 2 + \mu$ or $7 + \lambda = 3 - 2\mu$ Solve to obtain $\mu = 1$ or $\lambda = -2$ Obtain coordinates (5, 3, 5)	B1 B1 M1 A1 A1 (5)

Question Number	Scheme	Marks
8(a)	$\frac{dx}{dt} = -3a\sin 3t$, $\frac{dy}{dt} = a\cos t$ therefore $\frac{dy}{dx} = \frac{\cos t}{-3\sin 3t}$	M1 A1
	When $x = 0, t = \frac{\pi}{6}$	B1
	Gradient is $-\frac{\sqrt{3}}{6}$	M1
	Line equation is $(y - \tfrac{1}{2}a) = -\frac{\sqrt{3}}{6}(x - 0)$	M1 A1
		(6)
(b)	Area beneath curve is $\int a\sin t(-3a\sin 3t)dt$	M1
	$= -\frac{3a^2}{2}\int(\cos 2t - \cos 4t)dt$	M1
	$-\frac{3a^2}{2}\left[\tfrac{1}{2}\sin 2t - \tfrac{1}{4}\sin 4t\right]$	M1 A1
	Uses limits 0 and $\frac{\pi}{6}$ to give $\frac{3\sqrt{3}a^2}{16}$	M1 A1
	Area of triangle beneath tangent is $\frac{1}{2}\times\frac{a}{2}\times\sqrt{3}a = \frac{\sqrt{3}a^2}{4}$	M1 A1
	Thus required area is $\frac{\sqrt{3}a^2}{4} - \frac{3\sqrt{3}a^2}{16} = \frac{\sqrt{3}a^2}{16}$	A1
		(9)
N.B.	The integration of the product of two sines is worth 3 marks (lines 2 and 3 of scheme to part (b)) If they use parts	
	$\int \sin t \sin 3t dt = -\cos t \sin 3t + \int 3\cos 3t \cos t dt$	M1
	$= -\cos t \sin 3t + 3\cos 3t \sin t + \int 9\sin 3t \sin t dt$	
	$8I = \cos t \sin 3t - 3\cos 3t \sin t$	M1 A1